"A simple model of firm heterogeneity, international trade, and wages," Stephen Ross Yeaple (2005)

Firm Heterogeneity Assumptions:

- Instead of Eaton and Kortum type of randomly assigned technology, firms in Yeaple begin as homogeneous firms but subsequently choose their technology among three type : $\{Y, L, H\}$
- The only input to production is labor.

Market Structure

- Labor market is perfectly competitive
- Market for Y perfectly competitive
- Market for X monopolistic competitive

Variables

$$
\begin{aligned}
M &= \text{continuum of workers with mass M} \\
Z &= \text{workers are differentiated by skill level Z} \\
G(Z) &= \text{distribution of skills in the population} \\
\rho_j(Z) &= \text{the amount of good a worker of skill Z can produce with technology j, where j} \in \{Y, L, H\} \\
& \quad \text{and } \rho \text{ is increasing in Z} \\
P_x &= \text{price index of composite good X}
\end{aligned}
$$

Closed Economy Equilibrium

The equilibrium is defined as the $\{Z_1, Z_2, P_x, N_x, N_y\}$, such that given the prices consumers maximize utility, and firms maximize profits, and the cuttoffs Z_1, Z_2 uniquely determine the optimal allocation of workers to each sector and use of technology. From that, we know the total output for each sector and variety (each firm produce different varieties), thus number of firms working in the sector Y, H, and L.

Consumer Preference

Consumer preference is given by a Cobb-Douglas preference over a homogeneous good Y, and a composite good X, which is combined by CES.

$$
\begin{aligned}
U &= (1-\beta)\ln Y + \beta \ln X \\
\text{where } X &= \left[\int_0^N x(i)^\alpha di\right]^{1/\alpha} \\
\text{and } \sigma &= \frac{1}{1-\alpha} > 1
\end{aligned}
$$

The aggregate price of good X is:

$$
P_x = \left[\int_0^N p(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}
$$

Demand for any type i is then:

$$x(i) = \left(\frac{\beta E}{P_x}\right)\left(\frac{p(i)}{P_x}\right)^{-\sigma}$$

where $\frac{\beta E}{P_x}$ is total spending on the composite good X

Firm Production

A firm using technology j hiring workers produce with unit cost C_j. Note that this unit cost is the same for the firms using the same technology, and does not depend on worker skill level because in a perfectly competitive labor market, the wage distribution over Z adjusts to equalize the unit costs of all firms using the same technology. And recall that $\rho(.)$ is inceasing function in Z.

$$C_j = \frac{W(Z)}{\rho_j(Z)}$$

Solving for $\mathbf{Z}_1$ and $\mathbf{Z}_2$

Once we know $\{Z_1, Z_2\}$ we will know the optimal allocation of workers to each of the three sectors and all the other equilibrium variables. So we first solving for these two cutoffs.

We need equations that will allow us to solve for these cutoff. First, we need to find the distribution of wages:

$$W(Z) = \begin{array}{cc} C_y\rho_y(Z) & 0 \le Z \le Z_1 \\ C_L\rho_L(Z) & Z_1 \le Z \le Z_2 \\ C_H\rho_H(Z) & Z_2 \le Z \end{array}$$

Equalization of wages/unit costs across the cutoffs give us the following equations:

$$\begin{aligned} C_y &= P_y = 1 \\ C_L &= \frac{\rho_y(Z_1)}{\rho_L(Z_1)} < 1 \\ C_H &= \frac{\rho_L(Z_2)}{\rho_H(Z_2)}\frac{\rho_y(Z_1)}{\rho_L(Z_1)} < C_L \end{aligned}$$

From this relation, we can already observe one fact:

Firms that use the High technology must earn higher revenues than firms that use Low technology.

This is because by CES, the ratio of firm revenue is only a ratio of unit costs:

$$\begin{aligned} \frac{R_H}{R_L} &= \left(\frac{P_H}{P_L}\right)^{1-\sigma} = \left(\frac{C_H}{C_L}\right)^{1-\sigma} > 1 \\ \text{where } \frac{C_H}{C_L} &> 1 \\ \text{and } \sigma &> 1 \end{aligned}$$

Now, to solve for Z_1 and Z_2, we know that free entry / zero profit condition guarantees that revenues must equal to the total labor costs:

$$\begin{aligned}
\text{(Zero Profit Condition)} \quad &\Rightarrow \quad R = \text{total costs} \\
Px &= C(x+F) \qquad (1) \\
Px - Cx &= CF \qquad (2) \\
xC(\frac{1}{\sigma-1}) &= CF \qquad (3) \\
x\frac{\sigma-1}{\sigma}P(\frac{1}{\sigma-1}) &= CF \qquad (4) \\
\frac{R}{\sigma} &= CF \qquad (5)
\end{aligned}$$

If both H and L firms satisfy zero profit condition, we have an equation that will eventually allows us to pin down Z_2 :

$$\frac{R_H}{R_L} = \frac{C_H F_H}{C_L F_L}$$

Note that from the CES preference, we have two equations of $\frac{R_H}{R_L}$:

$$\begin{aligned}
(1) \quad \frac{R_H}{R_L} &= \frac{C_H F_H}{C_L F_L} \\
(2) \quad \frac{R_H}{R_L} &= \left(\frac{C_H}{C_L}\right)^{1-\sigma}
\end{aligned}$$

Solving for $\frac{C_H}{C_L}$ gives:

$$\begin{aligned}
\left(\frac{C_H}{C_L}\right)^{1-\sigma} &= \frac{C_H F_H}{C_L F_L} \\
\frac{C_H}{C_L} &= \left(\frac{C_H F_H}{C_L F_L}\right)^{\frac{1}{1-\sigma}} \\
\left(\frac{C_H}{C_L}\right)^{\frac{-\sigma}{1-\sigma}} &= \left(\frac{F_H}{F_L}\right)^{\frac{1}{1-\sigma}} \\
\frac{C_H}{C_L} &= \left(\frac{F_H}{F_L}\right)^{\frac{1}{-\sigma}}
\end{aligned}$$

Substitute for $\frac{C_H}{C_L}$ in terms of Z_2 :

$$\boxed{\frac{C_H}{C_L} = \frac{\rho_L(Z_2)}{\rho_H(Z_2)} = \left(\frac{F_H}{F_L}\right)^{\frac{1}{-\sigma}} \quad \text{This pins down } Z_2}$$

Now we need another equation to pin down Z_1. We need an equation that connects Z_1 and Z_2. Here we use the market clearing condition for good Y.

(Demand for Y) $\Rightarrow$ Y=(1-β)E=(1-β)M$\overline{w}$

(Supply of Y) $\Rightarrow$ Y= wages paid to workers in Y=total ouput in that sector = M $\int_0^{Z1} \rho_y(Z)dG(Z)$

where:

$$\overline{w} = \int_0^{Z1} \rho_y(Z)dG(Z) + C_L \int_{Z1}^{Z2} \rho_L(Z)dG(Z) + C_H \int_{Z2}^{\infty} \rho_H(Z)dG(Z)$$

And the supply condition came from (I think):

$$\begin{aligned} & \text{The total good produced in the economy is} \\ M\bar{x} &= \text{\# of ppl producing Y} \cdot \ \rho_y + \text{\# of ppl producing } x_H \cdot \ \rho_H + \text{\# of ppl producing } x_L \ \cdot \rho_L \\ &= M(\rho_y(\text{share of pop producing y}) + \rho_H(\text{share of pop producing H}) + \rho_L(\text{share of pop producing L})) \\ &= M\int_0^{Z1} \rho_y(Z)dG(Z) + M\int_{Z1}^{Z2} \rho_L(Z)dG(Z) + M\int_{Z2}^{\infty} \rho_H(Z)dG(Z)) \\ &= Y + X_H + X_L \end{aligned}$$

Combining gives the market clearing condition for good Y:

$$\text{(1-}\beta\text{)M}\left(\int_0^{Z1} \rho_y(Z)dG(Z) + C_L \int_{Z1}^{Z2} \rho_L(Z)dG(Z) + C_H \int_{Z2}^{\infty} \rho_H(Z)dG(Z)\right) = \text{M}\int_0^{Z1} \rho_y(Z)dG(Z)$$

Defining:

$$\begin{aligned} S(Z_1) &= C_L \ \ \text{(note this is decreases in } Z_1\text{)} \\ A(Z_2) &= \frac{C_H}{C_L} \ \ \text{(note this is decreases in } Z_2\text{)} \end{aligned}$$

Dividing the former long expression by C_L and isolating terms:

$$\frac{1}{S(Z_1)}\int_0^{Z1} \rho_y(Z)dG(Z) = \frac{(1\text{-}\beta)}{S(Z_1)}\left(\int_0^{Z1} \rho_y(Z)dG(Z)\right) + (1\text{-}\beta)\int_{Z1}^{Z2} \rho_L(Z)dG(Z) + (1\text{-}\beta)A(Z_2)\int_{Z2}^{\infty} \rho_H(Z)dG(Z)$$

$$\boxed{\frac{\beta}{(1\text{-}\beta)S(Z_1)}\left(\int_0^{Z1} \rho_y(Z)dG(Z)\right) = \int_{Z1}^{Z2} \rho_L(Z)dG(Z) + A(Z_2)\int_{Z2}^{\infty} \rho_H(Z)dG(Z)}$$

This equation gives us the second equation that pins down Z_1 after pinning down Z_2. Moreover, it suggests if Z_1 increases, Z_2 neccessarily decreases.

Interpretation

An increase of Z_1 paired with a decrease in Z_2 can be interpreted as the reallocation of workers in each sector. It means that the L sector (moderately skilled sector) is squeezed when Z_1 increases; employment in both the Y and H sector increases, and employment in the L sector decreases. The L sector is interpreted as the manufacture sector by the author.

Closing the Model

We can derive the rest of the equilibrium variables $\{N_H, N_L, P_x\}$:

We need to find x_H:

$$\begin{aligned} \text{(We have derived earlier from free entry)} \quad \frac{R_H}{\sigma} &= C_H F_H \\ x_H &= \frac{\sigma C_H F_H}{p_H} \\ x_H &= \frac{\sigma C_H F_H}{\frac{\sigma}{\sigma-1} C_H} \\ x_H &= (\sigma - 1)F_H \end{aligned}$$

The important part is to notice that all the firms using technology H must produce the same output x_H since they face the same price and share the same unit costs (although they might employ different Z, recall that wage and ρ adjust accordingly such they the marginal cost must be the same for all firms, just look at the equation $C_H = \frac{W(Z)}{\rho(Z)}$) In other words, the total effective labor required for all firms with this technology is the same.

Now we also know the total amount of good produced by technology H in the whole economy:

$$X_H = M \int_{Z2}^{\infty} \rho_H(Z) dG(Z)$$

Note that all firms actually produce $\widetilde{x_H} = x_H + F_H$ due to the fixed costs (expressed in terms of unmarketable goods). Divide the total output by this give N_H :

$$\boxed{N_H = \frac{M}{\sigma F_H} \int_{Z2}^{\infty} \rho_H(Z) dG(Z)}$$

Similarly for N_L :

$$\boxed{N_L = \frac{M}{\sigma F_L} \int_{Z1}^{Z2} \rho_L(Z) dG(Z)}$$

And for the price index for the composite good X, P_x :

$$\begin{aligned}
P_x &= \left[\int_0^N p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \quad \text{(note that } \frac{1}{\alpha} = \frac{\sigma}{\sigma - 1} \text{is the markup} \\
&= \frac{1}{\alpha} \left[\int_0^N c(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \\
&= \frac{1}{\alpha} \left[\int_0^{Nh} C_H^{1-\sigma} + \int_0^{Nl} C_L^{1-\sigma} + \int_0^{Ny} C_y^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
&= \frac{1}{\alpha} \left[N_H C_H^{1-\sigma} + N_L C_L^{1-\sigma} + \int_0^{Ny} C_y^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
&= \frac{1}{\alpha} \left[\frac{M}{\sigma F_H} \left(\int_{Z2}^{\infty} \rho_H(Z) dG(Z) \right) C_H^{1-\sigma} + \frac{M}{\sigma F_L} \left(\int_{Z1}^{Z2} \rho_L(Z) dG(Z) \right) C_L^{1-\sigma} + N_y \right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

Using:

$$\begin{aligned}
A(Z_2) &= \frac{C_H}{C_L} \\
S(Z_1) &= C_L \\
\frac{C_H}{C_L} &= \left(\frac{F_H}{F_L} \right)^{\frac{-1}{\sigma}} \\
& \text{and that long equation with } \mathrm{Z}_1 \text{ and } \mathrm{Z}_2
\end{aligned}$$

Dividing and multiplying by $C_L^{1-\sigma}$:

$$= \frac{C_L}{\alpha}\left[\frac{M}{\sigma F_H}\left(\int_{Z2}^{\infty}\rho_H(Z)dG(Z)\right)A(Z_2)^{1-\sigma}+\frac{M}{\sigma F_L}\left(\int_{Z1}^{Z2}\rho_L(Z)dG(Z)\right)+\frac{N_y}{C_L^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{S(Z_1)}{\alpha}\left[\frac{M}{\sigma F_H\left(\frac{F_H}{F_L}\right)^{-1}}\left(\int_{Z2}^{\infty}\rho_H(Z)dG(Z)\right)A(Z_2)+\frac{M}{\sigma F_L}\left(\int_{Z1}^{Z2}\rho_L(Z)dG(Z)\right)+\frac{N_y}{C_L^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{S(Z_1)}{\alpha}\left[\frac{M}{\sigma F_L}\left(\int_{Z2}^{\infty}\rho_H(Z)dG(Z)\right)A(Z_2)+\frac{M}{\sigma F_L}\left(\int_{Z1}^{Z2}\rho_L(Z)dG(Z)\right)+\frac{N_y}{C_L^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{S(Z_1)}{\alpha}\left[\frac{M}{\sigma F_L}\frac{\beta}{(1\text{-}\beta)S(Z_1)}\left(\int_{0}^{Z1}\rho_y(Z)dG(Z)\right)\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{S(Z_1)^{\frac{\sigma}{\sigma-1}}}{\alpha}\left[\frac{M}{\sigma F_L}\frac{\beta}{(1\text{-}\beta)}\left(\int_{0}^{Z1}\rho_y(Z)dG(Z)\right)\right]^{\frac{1}{1-\sigma}} \quad \text{(dunno where the 3rd term go)}$$

Interpretations

From the price index of the composite good, we can conclude the following relations between the cutoff Z and Prices:

(1) $Z_1 \uparrow$,(and $Z_2 \downarrow$) unambiguously $P_x \downarrow$ because $S(Z_1) \downarrow$ and the integration term gets smaller; In other words, when more firms are using the high techology, and the moderately skilled sector get squeezed, prices of the composite good will decrease

(2) As $M \uparrow$, $P_x \downarrow$; In other words, when size of the country increases, price of the composite good decrease. This is probably due to cheaper labor costs therefore more firms are able to accept the fixed costs to adopt the high technology

(3) As $\sigma \uparrow$ (elasticity of substitution increases), an increase in the tradable sector will produce larger decline in price – the effects would be bigger. This is because people are more responsive to decline in prices.

Open economy

The modifications to the closed economy equilibrium are:

- Y is not traded, so market clearing equation for Y is the same
- trade is costly, with fixed cost F_x and variable cost $\tau > 1$, and relative to non-trade revenue, revenue with trade is reduced by proportion $\tau^{1-\sigma}$, as shown here:

$$\frac{R_t}{R} = \left(\frac{C\tau}{C}\right)^{1-\sigma} = \tau^{1-\sigma}$$

- Two identical economies trade⇒demand are the same. So if a firm supplies both domestic and foreign goods, his revenue is $R + R\tau^{1-\sigma} = R(1+\tau^{1-\sigma})$
- The papers says (I can't figure out why) that no firms export if $F_H < F_X\tau^{\sigma-1}$and all firms export if $F_L > F_X\tau^{\sigma-1}$. The case of interest is when all H firms export and all L don't export

$$F_H > F_X\tau^{\sigma-1} > F_L$$

Assuming that the above condition holds and all H firms export, the equation that pinned down Z_2 now change because the zero profit condition changed:

$$\frac{R_H(1+\tau^{1-\sigma})}{R_L} = \left(\frac{C_H}{C_L}\right)^{1-\sigma}(1+\tau^{1-\sigma}) = \frac{C_H}{C_L}\frac{(F_H+F_X)}{F_L}$$

Solving for $\frac{C_H}{C_L}$ in terms of Z_2 the new zero profit equilibrium condition is

$$\boxed{\frac{C_H}{C_L}=\frac{\rho_L(Z_2)}{\rho_H(Z_2)}=\mathrm{A}(\mathrm{Z}_2)=\left(\frac{F_H+F_X}{F_L(1+\tau^{1-\sigma})}\right)^{-\frac{1}{\sigma}}}$$

Results

What is the effect of opening to trade on the cutoff? $Z_2 \downarrow$

Proof:

$$\begin{aligned} A(Z_2) &= \left(\frac{F_H+F_X}{F_L(1+\tau^{1-\sigma})}\right)^{-\frac{1}{\sigma}} \\ &> \left(\frac{F_H+F_H\tau^{1-\sigma}}{F_L(1+\tau^{1-\sigma})}\right)^{-\frac{1}{\sigma}} = \left(\frac{F_H(1+\tau^{1-\sigma})}{F_L(1+\tau^{1-\sigma})}\right)^{-\frac{1}{\sigma}} \\ &= \left(\frac{F_H}{F_L}\right)^{-\frac{1}{\sigma}} \quad \text{(this is the non trade condition)} \end{aligned}$$

Since $A(Z_2)$ is decreasing in Z_2, and the RHS is increasing in Z_2 , thus Z_2 must decrease. In other words, trade would lead to increase adoption of high technology and an increase in the employment in non-tradable sector. The moderately skilled sector will shrink. (We can see that N_L under trade will be smaller).

The number of firms in high technology sector is

$$N_H = \frac{M}{\sigma(F_H+F_X)}\int_{Z2}^{\infty}\rho_H(z)dG(Z)$$

Here we can see two offsetting effects:

1) Z_2 decreases, which increases N_H

2.) F_X decreases N_H

It is unclear whether N_H increases as a result of trade.

Effect of Trade Liberalization, a la decrease in τ or decrease in F_X

We want to know the effect of trade liberalization on:

- share of labor force working in each sector
- revenue per worker in each sector
- unit costs of production
- trade volume
- wages

1. Share of labor force working in each sector

From the equilibrium condition for Z_2,

$$A(Z_2) = \left(\frac{F_H+F_X}{F_L(1+\tau^{1-\sigma})}\right)^{-\frac{1}{\sigma}}$$

we can see that a decrease in τ will lead to decrease in Z_2. This means that Z_1 also increases. This pair of effects means that employment in high skill sector increases, increasing the number for firms that enter export market. It also rases the wage of the most highly skilled workers in terms of the numeraire (why???), leaving the wage of all other workers unchanged. Total GDP rises, which means that demand for non-tradable goods Y also rises, thus employment in sector Y increases.

2. Average revenue per worker (or observed labor productivity)
In Y sector, average revenue per worker is given by

$$\frac{R_y}{\text{\# of workers in Y}} = \frac{Y}{M\int_0^{Z1} dG(Z)} = \frac{1}{G(Z_1)}\int_0^{Z1} \rho_Y(Z)dG(Z)$$

The author claims that this is increasing in Z_1. Thus average revenue for workers in non-tradable sector increases. The intuition is that reduction in trade costs pushes least productive workers in X sector into Y sector, but since they are above average productivity in Y sector, this increases the value of output per worker in the Y sector.

In the X sector, average revenue per worker is given by

$$\frac{\beta}{1-\beta}\frac{1}{1-G(Z_1)}\int_0^{Z1} \rho_Y(Z)dG(Z)$$

The author claims that this is strictly increasing in Z_1. Thus average output in sector X also rises. This is because revenues increase with decrease in trade costs. It matches empirical evidence that reduction in transport costs is associated with greater average sales revenue per labor input.

3. Unit costs of production
Recall the following relations

$$\text{(unit cost definition and equalization of wages across } Z_1) \quad C_L = \frac{\rho_y(Z_1)}{\rho_L((Z_1)} \quad C_H = \frac{\rho_y(Z_1)}{\rho_L((Z_1)}\frac{\rho_L(Z_2)}{\rho_H((Z_2)}$$

$$\text{(Comparative advantage assumption)} \quad \frac{\partial\rho_H(Z)}{\partial(Z)}\frac{1}{\rho_H(Z)} > \frac{\partial\rho_L(Z)}{\partial(Z)}\frac{1}{\rho_L(Z)} > \frac{\partial\rho_Y(Z)}{\partial(Z)}\frac{1}{\rho_y(Z)} > 0$$

With trade Z_1 increases, and the comparative advantage condition tells us then that C_L must decrase. From the zero profit condition for C_L,

$$\text{(Zero profit condition)} \quad R_L = \sigma C_L F_L$$

We can also see that revenues of L firms must decrease because C_L has decreased.

Similarly, we can observe from the zero profit condition of H firms that revenues for H firms must increase, and unit costs

$$\text{(Zero profit condition)} \quad R_H(1+\tau)^{1-\sigma} = \sigma C_H(F_H + F_X)$$

The author says that it can be shown that domestic sales of an H firm, R_H must fall with decrease in trade costs.

4. Trade Volume
Since N_H and the export revenue of each exporter ($R_H\tau^{1-\sigma}$) have risen, trade volume between two identical countries must also increase.

5. Wage Distribution
The wage distribution is described by three equations, which are functions of C_L and C_H, which in turn are functions of $Z_1$and Z_2. Since the two cuffoffs have shifted, we have to regraph the distribution. The slopes of the 3 sectors are still the same, but since the thresholds have shifted, you can clearly see that the moderately skilled people have lower wage in terms of the numeraire good , and the wage of the people working in sector Y has not changed, by the wage of people working in the H technology sector has increased.