Disasters and Incumbent Electoral Fortunes: No Implications for Democratic Competence*

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Abstract

A recent empirical literature shows that incumbent electoral fortunes suffer following negative events incumbents cannot control. These results have been taken to show that voters are insufficiently rational to competently hold elected officials accountable. Without disputing the empirical findings, we dispute these interpretations. Our argument has two steps. First, we present a formal model that shows irrational voters can actually enhance, rather than diminish, accountability. Rational voters must use forward-looking optimal reelection rules, and, quite generally, such rules are not accountability-maximizing reelection rules. Second, we show voter irrationality is not implied by the observation that incumbents suffer electorally following negative events outside of their control. In our model, voters are fully rational and incumbent electoral fortunes suffer, on average, following disasters because disasters increase voter information, which harms advantaged incumbents. Taken together, our models show that the empirical evidence entails neither of the conclusions drawn by the existing literature.

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Electoral accountability—the use of elections to create incentives for politicians to take costly actions that benefit voters—is a cornerstone of the theory of democracy (Key, 1966; Pitkin, 1967; Fiorina, 1981; Manin, Przeworski and Stokes, 1999). Disturbingly, a recent empirical literature calls into question the idea that voters are sufficiently rational to competently hold elected officials to account (Achen and Bartels, 2004; Wolfers, 2009; Leigh, 2009; Healy, Malhotra and Mo, 2010).[1] These papers show that the electoral fortunes of incumbent politicians seem to suffer following negative events—natural disasters, world economic downturns, and even sporting losses—that incumbents cannot control.

The standard interpretation of these findings is two-fold.

First, the fact that, on average, incumbents suffer electorally following events outside their control is taken to indicate that voters must be irrational. For instance, Achen and Bartels (2004, pp.7–8) write:

> [I]ncumbents will pay at the polls for bad times, even in situations where objective observers can find little rational basis to suppose that those incumbents have had any part in producing the voters' pain ... To the extent that voters engage in sophisticated attributions of responsibility they should be entirely unresponsive to natural disasters, at least on average; to the extent that they engage in blind retrospection, they should exhibit "systematic attribution errors"...

Second, it is argued that this voter irrationality undermines the accountability-based normative argument for democracy. Again, Achen and Bartels (2004, pp.35–36) claim:

> Democracy must be defended some other way, if it is to be defended at all. ... real voters often have only a vague, more or less primitive understanding of the connections (if any) between incumbent politicians actions and their own pain or pleasure. As a result, rational retrospective voting is harder than it seems, and blind retrospection sometimes produces consistently misguided patterns of electoral rewards and punishments.

Without disputing the validity of the empirical findings, we dispute these interpretations. In so doing, we also illustrate a methodological point. An important, but under-appreciated, role for formal theory is to elucidate which conclusions do, and which do not, follow from existing empirical results. This role is particularly crucial when empirical findings seem to straightforwardly suggest an interpretation with important substantive implications, while

[1] Though see Healy and Malhotra (2009), who find an insignificant and small relationship between disaster damage and incumbent vote share.

a careful analysis of the underlying mechanisms suggests that those conclusions are not entailed by the data. We show that the literature on the relationship between disasters and incumbent electoral fortunes is such a case. We do so in two steps.

First, we argue that the leap from voter irrationality to the conclusion that electoral rewards and punishments are "consistently misguided" rests on a fundamental confusion about the nature of rational retrospective voting. Quite generally, rational voters must vote in a forward looking way that selects good types, not in a backward looking way that maximizes incentives (Fearon, 1999). That is, a rational voter votes retrospectively—i.e., based on past performance—only because that past performance is informative about expected future performance. As Fearon (1999) shows, this is true as long as the past provides even a tiny amount of information about the future. Moreover, the reelection rule that selects good types need not be the reelection rule that maximizes the incumbent's incentives to take costly actions that benefit voters nor need it be the reelection rule that maximizes the voters ex ante expected utility. These facts create the possibility that the level of accountability could be increased if voters were less rational.

We provide a formal model of voters who are irrational in the sense of having only a "primitive understanding of the connections (if any) between incumbent politicians' actions and their own pain or pleasure." We show that such voters can actually enhance, rather than diminish, the level of accountability. In particular, we demonstrate that the voting rule used by irrational voters who vote based on past performance with no reference to the future can sometimes provide stronger incentives to the politician than the voting rule used by rational voters. We then show that a similar conclusion holds when voters are forward looking Bayesian learners (as in Achen (1992) and Bartels (1993)) who make the mistake of holding incumbents responsible for events they should know are not affected by incumbent actions. Together, these models show that voter irrationality does not imply a necessary weakening of accountability.

Although our first results show that empirical evidence of voter irrationality has no particular implications for the quality of electoral incentives, the question of whether existing empirical evidence implies that voters are irrational is still of interest. The second step in our argument is to show that most of the evidence presented in the literature initiated by Achen and Bartels (2004) fails to have any implications for the debate over voter rationality. That is, voter irrationality is not implied by the observation that, on average, incumbents suffer electorally following negative events outside of their control.

To do so, we provide a formal model in which voters are fully rational in the sense of engaging in "sophisticated attributions of responsibility." That is, voters do not irrationally respond to negative events. Nonetheless, on average, incumbent reelection rates and vote shares are lower following natural disasters. Hence, the empirical fact that incumbent electoral fortunes suffer, on average, following such events does not entail the conclusion that voters are irrational.

The basic mechanism underlying this model is informational. Natural disasters make outcomes more informative about the competence of incumbent politicians. For instance, suppose incumbents are responsible for infrastructure. During normal times there may be little information about how good a job the incumbent did. But during a hurricane or tornado, the voters learn a lot about whether the incumbent did a good job overseeing infrastructure maintenance.

Why does increased information hurt incumbent electoral fortunes? On average, in our model, incumbents are advantaged in elections. This advantage arises endogenously through electoral selection (Zaller, 1998; Ashworth and Bueno de Mesquita, 2008). As such, the less new information the voters get, the less likely they are to learn enough to make them want to replace their incumbents. However, when there is lots of information available (e.g., following a disaster), it becomes more possible for a piece of information to be sufficiently informative to overcome the voters' initial tendency to support the incumbent. To see the intuition, consider a voter who thinks the incumbent is good and learns nothing. Such a voter will support the incumbent. But if that voter learns something, there is at least some chance that what he learns will be sufficiently bad that he will replace the incumbent.

The mechanism underlying our argument is substantively motivated and is consistent with empirical scholarship showing that voters are more responsive to outcomes in high information environments than in low information environments (Berry and Howell, 2007; Ferraz and Finan, 2008; Snyder and Strömberg, 2010). The model also generates new empirical predictions which could be tested against data. In particular, the model predicts the following:

> Disasters (or other shocks to voter information) increase the electoral fortunes of incumbents whom, ex ante, the voters believe are low competence and decrease the electoral fortunes whom, ex ante, the voters believe are high competence.

These predictions suggest ways in which an empirical researcher could look for evidence that behavior that, prima facie, seems to indicate voter irrationality might, in subtle ways, be consistent with voter rationality, as in Healy and Malhotra (2010).

It is important to note that we are not arguing that voters are in fact rational. In our view, the fact that college football losses—which are unlikely to change the informational environment—seem to have a small negative effect on incumbent electoral fortunes constitutes evidence of at least some amount of voter irrationality (Healy, Malhotra and Mo, 2010). Our point is that the evidence of a large negative relationship between natural disasters and incumbent electoral performance does not similarly constitute evidence of a significant amount of voter irrationality. Indeed, we show that relationship follows directly from changes in the informational environment in a straightforward application, with rational voters, of the canonical model of voter learning used by Achen (1992), Bartels (1993), and many others.

1 Irrational Voters and Accountability

The fundamental normative claim made in the empirical literature rests on what we believe is a mistaken understanding of the implications of the theoretical literature on accountability. In particular, the empirical literature asserts that evidence of voter irrationality implies a breakdown in accountability, with important implications for democratic theory. Here we show that voter irrationality need not imply anything of the sort. Indeed, it is entirely possible for irrational voters to create more accountability—i.e., stronger incentives for politicians to take costly actions on behalf of voters—than fully rational voters.[2]

The motivating thought behind the critique of democracy seems to be that voters, behaving rationally, will surely maximize accountability. And, by extension, voters behaving irrationally must weaken accountability. But this thought rests on a misunderstanding of the relationship between voter rationality and accountability-maximizing retrospective voting. As Fearon (1999) explained over a decade ago, rational voters are not in the business of creating optimal incentives. They are in the business of electing politicians whom they believe will produce good outcomes in the future. They vote retrospectively only because past performance is informative about future performance. As such, there is no reason to believe that the rational voting rule—the rule that maximizes a voter's future payoffs—will be the accountability-maximizing voting rule—the rule that maximizes a voter's past payoffs.

This point puts an interesting twist on arguments about the link between voter ratio-

[2]Notice, increased accountability can increase or decrease voter welfare, depending on context. For instance, stronger incentives can increase the provision of public goods or encourage other policies that enhance voter welfare. But they can also create incentives for pandering (Canes-Wrone, Herron and Shotts, 2001) or distort effort toward easily observable tasks (Ashworth, 2005) in ways that can harm social welfare. Following the empirical literature to which we are responding, we focus on cases where accountability is beneficial.

nality and voter competence. The literature views voters as competent if they behave in a way that maximizes politician incentives (i.e., accountability). The literature then critiques democracy on the grounds that voters are irrational and, thus, incompetent. However, if the argument above is correct, then voter rationality and voter competence are actually mutually exclusive. Rational voters do not maximize incentives. And competent voters must be behaving irrationally.

In this section, we start by demonstrating this point in a simple model. We characterize the incentive-maximizing voting rule, and show that a rational voter typically does not use this voting rule in equilibrium. We use this fact to show that irrational voters can actually improve accountability.

Modeling voter irrationality is conceptually tricky. There are many ways that voters might be irrational. To make our main point, we focus on a kind of irrational voter whom we call a *pain-pleasure voter,* following Achen and Bartels's (2004) suggestion that:

> The central fact about democracies is that the voters understand little beyond their own and their community's pain and pleasure... real voters often have only a vague, more or less primitive understanding of the connections (if any) between incumbent politicians actions and their own pain or pleasure.

We then show that the point also holds in a model of irrational voters who correspond to the empirical literature's more specific claim that voters hold politicians responsible for observable events that the voters should know are outside of the politicians' control.

1.1 Rational Voters Don't Maximize Incentives

The basic set up of the model is as follows. There is a voter, an incumbent politician (I), and a challenger (C). There are two periods. Each politician, j, is characterized by an ability (θ_j). In each period, t, the politician in office chooses a level of effort, $a_t \in \{\underline{a}, \overline{a}\}$, where $\overline{a}$ is interpreted as working hard and $\underline{a} < \overline{a}$ is interpreted as shirking.

The outcome in a period t is an additive function of three factors: the competence of the politician in office (θ_t), the effort of the politician in office (a_t), and unobservable noise (ϵ_t). The outcome in period t, then, is:

$$\pi_t = \theta_t + a_t + \epsilon_t.$$

Each ϵ is drawn independently from a normal distribution with mean 0 and variance σ_ϵ^2.

Politician j's ability (θ_j) is drawn from a normal distribution with mean m_j and variance σ_j^2. (The θ's are independent of each other and of the ϵ's.) We assume that the incumbent has higher expected ability than the challenger. In particular, we assume that $m_C = m$ and $m_I = m + r$, where $r > 0$ represents the incumbent's reputational advantage. Although assumed exogneous here, this incumbency advantaged can be micro-founded in an initial election, as we will show in Section 2.

All random variables are independent. No player observes the θ's or the ϵ's. The actions of the incumbent are only observable by the incumbent.

The voter's payoff is the sum of the two outcomes:

$$\pi_1 + \pi_2.$$

Each politician gains a benefit of 1 for being in office and suffers a cost $k \in (0, 1)$ from high effort. The cost of shirking is 0.

The timeline of the game is as follows:

1. Nature draws all random variables.
2. The incumbent chooses whether to exert effort.
3. The voter observes the outcome (π_1) and then chooses whether to elect the incumbent or the challenger.
4. The winner of election chooses effort.
5. π_2 is observed and the game ends.

We are interested in whether rational or irrational voters of the pain-pleasure variety do a better job of creating accountability. To answer this question, we ask for which performance thresholds the incumbent will choose high first period effort. We then discuss conditions under which an irrational voter can use such a threshold, but the rational voter cannot.

1.1.1 Incumbent Incentives

Suppose the incumbent believes the voter will reelect her if and only if the outcome, π_1, is better than some performance threshold, $\overline{\pi}$. (Different types of voters will determine this performance threshold differently, as we will see.) This means the incumbent believes she will be reelected if and only if:

$$\theta_1 + a_1 + \epsilon_1 \geq \overline{\pi}.$$

The left-hand side of this inequality is a normal random variable with mean $m + r + a_1$ and variance

$$\sigma^2 = \sigma_\theta^2 + \sigma_\epsilon^2.$$

Thus, an incumbent, facing a voter who uses performance threshold $\overline{\pi}$, believes that if she chooses effort a_1 she is reelected with probability

$$1 - \Phi\left(\frac{\overline{\pi} - a_1 - m - r}{\sigma}\right),$$

where Φ is the cdf of the standard normal distribution.

Given this, the incumbent will choose high effort if and only if:

$$1 - \Phi\left(\frac{\overline{\pi} - \overline{a} - m - r}{\sigma}\right) - k \geq 1 - \Phi\left(\frac{\overline{\pi} - \underline{a} - m - r}{\sigma}\right).$$

Rearranging, choosing high effort is the incumbent's best response to $\overline{\pi}$ if and only if:

$$\Phi\left(\frac{\overline{\pi} - \underline{a} - m - r}{\sigma}\right) - \Phi\left(\frac{\overline{\pi} - \overline{a} - m - r}{\sigma}\right) \geq k. \tag{1}$$

We can think of the left-hand side of Condition 1 as representing the strength of the incentives created by a performance threshold $\overline{\pi}$. Formally, say that reelection cutoff $\overline{\pi}'$ *creates stronger incentives than reelection cutoff* $\overline{\pi}''$ if

$$\Phi\left(\frac{\overline{\pi}' - \underline{a} - m - b}{\sigma}\right) - \Phi\left(\frac{\overline{\pi}' - \overline{a} - m - b}{\sigma}\right) > \Phi\left(\frac{\overline{\pi}'' - \underline{a} - m - b}{\sigma}\right) - \Phi\left(\frac{\overline{\pi}'' - \overline{a} - m - b}{\sigma}\right).$$

To understand this definition, notice the following two implications of a performance threshold $\overline{\pi}'$ creating stronger incentives than a different performance threshold $\overline{\pi}''$. First, if the incumbent wants to choose high effort when costs are k and the voter uses $\overline{\pi}''$, then the incumbent also wants to choose high effort when costs are k and the voter uses $\overline{\pi}'$. Second, there exists an open set of costs such that, for any cost in that set, the incumbent wants to choose high effort if the voter uses $\overline{\pi}'$, but not if the voter uses $\overline{\pi}''$.

Say that π^* is an *incentive-maximizing performance threshold* if no other performance threshold creates stronger incentives than π^*. Clearly, an incentive maximizing performance threshold solves the following maximization problem:

$$\max_{\overline{\pi}} \Phi\left(\frac{\overline{\pi} - \underline{a} - m - r}{\sigma}\right) - \Phi\left(\frac{\overline{\pi} - \overline{a} - m - r}{\sigma}\right).$$

Lemma 1 *The performance threshold*

$$\pi^* = \frac{\overline{a} + \underline{a}}{2} + r + m$$

is the unique incentive maximizing rule.

(Omitted proofs are in the appendix.)

1.1.2 Rational Voter

Now let's see what performance threshold a rational voter uses. In the second period, regardless of who is in office, the incumbent will shirk. As such, the expected payoff to the voter of reelecting the incumbent is:

$$\overline{m}_I = \mathbb{E}[\theta_I | \pi_1].$$

The expected payoff to the voter of electing the challenger is m. Thus a rational voter reelects the incumbent if and only if:

$$\overline{m}_I \geq m.$$

Suppose the voter believes that, in the first period, the incumbent chose effort a^*. Then the voter believes that $\pi_1 - a^*$ is a normal random variable with mean θ_I and variance $\sigma_\theta^2 + \sigma_\epsilon^2$. Standard facts about normal learning imply that the voter's posterior beliefs about the incubment's competence are normally distributed with mean

$$\overline{m}_I = \lambda(\pi_1 - a^*) + (1 - \lambda)(m + r)$$

and variance

$$\lambda \sigma_\epsilon^2,$$

with

$$\lambda = \frac{\sigma_\theta^2}{\sigma_\epsilon^2 + \sigma_\theta^2}.$$

Using these facts, we can rewrite the rational voter's reelection rule in terms of outcomes rather than posterior beliefs. In particular, since the rational voter reelects if and only if $\overline{m}_I \geq m$, she reelects if and only if:

$$\lambda(\pi_1 - a^*) + (1 - \lambda)(m + r) \geq m,$$

which is equivalent to:

$$\pi_1 \geq m + a^* - \frac{1-\lambda}{\lambda} r.$$

We define a function, $\overline{\pi}_R(a^*, r)$, which gives the performance threshold used by a rational voter as a function of the effort he believes the incumbent took and the incumbent's reputational advantage:

$$\overline{\pi}_R(a^*, r) \equiv m + a^* - \frac{1-\lambda}{\lambda} r.$$

Hence, there is an equilibrium in which the incumbent takes high effort, if, when the voter uses the performance threshold $\overline{\pi}_R(\overline{a}, r)$, the incumbent wants to take high effort.

1.1.3 Pain-Pleasure Voters Can Improve Accountability

A plain-pleasure voter does not rationally update beliefs and vote with an eye toward the future. Rather, such a voter supports the incumbent as long as the incumbent provided a good enough outcome. We formalize this by letting pain-pleasure voters use an arbitrary standard $\overline{\pi}$, reelecting the incumbent if and only if $\pi_1 \geq \overline{\pi}$.

Given the argument above, in order to prove that it is possible for pleasure-pain voters to improve accountability relative to rational voters, it suffices to show that the rational voter is not using the incentive maximizing performance threshold. Then, a pleasure-pain voter who does so improves accountability.

Recall that, for a fixed r and m, in an equilibrium with high effort, the rational voter uses the performance threshold:

$$\overline{\pi}(\overline{a}, r) = m + \overline{a} - \frac{1-\lambda}{\lambda} r.$$

From Lemma 1, the incentive maximizing performance threshold is:

$$\pi^* = \frac{\overline{a} + \underline{a}}{2} + r + m.$$

Quite generally, these are not equivalent, so a pain-pleasure voter can improve accountability relative to a rational voter. This immediately implies the following result:

Proposition 1 *If $\frac{\overline{a}-\underline{a}}{2} \neq \frac{r}{\lambda}$, then there exists an open set of pain-pleasure voters who provide stronger incentives than the rational voter.*

Proof. Follows directly from Lemma 1 and the argument in the text. ∎

As we've already indicated, the reason that irrational voters can provide stronger incentives than rational voters is because rational voters are forward looking. As such, they do not choose reward-punishment schemes to maximize incentives for incumbents. They choose reelection rules to maximize future payoffs for voters.[3]

One particularly stark way of seeing the intuition is to think about what happens as the incumbent's reputational advantage increases. First, consider the rational voter's rule. As the incumbent's reputational advantage increases, the rational voter's cutoff rule decreases. That is, the higher quality the voter believes the incumbent is, ex ante, the lower performance standard he holds the incumbent to. The logic behind this decreasing performance standard is as follows. When the incumbent has a strong reputation, only a really terrible performance can drag her reputation down below the reputation of the challenger. So, when the incumbent has a strong reputation, a rational voter must reelect her even following pretty bad outcomes.

Compare this to the irrational, incentive-maximizing cutoff rule. As the incumbent's reputational advantage increases, this cutoff rule increases—moving in the opposite direction from the rational rule. To see why, think about the incumbent's incentives for effort, given a fixed cutoff rule. As the incumbent's expected ability goes up, the incumbent thinks it more and more likely that she exceeds any fixed cutoff rule, even without putting in effort. As such, the higher the incumbent's ability, the weaker her incentives. To keep giving the incumbent incentives to work hard, as the incumbent's expected ability increases, the incentive-maximizing rule must raise the bar higher and higher.

This comparative static highlights how different rational behavior and incentive-maximizing behavior are. Not only are the two rules typically different, but they respond in opposite directions as incumbent reputation changes.

1.2 Voters Who Fail to Filter

The model above showed that voter irrationality can sometimes enhance accountability. It did so for a particular model of irrational voters—ones who respond to pleasure and pain in a mindless way. However, the empirical literature makes a more specific claim about ways in which voters behave irrationally. In particular, the literature claims that voters hold

[3]We've focused on the fact that the rational voter does not use a performance threshold that maximizes first period incentives—i.e., accountability. The rational voter also need not be using the performance threshold that maximizes his ex ante expected utility. In particular, consider the case where equilibrium with the rational voter involves low effort. A sufficiently impatient voter will be better off (from the ex ante perspective) using a rule that sub-optimally selects good types in order to induce high effort in the first period. Of course, such a rule is not sequentially rational, but could be used by a pain-pleasure voter.

politicians responsible for observable events that the voters should know are outside of the politicians' control. Here we show that the point above stands when we model voters who have this particular form of irrationality.

The model is virtually identical to that above, with the exception that the outcome in a period t is now a function of observable events not affected by the politician in office (ν_t) in addition to the three factors from the previous model—the competence of the politician in office (θ_t), the effort of the politician in office (a_t), and unobservable noise (ϵ_t). The outcome in period t, then, is:

$$\pi_t = \theta_t + a_t + \nu_t + \epsilon_t.$$

Each ν is observable and drawn independently (of all other random variables) from a normal distribution with mean 0 and variance σ_ν^2. All other random variables are as above.

Payoffs are as before.

The timeline of the game is as follows:

1. Nature draws all random variables.
2. Incumbent chooses whether to exert effort.
3. Voter observes the outcome (π_1) and the observable events not due to the incumbent (ν_1) and then chooses whether to elect the incumbent or challenger.
4. The winner of election chooses effort.
5. π_2 and ν_2 are observed and the game ends.

First, consider what happens with rational voters. Rational voters behave just as in the previous model, with one exception. In addition to filtering out expected incumbent effort prior to using the outcome to update beliefs about competence, rational voters also filter out observable events not affected by the politician in office. That is, rational voters, here, update their beliefs just as before, except the unbiased signal in this model is $\pi_1 - a^* - \nu_1$. The equilibrium incentives for the incumbent are exactly as before.

Next, consider what happens with an irrational voter who fails to filter the observable event, treating $\pi_1 - a^*$ as an unbiased signal of incumbent competence, rather than $\pi_1 - a^* - \nu_1$. The analysis of voter electoral behavior is very similar to that with rational voters. Of course, voters who fail to filter use the "wrong" cutoff rule in the sense of sometimes electing incumbents who are worse in expectation than the challenger or not reelecting incumbents who are better in expectation than the challenger. From the incumbent's perspective, this failure to filter events that are not in the incumbents control is equivalent to adding noise to the voter's decision, since the voter will vote based on events which, from the incumbent's perspective, are random luck.

The key result for us is that it is possible that the irrational rules used by these types of voters can provide stronger incentives for effort by the incumbent than does the rule used by a rational voter. The reason is just as above. The rule used by rational voters is optimal with respect to selecting good types. It is not optimal with respect to providing incentives, since rational voters are forward looking. Our irrational voters—who are also forward looking but use information incorrectly—use the wrong rule with respect to the goal of selecting good types. However, the fact that the rational voter's rule is typically not optimal with respect to providing incentives, creates the possibility that an irrational rule does a better job of providing incentives. Indeed, the next result shows that is true when the incumbent's reputational advantage is strong.

Proposition 2 *For a sufficiently large incumbent reputational advantage r, incentives are stronger with an irrational voter who fails to filter than with a rational voter.*

2 Disasters and Rational Voters

Now we turn to a different model to illustrate that the evidence from the empirical literature is, in fact, consistent with fully rational voters. Thus, neither of the interpretive claims in the empirical literature—(1) that voter irrationality necessarily reduces accountability or (2) that the fact that incumbent electoral fortunes decline on average following disasters implies that voters are irrational—are warranted.

There is an electoral district made of up a continuum of voters and four politicians: d_1, d_2, r_1, and r_2. Each politician, j, has a competence θ_j that is drawn from a normal distribution with mean zero and variance σ_θ^2.

At the beginning of the game, there is an electoral campaign between candidates d_1 and r_1. Prior to voting, the voters receive a public signal about each candidate as a result of the campaign. The signal about candidate j from this first campaign is:

$$s_j^1 = \theta_j + \eta_j^1,$$

where the η_j are independently drawn from a normal distribution with mean zero and variance σ_η^2.

After the campaign, the voter elects either d_1 or r_1. Whichever candidate wins the election is the executive in office for governance period 1. Denote the ability of this first period incumbent by θ_I^1. During this governance period, the world can be in one of two states: good or bad. Denote the state of the world in governance period 1 by $\omega^1 \in \{G, B\}$.

The bad state is different from the good state in two ways. First, all else equal, when the state is bad, the expected outcome is worse. Second, when the state is bad, the outcome is more sensitive to incumbent competence. The first of these difference captures the basic idea of what it means for the state of the world to be bad. The second of these differences captures the idea, discussed in the introduction, that bad states (i.e., times of disaster) reveal more information about incumbents than do good states.

To formalize, let ϵ^t be a random noise term distributed normally with mean zero and variance σ_ϵ^2. In the good state, the outcome is given by:

$$\pi_G^1 = \theta_I^1 + \epsilon^1.$$

In the bad state, the outcome in period 1 is given by:

$$\pi_B^1 = -\alpha + \beta\theta_I^1 + \epsilon^1$$

with $\alpha > \beta > 1$. The α term represents the fact that things are bad in the bad state. The β term represents the fact that outcomes are more sensitive to incumbent competence in the bad state.

Following the first governance period, there is an election between the incumbent and the other party's challenger. That is, if the first period incumbent is d_1, then the challenger in the second election is r_2. If the first period incumbent is r_1, then the challenger in the second election is d_2. (Sadly, there is never an $r_2 d_2$ election.) For simplicity, there is no campaign signal prior to voting in this second election. (All the qualitative results continue hold in a model where the voter again receives a campaign signal about each candidate as in the first election.)

The winner serves as the executive during the second governance period and faces the same technology as in the first governance stage. Then the game ends.

Each voter is indexed by b, a *bias* in favor of party d. These biases are distributed normally with mean zero and variance 1. If the executive in period t is from party d and the state is ω, then voter b's payoff in that period is:

$$\pi_\omega^t + b.$$

If the executive in period t is from party r and the state is ω, then voter b's payoff in that period is:

$$\pi_{\omega}^{t}.$$

A voter's total payoff is simply the sum of his per-period payoffs.

The timeline of the game is as follows:

1. Nature draws all the random variables.
2. The first election is held: The voter observes s_a^1 and s_b^1 and chooses a candidate.
3. All players observe the state of the world, $\omega^1 \in \{G, B\}$ and the first governance outcome, $\pi_{\omega^1}^1$.
4. The second election is held.
5. All players observe the state of the world, $\omega^2 \in \{G, B\}$ and the second governance outcome, $\pi_{\omega^2}^2$.
6. The game ends.

We model a continuum of districts independently playing the game described above. Since behavior in all districts is the same, we will focus on a particular district for the analysis and return to the continuum interpretation in order to relate our results to empirical work.

2.1 Description of Equilibrium

Since there is a continuum of voters, no voter is ever pivotal. As is standard in models of elections with two candidates, we assume voters vote sincerely. Given this, it is straightforward that the winner of the election will be whichever candidate gets the support of the median voter—i.e., the voter with bias $b = 0$.

In the first election, the median voter will support whichever candidate generates the better campaign signal. This means that the expected ability of the winning candidate in district d is just the maximum of two normally distributed random variables. As such, its distribution is the distribution of the first order statistic of two normally distributed random variables. This distribution is obviously "better" than the prior distribution of these random variables.

Given this, think about the cross-sectional distribution (across districts) of voter beliefs about candidate quality prior to the first governance period. In all districts, voters believe

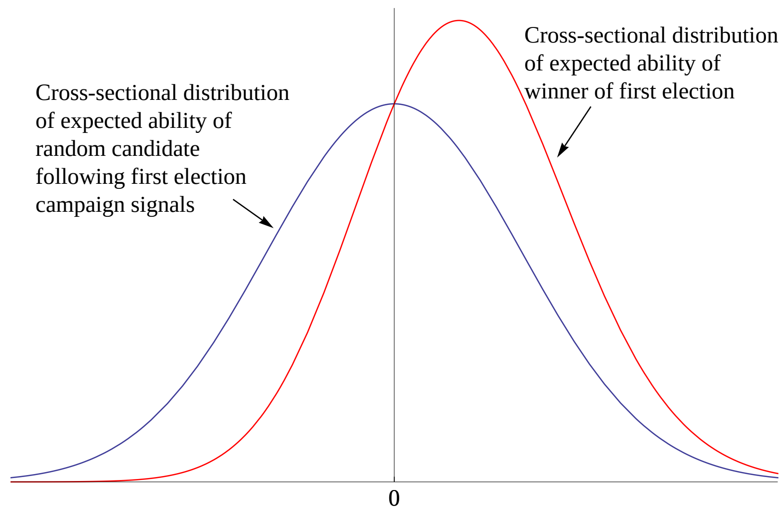

Figure 1: The blue curve is the density of a normal random variable. The red curve is the density of the maximum of two draws from that distribution.

that the future challenger has expected ability zero. In each district, the voters' beliefs about the incumbent's expected ability will depend on the first electoral signals. In most districts, the voters will think that the incumbent is of higher expected competence than the challenger. This is because in most districts the incumbent will have had a first period electoral signal greater than zero. Appealing to the law of large numbers, Figure 1 shows the cross-sectional distributions of expected incumbent abilities and expected abilities of random candidates (not conditional on winning). Incumbents with expected ability greater than zero are "ahead" and incumbents with expected ability less than zero are "behind". The key is that, in most districts, the incumbent is ahead.

We formalize this intuition with Lemma 2 in the appendix.

Why is this important? Since most incumbents go into the first governance period ahead of the future challenger, anything that reveals extra information about incumbents is bad for the expected electoral fortunes of incumbents. There are two ways to think about the idea of electoral fortunes: expected vote share and likelihood of winning the election. We will show that more information is bad for incumbent electoral fortunes in both senses.

The intuition is easiest to see for the likelihood of winning interpretation. Suppose the first governance period revealed no information. Then, in every district where the incumbent was ahead, she would win reelection and in every district where the incumbent was behind she would lose reelection. Since the incumbent is ahead in most districts, incumbents would win most elections. Now suppose the first governance period becomes informative. In some districts where the incumbent was ahead, the new information would harm her and she would lose the election. Of course, in some districts where the incumbent was behind, the new information would help her and she would win the election. But these effects do not exactly off-set. More incumbents are harmed than helped by the new information because there are many more districts where incumbents are ahead than behind. So increased information harms the overall electoral fortunes of incumbents. (A similar logic holds for the vote share interpretation.)

Let's see this somewhat more formally.

At the beginning of the first governance period the voters' beliefs are that the incumbent has ability that is normally distributed with mean m_I and variance σ_1^2. (See Lemma 2 in the appendix for a derivation.) Suppose the state of the world is good in the first governance period. Then the voters know that the first period outcome, π_G^1, is $\theta_I^1 + \epsilon_1$. Since this first period outcome is a normally distributed random variable with mean θ_I, standard results on Bayesian updating imply that the voters' posterior beliefs about the incumbent's ability are normally distributed with mean

$$\overline{m}_I = \lambda_G \pi_G^1 + (1 - \lambda_G) m_I,$$

where

$$\lambda_G = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_\epsilon^2}.$$

If the state is bad, the voters know that the first period outcome, π_B^1 is equal to $-\alpha + \beta\theta_I + \epsilon_1$. Thus

$$\frac{\pi_B^1 + \alpha}{\beta} = \theta_I + \frac{\epsilon_1}{\beta}$$

is a normally distributed random variable with mean θ_I. As such, standard results on Bayesian updating imply that the voters' posterior beliefs about the incumbent's ability are normally distributed with mean

$$\overline{m}_I = \lambda_B \left(\frac{\pi_B^1 + \alpha}{\beta} \right) + (1 - \lambda_B) m_I,$$

where

$$\lambda_B = \frac{\sigma_1^2}{\sigma_1^2 + \frac{\sigma_\epsilon^2}{\beta^2}}.$$

There is an important fact to notice in this updating. In the bad state, the voters observe the first period outcome, π_1^B and, before using it to update, add α to it. The voters do so because α represents the negative welfare consequences of the bad state that are entirely unrelated to the incumbent's competence. Thus, the voters, here, are not being fooled into blaming the incumbent for bad outcomes for which the incumbent is not responsible. Indeed, the voters are fully, rationally ignoring those negative consequences.

The median voter (who is unbiased) will support the incumbent if $\overline{m}_I \geq 0$. Given that, let's calculate the probability of reelection in each state of the world for an incumbent who enters the first governance period with expected ability m_I.

Notice, given m_I, prior to the governance period, the voters' eventual posterior belief, $\overline{m}_I$ is a random variable. As such, the probability the incumbent is reelected, given m_I, is just the probability that the random variable $\overline{m}_I$ is greater than zero. (As we will show below, that random variable has a normal distribution, with mean m_I.) Figure 2 shows this probability for two different values of m_I, one greater than zero (so that the incumbent is ahead going to the first governance period) and the other less than zero (so that the incumbent is behind going into the first governance period).

What we actually want to know is how the probability of the incumbent winning reelection differs in bad and good states of the world. Therefore, we need to calculate of the distribution of the random variable $\overline{m}_I$ conditional on each state.

First, consider the good state of the world. The incumbent is reelected if

$$\overline{m}_I = \lambda_G \pi_G^1 + (1 - \lambda_G) m_I \geq 0.$$

Since $\pi_G^1 = \theta_I + \epsilon_1$, it is a normally distributed random variable with mean m_I and variance $\sigma_1^2 + \sigma_\epsilon^2$. Given this, in the good state, $\overline{m}_I$ is a normal random variable with mean m_I and variance

$$\sigma_G^2 = \lambda_G^2(\sigma_1^2 + \sigma_\epsilon^2) = \frac{\sigma_1^4}{\sigma_1^2 + \sigma_\epsilon^2}.$$

So the probability of reelection, given that the state was good, for an incumbent who entered the first governance period with expected ability m_I is:

$$\Pr(\overline{m}_I \geq 0|G) = 1 - \Phi\left(\frac{-m_I}{\sigma_G}\right).$$

Next consider the bad state of the world. The incumbent is reelected if

$$\overline{m}_I = \lambda_B \left(\frac{\pi_1^B + \alpha}{\beta}\right) + (1 - \lambda_B) m_I \geq 0.$$

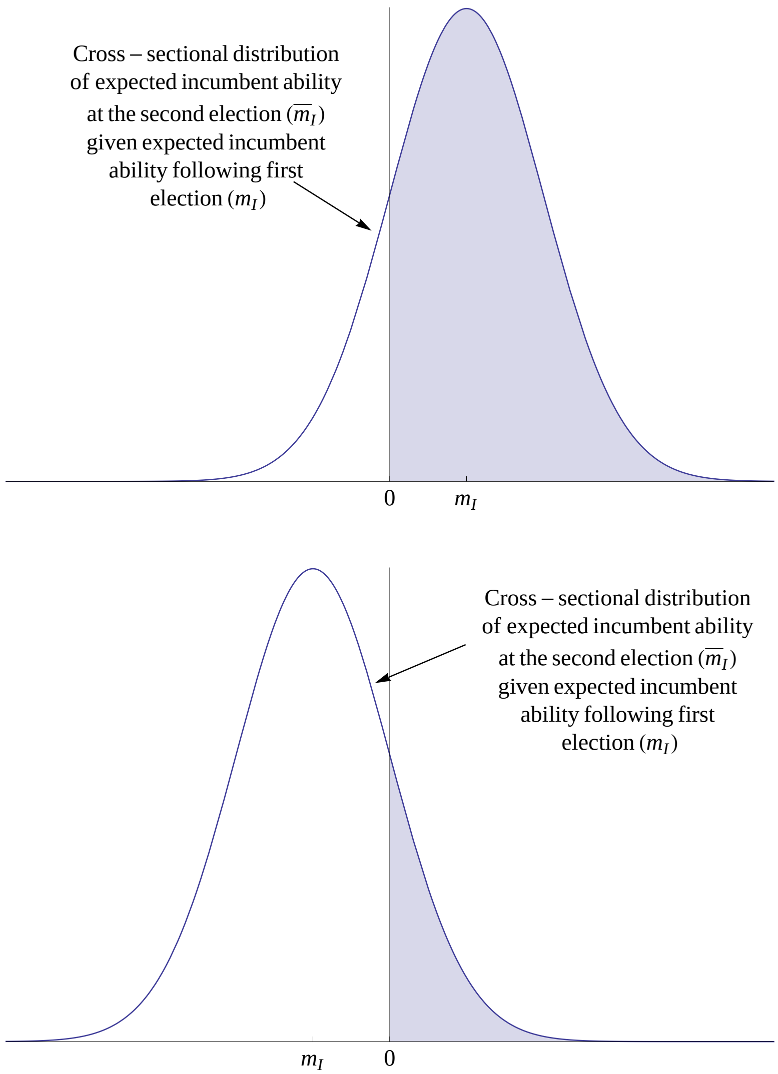

Figure 2: In each panel, the shaded area represents the probability the incumbent wins reelection given expected ability m_I following the first election.

Notice that $\frac{\pi_1^B+\alpha}{\beta}$ is a normally distributed random variable with mean m_I and variance $\sigma_1^2 + \frac{\sigma_\epsilon^2}{\beta^2}$. Given this, in the bad state, $\overline{m}_I$ is a normal random variable with mean m_I and variance:

$$\sigma_B^2 = \lambda_B^2 \left(\sigma_1^2 + \frac{\sigma_\epsilon^2}{\beta^2} \right) = \frac{\sigma_1^4}{\sigma_1^2 + \frac{\sigma_\epsilon^2}{\beta^2}}.$$

So the probability of reelection, given that the state was bad, for an incumbent who entered the first governance period with expected ability m_I, is:

$$\Pr(\overline{m}_I \geq 0|B) = 1 - \Phi\left(\frac{-m_I}{\sigma_B} \right).$$

As the derivations above show, in both the good and bad states, the random variable $\overline{m}_I$ is distributed normally with mean m_I. The only difference is the variance of the distribution. Thus, any difference in reelection probability between two states reflects differences in the variances. Those difference in turn reflect difference in information in the two states.

Comparing the two variances, $\beta > 1$ straightforwardly implies that $\sigma_B^2 > \sigma_G^2$. This reflects the fact that there is *more* information available in the bad state than in the good state. Why does a larger variance indicate more information? Remember, the distribution we are calculating is the distribution, prior to the first governance stage, of the random variable $\overline{m}_I$, which is the voter's posterior beliefs after observing the first governance stage. When the first governance stage is more informative, this distribution of the voter's posterior beliefs is more spread out (i.e., has higher variance) because when the voter learns more his beliefs move more in response to his observations.

So is reelection more likely under the good or the bad state? The answer depends on the voters' belief entering the first governance stage, m_I. Consider Figure 3. In the upper panel, the blue (smaller variance) distribution represents the distribution of posterior beliefs if the state is good. The red (higher variance) distribution represents the distribution of posterior beliefs if the state is bad. In this panel, the incumbent is ahead prior to the first governance period (i.e., $m_I > 0$). In the good state, the probability of incumbent reelection is the sum of the areas of regions 1 and 2. In the bad state, the probability of incumbent reelection is the sum of the areas of regions 2 and 3. The fact that region 1 has larger area than region 3 shows that, when the incumbent is ahead going into the first governance period, the probability of reelection is higher in the good state than in the bad state.

Now consider the lower panel of Figure 3. Again the blue (lower variance) distribution represents the good state and the red (higher variance) distribution represents the bad state. The only difference is that now the incumbent is behind prior to the first governance period (i.e., $m_I < 0$). In the good state, the probability of incumbent reelection is the sum of the areas of regions i and ii. In the bad state, the probability of incumbnet reelection is the sum of the areas of regions ii and iii. The fact that region i has smaller area than region iii shows that, when the incumbent is behind going into the first governance period, the probability of reelection is lower in the good state than in the bad state.

Remark 1 *An incumbent who enters the governance period ahead ($m_I > 0$) has a greater probability of reelection in the good state than in the bad state. An incumbent who enters the governance period behind ($m_I < 0$) has a greater probability of reelection in the bad state than in the good state.*

To see this, note that an incumbent who enter the governance period with expected competence m_I is reelected with probability $1 - \Phi(-m_I/\sigma_\omega)$. Since $\sigma_B > \sigma_G$, the result will follow from showing that $1 - \Phi(-m_I/\sigma)$ is decreasing in σ if $m_I > 0$ and is increasing in σ if $m_I < 0$.

Differentiate to get

$$\frac{\partial}{\partial \sigma}\left[1 - \Phi\left(\frac{-m_I}{\sigma}\right)\right] = -\phi\left(\frac{m_I}{\sigma}\right) \cdot \frac{m_I}{\sigma^2}.$$

This derivative has the opposite sign of m_I, establishing the result.

We've seen that some incumbents' expected electoral fortunes are helped by the bad state and others' expected electoral fortunes are helped by the good state. In particular, incumbents who go into the first governance period ahead are helped by the bad state and incumbents who go into the first governance period behind are helped by the good state.

The empirical literature shows that, on average, bad states are bad for incumbents. The question for us is, given the heterogeneous effects just identified, can we say anything about what happens on average. The answer is yes.

To see why, think about the cross-section of districts, each of which behaves as described above. As we've already shown, the cross-sectional distribution of m_I has a mean which is greater than zero. That is, on average, incumbents are ahead going into the first governance period. The reason, of course, is electoral selection. Incumbents have already won an election. Thus, the majority of incumbents have shown themselves to be higher expected quality than the average candidate.

So we've seen two facts. First, the bad state helps the expected electoral fortunes of incumbents who are behind and hurts the expected electoral fortunes of incumbents who are ahead. Second, most incumbents are ahead. As formalized in the following result,

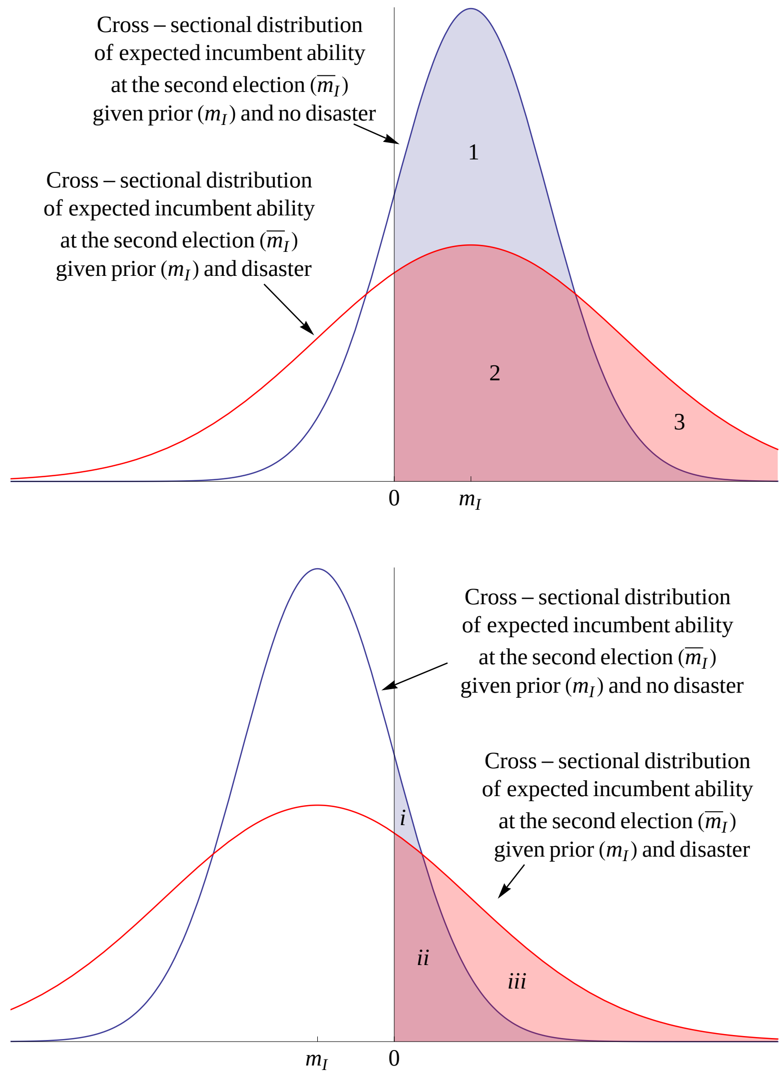

Figure 3: The upper panel shows that more information hurts incumbents who are ahead. The lower panel shows that more information helps incumbents who are behind.

combining these two facts leads to the conclusion that on average, the expected electoral fortunes of incumbents is worse in the bad state than in the good state.

Proposition 3 *The cross-sectional average probability of incumbent reelection is larger in the good state than in the bad state.*

Proposition 3 shows our key result for one interpretation of incumbent electoral fortunates, namely the likelihood of winning reelection. The same result holds for the other interpretation, expected vote share. The first step to seeing this is to derive an incumbent's expected vote share in our model. Consider first the case when d_1 is the incumbent. Voter b votes for the incumbent if

$$\overline{m}_{d_1} + b \geq 0.$$

Since b is distributed normally with mean 0 and variance 1, the share of voters who vote for the incumbent d_1 is:

$$1 - \Phi(-\overline{m}_{d_1}).$$

If r_1 is the incumbent, a voter b votes for her if:

$$\overline{m}_{r_1} \geq b.$$

Given this, the share of voters who vote for the incumbent r_1 is:

$$\Phi(\overline{m}_{r_1}) = 1 - \Phi(-\overline{m}_{r_1}).$$

Hence, in either case, the vote share of the incumbent, given the voters' posterior belief about the incumbent $\overline{m}_I$, is

$$1 - \Phi(-\overline{m}_I).$$

As in the discussion of the likelihood of reelection above, the key is that the distribution of voter posterior means is different depending on whether the state is bad or good, since this affects the amount of information. We can find the expected vote share of the incumbent by integrating the vote share given a posterior belief against the prior distribution of these beliefs. Recall, in a state ω, the prior distribution of voter posterior beliefs has mean m_I and variance σ_ω^2. Hence, the expected vote share of an incumbent who entered the first governance period with expected ability m_I, given a state ω, is:

$$\int [1 - \Phi(-\overline{m}_I)] \frac{1}{\sigma_\omega} \phi \left(\frac{\overline{m}_I - m_I}{\sigma_\omega} \right) d\overline{m}_I.$$

The result now follows from two facts. First, this expected vote share is decreasing in σ_ω (so that disasters are bad for expected vote share) when incumbents are ahead (i.e., $m_I > 0$) and is increasing in σ_ω when incumbents are behind (i.e., $m_I < 0$). Second, on average, incumbents are ahead. Hence, on average, disasters reduce expected incumbent vote share, as formalized in the next result.

Proposition 4 *The cross-sectional average incumbent vote share is larger in the good state than in the bad state.*

Propositions 3 and 4 show that our model, with fully rational voters, is consistent with existing empirical findings that natural disasters are associated, on average, with worse incumbent electoral fortunes. However, the reason is very different than the standard account. In the standard account, incumbents are harmed by disasters because voters irrationally punish incumbents for bad outcomes that were not within the incumbents' control. As we've already highlighted, that is not what is going on in our model. In our model, the incumbent is not responsible for the bad state (i.e., a natural disaster). Moreover, the voter knows this to be the case and rationally ignores the negative welfare consequences of the bad state when making his electoral choice. Nonetheless, the electoral fortunes of incumbents are worse, on average, in the bad state. The reason is that the bad state makes outcomes more responsive to incumbent competence, thereby revealing extra information about the incumbent, which is bad for the typical incumbent, who enters the first governance period ahead.

Finally, it is worth noting that this model has some additional empirical implications beyond the prediction that natural disasters harm expected incumbent electoral fortunes. Suppose that we observe proxies for beliefs about incumbent quality relative to likely challengers. A first prediction of the model is the existence of an incumbency advantage—the majority of incumbents will be ahead of likely challengers. A second prediction offers a more subtle hypothesis about the relationship between disasters and the electoral fortunes of incumbents. In particular, we predict heterogeneous effects, depending on whether the incumbent is ahead or behind. Natural disasters are expected, on average, to hurt incumbents who are ahead and to help incumbents who are behind.

3 Conclusion

An important, but under-appreciated, role for formal theory is to clarify which conclusions do and which do not follow from empirical findings. This role is particularly crucial when empirical findings seem to straightforwardly suggest an interpretation with important substantive implications, while a careful analysis of the underlying mechanisms suggests that those conclusions are not entailed by the data. In this paper, we have argued that the literature on the relationship between disasters and incumbent electoral fortunes is such a case.

Several empirical studies document that incumbent reelection rates drop following disasters that were outside the control of the incumbent. These findings have been widely viewed to support two conclusions. First, that voters are irrational. And, second, that this irrationality undermines an accountability-based defense of democratic legitimacy. We show that neither conclusion is entailed by the data.

First we show that, even if voters are behaving irrationality, this does not necessarily imply that accountability is weakened. Since rational voters typically do not choose accountability maximizing rules (since such rules are not sequentially rational) it is entirely possible that irrational voters provide a higher level of accountability. Indeed, the political agency literature has identified a variety of mechanisms by which rational reelection behavior creates distortions away from optimal accountability (Coate and Morris, 1995; Lohmann, 1998; Canes-Wrone, Herron and Shotts, 2001; Maskin and Tirole, 2004; Ashworth, 2005; Ashworth and Bueno de Mesquita, 2006; Besley, 2006; Gelbach, 2007; Daley and Snowberg, 2011).

Second, we show that the observation that incumbents suffer electorally following negative events outside of their control does not imply that voters are irrational. The argument is informational. We argue that disasters may increase the amount of information voters have about incumbents. Since there is an (endogenously derived) incumbency advantages, on average incumbents are ahead in future elections. As such, even though voters do not blame or punish incumbents for events outside the incumbents' control, the increased information associated with disasters hurts the average incumbent's electoral fortunes. It does so because, when there is lots of information available, it becomes more possible for a piece of information to be sufficiently informative to overcome the voter's initial tendency to support the incumbent.

The mechanism underlying our argument is substantively motivated and is consistent with empirical scholarship showing the voters are more responsive to outcomes in high information environments than in low information environments (Berry and Howell, 2007; Ferraz and Finan, 2008; Snyder and Strömberg, 2010). The model also generates new empirical predictions which we hope future empirical work will test against data. Most interestingly, the model predicts the following:

> Disasters increase the electoral fortunes of incumbents whom, ex ante, the voters believe are low competence and decrease the electoral fortunes whom, ex ante, the voters believe are high competence.

These predictions suggest ways in which empirical researchers could look for evidence that behavior that, prima facie, seems to indicate voter irrationality might, in subtle ways, be consistent with voter rationality. As such, our model not only shows that the existing evidence on natural disasters does not entail the conclusion that voters are irrational, it suggests empirical ways forward for evaluating which interpretation of the data is more likely to be right.

A Proofs

A.1 Proofs for Section 1

Proof of Lemma 1. First, note that

$$\lim_{\overline{\pi}\to\infty} \Phi\left(\frac{\overline{\pi}-\underline{a}-m-r}{\sigma}\right) - \Phi\left(\frac{\overline{\pi}-\overline{a}-m-r}{\sigma}\right) = 0$$

and

$$\lim_{\overline{\pi}\to-\infty} \Phi\left(\frac{\overline{\pi}-\underline{a}-m-r}{\sigma}\right) - \Phi\left(\frac{\overline{\pi}-\overline{a}-m-r}{\sigma}\right) = 0.$$

Now, differentiating with respect to $\overline{\pi}$, we have that an interior maximum must satisfy:

$$\frac{1}{\sigma}\left[\phi\left(\frac{\overline{\pi}^*-\underline{a}-m-r}{\sigma}\right) - \phi\left(\frac{\overline{\pi}^*-\overline{a}-m-r}{\sigma}\right)\right] = 0.$$

Since the arguments inside the normal pdf's are not equal, this can only hold if:

$$\frac{\overline{\pi}^*-\underline{a}-m-r}{\sigma} = -\left(\frac{\overline{\pi}^*-\overline{a}-m-r}{\sigma}\right),$$

which implies

$$\pi^* = \frac{\overline{a}+\underline{a}}{2} + r + m,$$

as required.

All that remains is to show that there is an interior maximum. Since we've shown that there is a unique critical point and that the function goes to zero in either direction, it suffices to show that the function is positive somewhere in the interior. To see this, substitute for π^* to get:

$$\Phi\left(\frac{\overline{a}-\underline{a}}{2\sigma}\right) - \Phi\left(\frac{\underline{a}-\overline{a}}{2\sigma}\right),$$

which is clearly positive, since $\overline{a} > \underline{a}$. ■

Proof of Proposition 2. With a rational voter, the incumbent believes she is reelected if:

$$\theta_I + a_1 + \epsilon_1 + \nu_1 \geq m + a^* - \frac{1-\lambda}{\lambda} r + \nu_1.$$

Simplifying and rearranging, she believes she is reelected if:

$$\theta_I + a_1 + \epsilon_1 \geq m + a^* - \frac{1-\lambda}{\lambda} r.$$

With an irrational voter, the incumbent believes she is reelected if:

$$\theta_I + a_1 + \epsilon_1 + \nu_1 \geq m + a^* - \frac{1-\lambda}{\lambda} r.$$

Simplifying and rearranging, she believes she is reelected if:

$$\theta_I + a_1 + \epsilon_1 + \nu_1 \geq m + a^* - \frac{1-\lambda}{\lambda} r.$$

Now, define

$$\hat{\pi} \equiv m + \overline{a} - \frac{1-\lambda}{\lambda} r.$$

Under rational voters, there is an equilibrium with high effort if and only if

$$\Phi\left(\frac{\hat{\pi} - \underline{a} - m - r}{\sqrt{\sigma_\theta^2 + \sigma_\epsilon^2}}\right) - \Phi\left(\frac{\hat{\pi} - \overline{a} - m - r}{\sqrt{\sigma_\theta^2 + \sigma_\epsilon^2}}\right)$$

and under irrational voters, there is an equilibrium with high effort if and only if

$$\Phi\left(\frac{\hat{\pi} - \underline{a} - m - r}{\sqrt{\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_\nu^2}}\right) - \Phi\left(\frac{\hat{\pi} - \overline{a} - m - r}{\sqrt{\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_\nu^2}}\right).$$

Thus, it suffices to show that for r sufficiently large,

$$F(r, \sigma) \equiv \Phi\left(\frac{\hat{\pi} - \underline{a} - m - r}{\sigma}\right) - \Phi\left(\frac{\hat{\pi} - \overline{a} - m - r}{\sigma}\right)$$

is increasing in σ.

By the fundamental theorem of calculus, we have:

$$F(r, \sigma) = \int_{-\overline{a}}^{-\underline{a}} \frac{1}{\sigma} \phi\left(\frac{\hat{\pi} + \tilde{a} - m - r}{\sigma}\right) d\tilde{a}.$$

We make use of the following claim.

Claim 1 *Fix $\sigma_1 > \sigma_2 > 0$. There exists an $x^*(\sigma_1, \sigma_2)$ such that for $|x| > x^*(\sigma_1, \sigma_2)$:*

$$\frac{1}{\sigma_1} \phi\left(\frac{x}{\sigma_1}\right) > \frac{1}{\sigma_2} \phi\left(\frac{x}{\sigma_2}\right).$$

Now choose $r^* > 0$ such that

$$|\hat{\pi} - \underline{a} - m - r^*| = x^*(\sqrt{\sigma_\theta^2 + \sigma_\epsilon^2 + \sigma_\nu^2}, \sqrt{\sigma_\theta^2 + \sigma_\epsilon^2}).$$

Then, for all $r > r^*$, the integrand is larger with irrational voters than with rational voters, for all $a \in [-\overline{a}, -\underline{a}]$, thus the integral is larger with irrational voters than with rational voters.

All that remains is to prove the claim.

Proof of Claim. We want

$$\frac{1}{\sigma_1}\phi\left(\frac{x}{\sigma_1}\right) > \frac{1}{\sigma_2}\phi\left(\frac{x}{\sigma_2}\right).$$

This is true if and only if:

$$\frac{\sigma_2}{\sigma_1} > \frac{\phi\left(\frac{x}{\sigma_2}\right)}{\phi\left(\frac{x}{\sigma_1}\right)}.$$

The right-hand side of this inequality equals

$$\frac{e^{-\frac{1}{2}\frac{x^2}{\sigma_2^2}}}{e^{-\frac{1}{2}\frac{x^2}{\sigma_1^2}}},$$

which is equal to

$$e^{\frac{x^2}{2}\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2}\right)}.$$

Substituting this into the inequality and taking logs, the claim is true if:

$$\log \frac{\sigma_2}{\sigma_1} > \frac{x^2}{2}\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2}\right).$$

The left-hand side is constant in x. The right hand side goes to $-\infty$ as x goes to ∞. ■

■

A.2 Proofs for Section 2

The proofs of Propositions 3 and 4 share a common structure, based on the following two Lemmas.

Lemma 2 *Let f denote the cross-sectional density of expected competences of the winners* *of the first election. Then incumbents tend to be ahead in the following sense:*

$$f(m) > f(-m)$$

for all $m > 0$.

Proof. We use the standard appeal to the law of large numbers for a continuum of iid random variables to equate the cross-sectional distribution with the prior distribution of a single instance.

In any given district, a candidate, j, generated a campaign signal, $s_j^1 = \theta_j + \eta_j^1$. This signal is normally distributed with mean θ_j. Since the voters' prior about j's competence was normal with mean zero, standard results say that the voters' posterior beliefs are normal with mean

$$m_j = \lambda_1 s_j^1$$

and variance

$$\sigma_1^2 = \lambda_1 \sigma_\eta^2,$$

with $\lambda_1 = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2}$. Moreover, standard results on Bayesian updating of normal random variables imply that the prior distribution of m_c is normal with mean 0 and variance

$$\hat{\sigma}_1^2 = \frac{(\sigma_\theta^2)^2}{\sigma_\theta^2 + \sigma_\eta^2}.$$

Since the median voter has bias $b = 0$, he elects the candidate in the first election with greater expected competence. This implies that $m_I = \max\{m_{l_1}, m_{r_1}\}$. Call the prior density of m_I, f. Since it is the density of the first-order statistic of two normally distributed random variables, it is given by:

$$f(m) = 2\phi\left(\frac{m}{\sigma_1}\right)\Phi\left(\frac{m}{\sigma_1}\right).$$

Now fix $m > 0$ and consider the ratio

$$\frac{f(m)}{f(-m)} = \frac{\phi\left(\frac{m}{\sigma_1}\right)\Phi\left(\frac{m}{\sigma_1}\right)}{\phi\left(\frac{-m}{\sigma_1}\right)\Phi\left(\frac{-m}{\sigma_1}\right)}.$$

Symmetry of the normal pdf implies $\phi(m/\sigma_1) = \phi(-m/\sigma_1)$. Monotonicity of the normal cdf implies $\Phi(m/\sigma_1) > \Phi(-m/\sigma_1)$. Together, these two facts imply that $f(m)/f(-m) > 1$, establishing the result. ■

Lemma 3 *Consider a function $h(\cdot)$ satisfying the following properties:*

1. $h(x) = -h(-x)$
2. $h(x) < 0$ *for* $x > 0$.

If $f(x) > f(-x)$ *for all* $x > 0$, *then*

$$\int_{-\infty}^{\infty} h(x) f(x)\, dx < 0.$$

Proof. Split the integral into two parts:

$$\int_{-\infty}^{0} h(x) f(x)\, dx + \int_{0}^{\infty} h(x) f(x)\, dx.$$

Make the change of variables in $x \mapsto -x$ in the first integral to get:

$$\int_{0}^{\infty} h(-x) f(-x)\, dx + \int_{0}^{\infty} h(x) f(x)\, dx.$$

Using $h(x) = -h(-x)$, this can be rewritten:

$$\int_{0}^{\infty} h(x) \left[f(x) - f(-x) \right]\, dx,$$

which is negative because, for $x > 0$, we have $h(x)$ is negative and $f(x) > f(-x)$. ■

Proof of Proposition 3. The cross-sectional average reelection probability in state ω is

$$\int_{-\infty}^{\infty} \left[1 - \Phi\left(\frac{-m_I}{\sigma_\omega} \right) \right] f(m_I)\, dm_I.$$

We need to show that this integral is larger when $\sigma_\omega = \sigma_G$ than when $\sigma_\omega = \sigma_B$. Since $\sigma_B > \sigma_G$, it suffices to show that:

$$\int_{-\infty}^{\infty} \left[1 - \Phi\left(\frac{-m_I}{\sigma} \right) \right] f(m_I)\, dm_I$$

is decreasing in σ.

Differentiate the integral with respect to σ to get

$$\int_{-\infty}^{\infty} -\phi\left(\frac{-m_I}{\sigma}\right)\left(\frac{m_I}{\sigma^2}\right) f(m_I)\, dm_I.$$

Now note that the function $h(m_I) = -\phi\left(\frac{-m_I}{\sigma}\right)\left(\frac{m_I}{\sigma^2}\right)$ has the two properties from Lemma 3 and that $f(m_I) > f(-m_I)$ by Lemma 2. Hence Lemma 3 implies the integral is negative, as required. ■

Proof of Proposition 4. Define

$$V(\sigma, m_I) = \int_{-\infty}^{\infty} [1 - \Phi(-\overline{m}_I)] \frac{1}{\sigma}\phi\left(\frac{\overline{m}_I - m_I}{\sigma_\omega}\right) d\overline{m}_I.$$

Conditional on an m_I, the expected incumbent vote share in state ω is $V(\sigma_\omega, m_I)$. Define $\Delta(m_I)$ as the difference in expected vote share in the good state relative to the bad state. That is:

$$\Delta(m_I) \equiv V(\sigma_G, m_I) - V(\sigma_B, m_I).$$

Lemma 4 *1. If $m_I > 0$, then $\Delta(m_I) < 0$.*

2. $\Delta(m_I) = -\Delta(-m_I)$.

Proof. It will be useful to study $V(\sigma, m_I)$ as a function of σ. Integrate by parts to get:

$$[1 - \Phi(-\overline{m}_I)]\, \Phi\left(\frac{\overline{m}_I - m_I}{\sigma}\right)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \Phi\left(\frac{\overline{m}_I - m_I}{\sigma}\right)\phi\left(\overline{m}_I\right)\, d\overline{m}_I.$$

The first term is zero, so this can be rewritten:

$$\int_{-\infty}^{\infty} -\Phi\left(\frac{\overline{m}_I - m_I}{\sigma}\right)\phi\left(\overline{m}_I\right)\, d\overline{m}_I.$$

Now differentiate with respect to σ to get

$$\int_{-\infty}^{\infty} \phi\left(\frac{\overline{m}_I - m_I}{\sigma}\right)\left(\frac{\overline{m}_I - m_I}{\sigma^2}\right)\phi\left(\overline{m}_I\right)\, d\overline{m}_I.$$

This can be rewritten as:

$$\int_{-\infty}^{m} \phi\left(\frac{\overline{m}_I - m_I}{\sigma}\right)\left(\frac{\overline{m}_I - m_I}{\sigma^2}\right)\phi\left(\overline{m}_I\right)\, d\overline{m}_I + \int_{m}^{\infty} \phi\left(\frac{\overline{m}_I - m_I}{\sigma}\right)\left(\frac{\overline{m}_I - m_I}{\sigma^2}\right)\phi\left(\overline{m}_I\right)\, d\overline{m}_I.$$

Now make a change of variables as follows. Let $y \mapsto \frac{\overline{m}_I - m_I}{\sigma}$. This implies that $\overline{m}_I \mapsto y\sigma + m_I$ and that $d\overline{m}_I \mapsto \sigma dy$. Hence, we can rewrite the derivative above as follows:

$$\int_{-\infty}^{0} \phi(y)\, y\phi(y\sigma + m_I)\; dy + \int_{0}^{\infty} \phi(y)\, y\phi(y\sigma + m_I)\; dy.$$

This can again be rewritten:

$$-\int_{0}^{\infty} \phi(y)\, y\phi(m_I - y\sigma)\; dy + \int_{0}^{\infty} \phi(y)\, y\phi(m_I + y\sigma)\; dy.$$

Gathering terms, this can be rewritten

$$\int_{0}^{\infty} \phi(y)\, y\left[\phi(m_I + y\sigma) - \phi(m_I - y\sigma)\right]\; dy.$$

Now we turn to the two enumerated points in the statement of the lemma.

1. Since $\sigma_B > \sigma_G$, it suffices for the derivative above to be negative. This is clear from the fact that the normal density is symmetric and single peaked around zero and the fact that, for $m_I > 0$, $|m_I + y\sigma| < |m_I - y\sigma|$.

2. Using the fundamental theorem of calculus, we can write:

$$\Delta(m_I) = \int_{\sigma_B}^{\sigma_G} \frac{\partial}{\partial\sigma} V(\sigma, m_I)\, d\sigma.$$

   Using the derivation above, this implies that

$$\Delta(m_I) = \int_{\sigma_B}^{\sigma_G} \int_{0}^{\infty} \phi(y)\, y\left[\phi(m_I + y\sigma) - \phi(m_I - y\sigma)\right]\; dy\, d\sigma.$$

   Hence, it suffices to show that

$$\left[\phi(m_I + y\sigma) - \phi(m_I - y\sigma)\right] = -\left[\phi(-m_I + y\sigma) - \phi(-m_I - y\sigma)\right],$$

   which follows from the symmetry of the normal density.

■

The cross-sectional average of the difference in vote share in the good and bad states is:

$$\int_{-\infty}^{\infty} \Delta(m_I) f(m_I)\, dm_I.$$

Lemma 4 implies that Δ has both properties from Lemma 3. Hence, the result follows from Lemmas 2 and 3. ■

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