

## Econometrics A

### Problem Set #4

**Date Due:** Thursday, May 15

1. Stock and Watson, Exercises 6.9, 6.11
2. Let  $A$  be an  $n \times n$  matrix and  $b \in \mathbf{R}^n$ . Suppose that the columns of  $A$  are linearly independent. Consider the system of equations

$$Ax = b . \tag{1}$$

Recall (from the class notes) that there must exist at least one solution to (1). In this exercise, you will prove that the solutions is in fact unique.

- (a) Let  $x^*$  and  $\tilde{x}$  be two distinct solutions to (1). Show that

$$Ax^* = A\tilde{x} . \tag{2}$$

- (b) Show that (2) contradicts the assumption that the columns of  $A$  are linearly independent. Conclude that there must exist exactly one solution to (1).
3. Consider the following  $3 \times 3$  matrix:

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} .$$

- (a) Find the determinant of  $A$ .
  - (b) Find  $A^{-1}$ .
4. Let  $(Y_1, X_{1,1}, X_{1,2}), \dots, (Y_n, X_{n,1}, X_{n,2})$  be an i.i.d. sample from  $(Y, X_1, X_2)$  satisfying

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U ,$$

where  $(1, X_1, X_2)$  is not perfectly colinear,  $E[Y^4] < \infty$  and  $E[X_j^4] < \infty$  for  $1 \leq j \leq 2$ . You the researcher wish to interpret this regression as the best linear predictor of  $Y$  given  $X_1$  and  $X_2$ .

- (a) How would you interpret  $U$ ?
- (b) Is it necessarily true that  $E[U] = 0$ ? What about  $E[X_1 U] = 0$  or  $E[X_2 U] = 0$ ?

(c) Show that

$$\text{Cov}[X_1, Y] = \beta_1 \text{Var}[X_1] + \beta_2 \text{Cov}[X_1, X_2] .$$

(d) Suppose the researcher estimates the equation

$$Y = \beta_0^* + \beta_1^* X_1 + U^*$$

by OLS. Show that

$$\hat{\beta}_1^* \xrightarrow{P} \beta_1 + \beta_2 \frac{\text{Cov}[X_1, X_2]}{\text{Var}[X_1]} .$$

(e) Under what conditions will it be true that  $\hat{\beta}_1^*$  is consistent for  $\beta_1$ ?

(f) Is it necessarily true that

$$\sum_{i=1}^n X_{i,j} \hat{U}_i^* = 0$$

for  $1 \leq j \leq 2$ ? What about

$$\sum_{i=1}^n \hat{U}_i^* = 0 ?$$

5. Levitt and Venkatesh investigate Chicago street gangs by modelling the determinants of the wages paid to “foot soldiers” in the gangs. They estimated the following regression,

$$wage = \beta_0 + \beta_1 war + \beta_2 large + U ,$$

where

$$\begin{aligned} wage &= \text{the hourly wage paid to the foot soldiers} \\ war &= \begin{cases} 1 & \text{if the gang is currently involved in a gang war} \\ 0 & \text{otherwise} \end{cases} \\ large &= \begin{cases} 1 & \text{if the gang is “large”} \\ 0 & \text{otherwise} \end{cases} , \end{aligned}$$

and found that

$$\hat{\beta}_0 = 1.83, \hat{\beta}_1 = 1.3, \hat{\beta}_2 = 4.07$$

(a) According to their estimates, what is effect of a gang war on wages?

(b) Why did they include a dummy variable for the gang being large but not also for the gang being small?

- (c) How would you modify their regression to allow for the effects of a gang war on wages to be different for members of large versus small gangs?

6. Consider the following model of the determinants of wages:

$$\begin{aligned} \log \text{wage} \\ = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{pareduc} \times \text{educ} + \beta_3 \text{experience} + \beta_4 \text{experience}^2 + U , \end{aligned}$$

where

*experience* = years of individual work experience

*pareduc* = sum of the mother's education and father's years of education

*educ* = years of own education .

- (a) What is the percent return on wages from another year of own education? Does it depend on the level of own education? Does it depend on parents' education? Does it depend on the level of work experience?
- (b) What is the percent return on wages from another year of work experience? Does it depend on the level of own education? Does it depend on parents' education? Does it depend on the level of work experience?