

Econometrics A

Problem Set #1

Date Due: Thursday, April 9

1. Stock and Watson, Exercises 2.6, 2.8, 2.9, 2.22
2. Let X denote the annual salary of scientists in the United States, measured in thousands of dollars. Suppose that $E[X] = 48.8$ and that the standard deviation of X is 12.1. Let Y denote the annual salary of scientists in the United States measured in dollars. What is $E[Y]$ and what is the standard deviation of Y ?
3. Let $X \sim \text{Bernoulli}(p)$. Define $Z = 3^X - 1$.
 - (a) Is Z a random variable? Why?
 - (b) Show that $E[Z] = 2p$.
 - (c) Show that $E[Z^2] = 4p$.
 - (d) What is the variance of Z ?

4. Suppose that X is a discrete random variable taking the values $-1, 0, 1$, with

$$P\{X = -1\} = P\{X = 0\} = P\{X = 1\} = 1/3 .$$

Let $Y = X^2$. Show that $\text{Cov}[X, Y] = 0$, but that X and Y are not independent.

5. Let GPA denote a random variable for the college student's grade point average, and SAT denote a random variable for the college student's SAT score. Suppose that there is the following relationship between GPA and SAT: $E[\text{GPA}|\text{SAT}] = .70 + .002\text{SAT}$.
 - (a) What is the expected GPA when SAT= 750? What is the expected GPA when SAT= 1500?
 - (b) If $E[\text{SAT}] = 1000$, what is $E[\text{GPA}]$?
6. Let $Y = a + bX + U$, where X and U are random variables and a and b are constants. Assume that $E[U|X] = 0$ and $\text{Var}[U|X] = X^2$.
 - (a) Is Y a random variable? Why?
 - (b) Is U mean independent of X ? Why? Is U independent of X ? Why?
 - (c) Show that $E[U] = 0$ and $\text{Var}[U] = E[X^2]$.
 - (d) Show that $E[Y|X] = a + bX$, and that $E[Y] = a + bE[X]$.
 - (e) Show that $\text{Var}[Y|X] = X^2$, and that $\text{Var}[Y] = b^2\text{Var}[X] + E[X^2]$.