Econometrics A

Practice Problems #1

1. Suppose that (X, Y) is a random vector, where $X \geq 1$, $Y \leq M$ (each with probability one) and $E[Y|X] = \theta X$. Let $Z = Y/X$.

   (a) Is Z a random variable? Why?

   (b) Show that $E[Z] = \theta$.

   (c) Let $(X_1, Y_1), ..., (X_n, Y_n)$ be an i.i.d. sample from (X, Y). Let $\hat{\theta}_n = \frac{1}{n}\sum_{i=1}^{n} \frac{Y_i}{X_i}$. Show that:

      i. $\hat{\theta}_n$ is an unbiased estimator of θ.

      ii. $\hat{\theta}_n$ is a consistent estimator of θ.

2. Let $X_1, X_2, ..., X_n$ be an i.i.d. sample from X, where $\text{Var}[X] < \infty$. Suppose you are interested in estimating $E[X]$. Consider two different estimators,

$$\begin{aligned} \hat{\theta}_{1,n} &= \bar{X}_n + \frac{1}{n} \\ \hat{\theta}_{1,n} &= \frac{1}{4}X_1 + \frac{3}{4}X_n \ . \end{aligned}$$

   (a) Show that $\hat{\theta}_{1,n}$ is a biased estimator of $E[X]$.

   (b) Show that $\hat{\theta}_{2,n}$ is an unbiased estimator of $E[X]$.

   (c) What is $\text{Var}(\hat{\theta}_{1,n})$?

   (d) What is $\text{Var}(\hat{\theta}_{2,n})$? How does it compare with the answer to part (c)?

   (e) Show that $\hat{\theta}_{1,n}$ is a consistent estimator of $E[X]$.

3. Let $X_1, X_2, ..., X_n$ be an i.i.d. sample from X, where X is a binary random variable. Assume that $p = P\{X = 1\}$ is such that $0 < p < 1$. Let

$$\gamma = \frac{p}{1-p}$$

   and define

$$\hat{\gamma}_n = \frac{\bar{X}_n}{1 - \bar{X}_n} \ .$$

   (a) Can you show that $\hat{\gamma}_n$ is an unbiased estimator of γ?

   (b) Show that $\hat{\gamma}_n$ is a consistent estimator of γ.

4. Suppose

$$Y = \beta_0 + \beta_1 X + U \ ,$$

where Y is a binary random variable. Suppose further that $E[U|X] = 0$ and $0 < \text{Var}[X] < \infty$.

(a) What is $E[Y|X]$? What is $P\{Y = 1|X\}$?

(b) What is $\text{Var}[Y|X]$?

(c) What is $\text{Var}[U|X]$? Is the model homoskedastic or heteroskedastic?

(d) Let $(Y_1, X_1), \ldots, (Y_n, X_n)$ be a i.i.d. sample from (Y, X). In addition to the assumptions above, suppose that $E[X^4] < \infty$. Assume that the sample size, n, is large.

i. How would you test the null hypothesis that $\beta_1 = 0$ versus the alternative that $\beta_1 \neq 0$ at the 5% significance level?

ii. How would you compute the p-value for the test in part (i)?

iii. Construct a (two-sided) confidence interval for β_1 at the 5% significance level?

iv. What coverage property does the interval in part (iii) satisfy?