Creating Incentives and Selecting Good Types Revisited*

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Abstract

The typical rational account of elections involves standard-setting: voters deciding to retain or replace an incumbent politician, based on whether the incumbent's performance meets a particular standard. While there is consensus that voters set standards, there is considerable debate about how voters determine which standard to apply. There are two views. The first is that voters set standards for the purpose of creating incentives. The second is that voters set standards for the purpose of selecting good types, i.e., that voters set standards for the purpose of selecting better performing politicians in the future. Proponents of this latter view argue that, if candidates differ from one another in ways that are relevant to voters' future payoffs, rational voters can only set standards for the purpose of selecting good types—that is, incentives are created only as a by-product of voters setting standards for the purpose of selecting good types. This has become the conventional wisdom in the literature. This paper challenges this viewpoint: We show that, even in the presence of candidate heterogeneity, rational voters may set standards for the direct purpose of creating incentives for politicians. Thus, voters can set standards both for the purpose of creating incentives and for the purpose of selecting good types.

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The typical rational account of elections involves standard-setting: voters deciding to retain or replace an incumbent politician, based on whether the incumbent's performance meets a particular standard. While there is consensus that voters set standards, there is considerable debate about how voters determine which standard to apply.

Traditionally, the literature focused on the idea that voters set standards to create incentives for politicians—i.e., to achieve what Pitkin (1967) calls *accountability*. As Pitkin writes:

> The point of holding [a politician] to account after he acts is to make him act in a certain way—look after his constituents, or do what they want. (1967, p. 57)

This is an *ex ante* rationale for standard setting: Voters use a particular standard because committing to that standard—before the politician has acted—gives the politician incentives to take actions that benefit the voters. We refer to this rationale as setting a standard **for the purpose of creating incentives**. (For models with standard setting for the purpose of creating incentives, see Barro (1973), Ferejohn (1986), Austen-Smith and Banks (1989), Seabright (1996) , Persson, Roland and Tabellini (1997), Persson and Tabellini (2000), and Shi and Svensson (2006), among others.)

A more recent literature focuses on voters setting standards for the purpose of obtaining better performing politicians in the future. When a politician's past performance rises above a particular standard, voters may believe that the politician is more likely than alternative candidates to provide good outcomes in subsequent terms. That is, voters may believe that a politician's past performance reveals information about her future performance, and so a politician who performs well in the past is more likely to perform well in the future. This is an *ex post* rationale for standard setting: Voters use a particular standard because, at the time of election, the standard maximizes their future expected payoffs, conditional upon the observed outcome. Following Fearon (1999), we refer to this rationale as setting a standard **for the purpose of selecting good types**. (For models with standard setting for the purpose of selecting good types, see Banks and Sundaram (1993, 1998), Alesina and Rosenthal (1995), Canes-Wrone, Herron and Shotts (2001), Maskin and Tirole (2004), Ashworth (2005), Besley (2006), Smart and Sturm (2006), Myerson (2006), Ashworth and Bueno de Mesquita (2006, 2008), Besley and Smart (2007), Canes-Wrone and Shotts (2007), Fox (2007), Gordon, Huber and Landa (2007), and Fox and Shotts (2009), among others.)

In an important paper, Fearon (1999) argues that, if candidates differ from one another in ways that are relevant to voters' future payoffs, rational voters can *only* set standards for the purpose of selecting good types. This is not to say that voters fail to create incentives.

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This argument has been highly influential in both formal political theory and normative democratic theory. (See, for instance, Manin, Przeworski and Stokes (1999); Persson and Tabellini (2000); Przeworski (2002); Mansbridge (2003, 2009); Besley (2006); Besley and Smart (2007); Rehfeld (2009).) It is a major contribution to the understanding of elections—it demonstrates that the presence of candidate heterogeneity has fundamental implications for the determinants of rational voter behavior. Yet, we will argue that the claim has been pushed too far. The presence of candidate heterogeneity has implications for the standards voters set. However, it is incorrect to conclude that, in the presence of candidate heterogeneity, incentives can only be created as a by-product of setting standards for the purpose of selecting good types. We show that, even in the presence of candidate heterogeneity, rational voters may set standards for the direct purpose of creating incentives for politicians.

At first glance, this debate might appear to be over a second-order concern—whether voters are directly motivated to create incentives or whether voters create incentives only as a by-product of a desire to select good types. But this debate has first-order consequences. In particular, it speaks to which standards voters adopt and, so, which actions politicians take. As such, it has implications for politician behavior and voter welfare. Likewise, our argument has first-order implications for the normative theory of democracy. In particular, contra the now conventional wisdom, Pitkin's (1967) notion of elections as mechanisms of accountability is compatible with rational voter behavior, even in the presence of candidate heterogeneity.

The paper proceeds as follows. Section 1 reviews and clarifies the two rationales for standard-setting by voters. In so doing, we show that voters can simultaneously set standards for the purpose of creating incentives and for the purpose of selecting good types if there are multiple equilibria satisfying two particular properties. Section 2 provides necessary and sufficient conditions for such equilibria to exist in simple examples of canonical agency models of elections. Section 3 characterizes necessary and sufficient conditions for such equilibria to exist in a much more general class of agency models of elections and applies those conditions to an important model from the literature on elections and public finance. Section 4 concludes.

1 Two Motivations for Standard-Setting

We start by reviewing how the literature has formalized the two rationales for standard-setting. We begin with standard-setting for the purpose of creating incentives (Section 1.1), and then turn to standard-setting for the purpose of selecting good types (Section 1.2). Next we explain the argument underlying the conventional wisdom that standard-setting for the purpose of creating incentives and standard setting for the purpose of selecting good types are incompatible (Section 1.3). We provide a conceptual clarification of what it means for voters to set standards for the purpose of creating incentives (Section 1.4) and use this to discuss how and why voters can simultaneously set standards for both purposes (Section 1.5). We then briefly discuss other approaches to the question of creating incentives versus selecting good types (Section 1.6).

1.1 Standard-Setting for the Purpose of Creating Incentives

Let us begin with an informal example of a game in which rational voters set standards for the purpose of creating incentives. Specifically, this is an example of a principal-agent relationship between a voter and a politician in which there is no heterogeneity among the politicians—i.e., a model with "pure moral hazard."

There is an incumbent politician and a voter. (We refer to a politician as "she" and a voter as "he".) The incumbent takes a costly action, which influences the outcome that the voter obtains. For instance, the politician might be choosing how much effort to exert. Effort is costly to the politician, but increases the likelihood that the voter will obtain a good outcome. Alternatively, the politician might be choosing a particular policy. The more distant a policy from the politician's ideal point, the more costly the action, but choosing such policies could result in a better outcome from the voter's perspective.[1]

The voter must decide whether to retain the politician or replace her with an identical challenger. The winner of the election once more undertakes a costly action. Again, more costly actions can lead to better (second-period) outcomes for the voter. At this point, the game is over (irrespective of who is in office).

Can voter behavior shape a politician's actions? In the second period, the answer is no—undertaking a costly action offers no potential electoral benefits, so the politician will not take a costly action. But in the first period, the answer may very well be yes. Suppose the politician anticipates that the voter will reelect her if and only if the outcome is

[1]Note, in the effort application, actions associated with higher cost are also associated with better outcomes. This need not hold in the policy application.

particularly good (from the voter's perspective). Then the politician may want to undertake a costly action, if the electoral benefits are sufficiently high. Undertaking such an action increases the probability that the voter's outcome is good and so increases the likelihood that the incumbent is reelected. This is related to what Mansbridge (2003, page 517) calls "anticipatory representation," in which "what appears to the representative to be a power relation ... works not forward, but backward, through anticipated reactions, from the voter."

Of course, this raises a question: If the politician thinks the voter is rational, can she anticipate that the voter will adopt such a retention rule? Put differently, is it credible for the voter to reelect the politician if and only if the outcome rises to some standard? The answer is yes. Recall, the incumbent and challenger are identical. So, at the point the voter actually makes his reelection decision, he is indifferent between reelecting vs. replacing the incumbent. With this, each of the voter's strategies is sequentially optimal—in particular, the retention rule that rewards good outcomes and sanctions bad outcomes is credible.

In sum, there is a Nash equilibrium in sequentially optimal strategies where the incumbent undertakes the costly action because she (correctly) anticipates that the voter will reelect her if and only if the voter's outcome is particularly good. This is essentially the solution identified in Barro (1973), Ferejohn (1986), and Persson and Tabellini (2000). (In Section 1.4, we revisit this example and argue that there is a missing ingredient.)

1.2 Standard-Setting for the Purpose of Selecting Good Types

To illustrate the idea that voters might set standards for the purpose of selecting good types, we again begin with an informal example. Specifically, this is an example where the past performance of politicians provides voters with information about politicians' distinct underlying characteristics—i.e., a model with "pure adverse selection."

Once again, there is an incumbent politician and a voter. After the first period, the voter again observes an outcome. But now (in this simple model) the politician takes no action. Instead, the outcome depends only on the politician's characteristics, or in the terminology of game theory, the politician's "type." For instance, the type can reflect the ability, honesty, independence, or policy preferences of politicians. The outcome is a signal of the politician's type. The signal may be noisy (e.g., if, say, the politician's ability influences but does not uniquely determine an outcome) or the signal may be perfectly informative (e.g., if, say, the politician's policy preference uniquely determines the policy implemented).

After observing the outcome, the voter forms an assessment about the politician's type.

(If the outcome is perfectly informative of the politician's type, the voter's assessment of the politician's type is correct—that is, the voter knows the politician's type. But, more generally, the voter need not know the politician's type.) Given this assessment, the voter decides whether or not to reelect the incumbent. In the second-period, the voter's outcome depends on the type of the politician in office. At this point, the game ends (irrespective of which politician is in office).

Consider the voter's electoral decision. From the voter's perspective, there is only one aspect that distinguishes the incumbent from the challenger—namely, the voter's assessment of each politician's type. Unlike the model of pure moral hazard, now the voter need not be indifferent between reelecting vs. replacing the incumbent at the time he makes his electoral decision. In particular, the voter may strictly prefer the incumbent (respectively, challenger) if the voter's assessment of the incumbent's type is sufficiently favorable (respectively, poor). Favorable outcomes may cause the voter to believe that the incumbent is likely to be a good type relative to the voter's expectation of the challenger. If this is the case, a rational voter will apply a standard which rewards good outcomes and sanctions bad outcomes. Now the voter applies such a standard not to create incentives for the politician, but because good outcomes lead the voter to believe the incumbent is likely to be a good type.

1.3 Are the Two Rationales Consistent?

Sections 1.1-1.2 describe models that exemplify the two rationales for standard setting by voters. Now, we ask, are these two rationales compatible? That is, can a voter simultaneously set standards for both purposes? Recently, an influential literature has argued no—while rational voters might like to adopt standards for the purpose of creating incentives, doing so is not credible when it is possible to set standards for the purpose of selecting good types. (See, e.g., Fearon (1999) and Besley (2006), among others.)

To see this argument, begin with the pure moral hazard model in Section 1.1, and suppose, now, that politicians can vary by type (even if only by a small amount). The logic is this: At election time, the incumbent's first-period actions are in the past. Moreover, in the second period, no politician has an incentive to undertake a costly action, as it offers no electoral benefit. So there is only one feature that distinguishes the incumbent from the challenger—namely, the voter's expectation of each politician's type. Now the voter may strictly prefer the incumbent (respectively, challenger) if the first period outcome is sufficiently favorable (respectively, poor).

That the voter might have different assessments of the incumbent's and challenger's types has important implications for voter behavior. A rational voter must be concerned about the implications of his electoral decision for future outcomes. Rehfeld (2009, page 220) puts the point well: "Intentionality and the power that we express when we cast a vote are always, by necessity, forward looking. Unless we were able to move back in time . . . votes for political representatives always amount to this intention, 'I intend that going forward you should be my representative.'"

Thus, even if a rational voter would like to commit ex ante to impose a particular standard because it creates good incentives, he may not be able to do so. A rational voter can only commit to those standards which will be consistent with his interests come election time. As such, he can only commit to electing a politician who can be expected to deliver the highest payoffs in the future.

Perhaps the clearest articulation of this argument comes from Fearon:

> If politicians do not vary in type, as presumed by the pure sanctioning view of elections, then voters are completely indifferent between candidates. . . But this indifference is fragile. Introduce *any* variation in politicians' attributes or propensities relevant to their performance in office, and it makes sense for the electorate to focus *completely* on choosing the best type when it comes time to vote. (1999, page 77)

Likewise, Besley writes:

> With pure moral hazard, voters are indifferent (ex post) between voting for the incumbent and a randomly selected challenger. Hence, they can pick a standard for incumbents to meet. . . Voters cannot commit to this voting rule, however, when the incumbent could be a good type even though they would prefer to commit to the voting rule that is used under moral hazard. This finding suggests that the pure moral hazard model is rather fragile to a small variation in the model to include some good types of politicians. (2006, pages 192–193)

Fearon's (1999) and Besley's (2006) models suggest that the two rationales for standard-setting are indeed inconsistent. The literature has taken these models to imply that these two rationales are quite generally inconsistent. (See, e.g., Manin, Przeworski and Stokes (1999), Persson and Tabellini (2000), Przeworski (2002), Mansbridge (2003, 2009), Myerson (2006), Besley and Smart (2007), Rehfeld (2009), etc.) The now standard view is that a rational explanation of standard setting can involve sanctioning for the purpose of creating incentives only if politicians do not differ by type. And, the argument continues, since, in reality, politicians do differ by type, a rational explanation of standard setting cannot involve sanctioning for the purpose of creating incentives.

Although this critique has gained considerable currency, we believe that it is incorrect. There is no fundamental conflict between the two rationales for standard setting. Specifically, we will see that a rational explanation of standard setting may very well involve sanctioning for the purpose of creating incentives, even if politicians differ by type. The key to understanding why is to conceptualize "standard setting for the purpose of creating incentives" at a more basic level. This is where our contribution begins.

1.4 Sanctioning for the Purpose of Creating Incentives, Revisited

Return to the example from Section 1.1. There we identified an equilibrium in which the voter sets a standard that induces the incumbent to take a costly action. Under that analysis, the incumbent takes this action because she thinks the voter will reelect her if and only if the outcome is particularly good. But this is not yet a complete analysis. We have specified one equilibrium of the game, but there are in fact many equilibria.[2] Let's see why.

At the point that the voter actually makes his reelection decision, he is indifferent between electing the incumbent or the challenger. (The key here is that all politicians are *ex post* identical.) As such, each of the voter's strategies is sequentially optimal. Given this, there are many (sequentially optimal) equilibria of the game, corresponding to different reelection strategies for the voter. We already said that there is one equilibrium where the incumbent chooses a costly action and expects the voter to reelect her if and only if her performance is particularly good. But there is another equilibrium where the incumbent does not undertake a costly action and expects that the voter will reelect her regardless of performance.

While the voter is indifferent between any of his strategies, he is not indifferent amongst all these equilibria. For instance, the voter may strictly prefer the equilibrium where the incumbent chooses a particular costly action—e.g., high effort. This is a key, and somewhat subtle, point: There is a distinction between the voter being indifferent between all of his strategies (which he is) and being indifferent between all the equilibria those strategies induce (which he is not).

To be more concrete, consider the case where the voter strictly prefers an equilibrium in which the incumbent exerts high effort over an equilibrium in which she exerts no effort. If the voter could choose among equilibria, he would choose an equilibrium where he rewards

[2] The literature sometimes incorrectly identifies this specific equilibrium as the unique equilibrium of such games.

good outcomes and the incumbent chooses high effort. More generally, if the voter could choose an equilibrium, he would choose one that maximizes his utility from an *ex ante* perspective. Put differently, he would choose a particular standard *because* it provides good incentives for the incumbent.

In sum: The voter would like to commit to using a standard that induces the politician to take an action that is beneficial to the voter. Such a standard is only credible if it is part of an equilibrium in sequentially optimal strategies. In the pure moral hazard game there are multiple equilibria in sequentially optimal strategies. The voter's choice of which standard to apply amounts to selecting an equilibrium of the game. Which equilibrium is selected is motivated by *ex ante* considerations—giving the politician good incentives. Thus, the voter sets his standard for the purpose of creating incentives. This logic (at least implicitly) underlies the analysis in Barro (1973), Ferejohn (1986), Austen-Smith and Banks (1989), Seabright (1996), Persson, Roland and Tabellini (1997), Persson and Tabellini (2000), Shi and Svensson (2006), and others.

1.5 Creating Incentives and Selecting Good Types Are Not Mutually Exclusive Motivations

The discussion in Section 1.4 highlights the relationship between standard-setting for the purpose of creating incentives and the existence of multiple equilibria. In the pure moral hazard model, voters choose the reelection rule that gives the politician the best *ex ante* incentives. Doing so is credible because this reelection rule is part of an equilibrium in sequentially optimal strategies. Many other voting rules are also part of equilibria in sequentially optimal strategies. The reason the voter chooses this particular rule is for the purpose of creating good incentives.

Note, then, in the pure moral hazard model of elections, setting standards for the purpose of creating incentives is, in a sense, about the voter selecting an equilibrium from amongst the set of sequentially optimal—that is, credible—equilibria of the game.

Now consider an agency model of elections with moral hazard and adverse selection. This was the case we considered in Section 1.3, where the candidates both differ in type and choose actions. There it seemed that setting standards for the purpose of selecting good types crowds out setting standards for the purpose of creating incentives. But now we see that, formally, setting standards for the purpose of creating incentives is about selecting an equilibrium from amongst the set of sequentially optimal equilibria. So, whether the argument in Section 1.3—that the two rationales are inconsistent—is correct depends on whether there are at least two (sequentially optimal) equilibria satisfying the following properties:

(i) The voter selects good types in both equilibria.

(ii) The voter strictly prefers one equilibrium to the other.

If there are at least two such equilibria, then the voter can set standards for both purposes—selecting good types and creating incentives for the politicians to take costly actions. If not, then, whenever politicians differ by type, the voter can only credibly set standards for the purpose of selecting good types.

This raises the question: Do there exist two equilibria satisfying (i)-(ii)? This is the question addressed in the formal treatment of the paper (Sections 2-3). There, we show that the answer can be yes. Before turning to our formal analysis, let us informally review how it is possible for two equilibria satisfying (i)-(ii) to exist.

Recall, when the voter observes the first-period outcome, he updates his beliefs (about the incumbent's type), given his prior belief (about the incumbent's action and incumbent's type). Quite generally, this will lead the voter to have a unique best reply—if, *ex post*, the incumbent (respectively, challenger) looks like a higher type, he should reelect (respectively, replace) the incumbent. Only in a very special case—arguably, only in a degenerate case—will the voter be indifferent between the incumbent and the challenger. But, the fact that the voter may have a unique best response, given his prior and his first-period observation, does not imply that there is a unique equilibrium in sequentially optimal strategies. Different equilibria can—and must—be associated with different prior beliefs over incumbent actions. So, even if the voter is not indifferent *ex post*, there may still be multiple equilibria of the game.

Let's rephrase this last point: In equilibrium, the voter correctly anticipates the actions the politicians will play. So, if two different standards induce distinct behavior by the incumbent, then they will necessarily be associated with different prior beliefs on the voter's part. In these situations, the voter's posterior beliefs will also differ. As such, there may very well be multiple (sequentially optimal) equilibria.

In the main text, we formalize this idea. To do so, we study the canonical agency model of elections discussed above. Within the class of such models, we fix a particular formalization of politician type, action, and outcome. (Section 4 discusses the possibility of extending the analysis to other formalizations.) In particular, the outcome is the quality of policy generated and this is an increasing function of the incumbent's type and (costly) action. (So, type is best thought of as ability or competence.) Within this framework, the voter selects good types in every equilibrium.[3] We provide sufficient conditions on the informational and production environment so that there are multiple (sequentially optimal) equilibria and the voter strictly prefers one of those equilibria over another. It follows that these are sufficient conditions so that there are multiple (sequentially optimal) equilibria satisfying (i)-(ii) from above. As such, these are sufficient conditions for voters to simultaneously set standards for the purpose of creating incentives and for the purpose of selecting good types.

We perform this exercise for two versions of the canonical agency model of elections: The first is a model with symmetric uncertainty, where neither the politician nor the voter knows the politician's type. (A variety of models in the literature assume symmetric uncertainty. See, e.g., Persson and Tabellini (2000), Ashworth (2005), Ashworth and Bueno de Mesquita (2006).) The second is a model with asymmetric uncertainty, where the politician knows her type but the voter does not. (A variety of models in the literature assume asymmetric uncertainty. See, e.g., Banks and Sundaram (1993), Canes-Wrone, Herron and Shotts (2001), Maskin and Tirole (2004), Besley (2006), Myerson (2006), and Canes-Wrone and Shotts (2007).)

1.6 Other Approaches

Return to the example from Section 1.3. There, both types of politician choose low effort in the second period. As such, the voter can focus exclusively on electing the highest type politician. But, suppose, instead, the game has a longer time horizon. Now there are at least two additional, potential sources of multiple equilibria: First, at the time of the voter's first electoral decision, he must evaluate both the expected type and the expected future behavior of a candidate. Second, the infinite length of the game offers a richer set of credible punishments. Indeed, there are multiple equilibria in many models of elections with an infinite time horizon (Banks and Sundaram, 1993; Duggan, 2000; Snyder and Ting, 2008; Banks and Duggan, 2006; Meirowitz, 2007; Schwabe, 2009).

The fact that there may be multiple equilibria in games with infinite time horizons does not necessarily imply that voters can simultaneously set standards for the purpose of selecting good types and for the purpose of creating incentives. For that, there must be at least two equilibria satisfying points (i) and (ii) from Section 1.5.

Are there multiple equilibria satisfying these points in these infinite time horizon games? The literature does not directly address this question. (Arguably, the literature has not

[3]We restrict attention to monotonic equilibria, in which case this obtains.

done so precisely because the connection between equilibria that satisfy these conditions and standard setting for the purpose of creating incentives has not previously been fully understood.)

In fact, it is not clear whether or not there are multiple equilibria satisfying points (i) and (ii). To see why, let us return to one of the potential sources of multiplicity mentioned above: In the infinite horizon model, the voter must evaluate both the expected type and the expected future behavior of a candidate. As such, it is not clear that the voter always votes for the highest (expected) type politician in the first election: For instance, the voter may not if politicians with high (expected) type do not work as hard in the second period.[4]

Indeed, existing results suggest that finding infinite horizon games that support such equilibria may not be trivial. Schwabe (2009) shows that, in his model of repeated elections, the set of Markov perfect equilibria cannot satisfy points (i) and (ii); in particular, politician behavior and, thus, voter welfare is the same in any Markov perfect equilibrium. Meirowitz (2007), Snyder and Ting (2008), and Schwabe (2009) show that, in their models, equilibria exist in which the voter need not select good types. The key is that the voter can be indifferent between different types of politicians. (In Snyder and Ting (2008) this is because equilibrium outcomes are constant in type for incumbent's of sufficiently high type. In Meirowitz (2007) and Schwabe (2009) this is because different types of politicians choose precisely the behavior that makes the voter indifferent between them.) So, sequential optimality does not require the voter to select good types.

Meirowitz's (2007) and Schwabe's (2009) results highlight a distinct issue in Fearon's (1999) argument. Based on a two-period model, Fearon (1999) asserted that, when there is heterogeneity in politician type, any equilibrium involves voters selecting good types. By considering models that capture the infinitely repeated nature of many real electoral environments, Meirowitz (2007) and Schwabe (2009) show that this claim is incorrect.

In this paper we focus on a generalization of the two period model of elections from Section 1.3. By focusing on a model with a short time horizon, we constrain ourselves to an environment in which rational voters do want to select good types—in any equilibrium of our model, voters set standards for the purpose of selecting good types. So, for us, the only question is whether there are multiple equilibria that differ in terms of voter welfare. If there are, then voters can also set standards for the purpose of creating incentives.

[4]If the voter's payoffs are not monotonic in politician type, then the voter's best response correspondence may not be monotonic in beliefs. If this is the case, then politicians may not always want to appear to be high types, introducing potential non-monotonicities into the politicians' best response correspondences.

2 An Example

This section focuses on a simple example of a canonical agency model of elections. Within the context of the example, we provide a necessary and sufficient condition for the existence of multiple equilibria satisfying conditions (i) and (ii) from Section 1.5. As such, it is an example where voters can set standards both for the purpose of selecting good types and for the purpose of creating incentives.

There are three players: An Incumbent (I), Challenger (C) and Voter (V). We refer to each Politician (P) as "she" and the Voter as "he." Each Politician P is either of high type, viz. $\overline{\theta}$, or of low type, viz. $\underline{\theta}$, where $\overline{\theta} > \underline{\theta}$. Write $\Pr(\overline{\theta})$ for the probability that a Politician is of high type. This probability is commonly understood by the players.

In each period, the Politician in office can take one of two actions, viz. $\overline{a}$ or $\underline{a}$, where $\overline{a} > \underline{a}$. Both the Politician's type and action affect the level of public goods produced for the voter. If the Politician is of type θ and chooses an action a, the level of public goods produced would simply be $f(a, \theta)$, absent any idiosyncratic noise. The function f is strictly increasing in type (θ) and actions (a). We refer to the function f as the **production function.** However, the total level of public goods produced does not only depend on the production function—it also depends on random noise. Specifically, if in period t the politician in office is of type θ, she chooses action a_t, and the random noise is σ_t, the level of public goods produced in that period is

$$f(a_t, \theta) + \sigma_t.$$

Each σ_t is the realization of a mean zero, normally distributed random variable. (Appendix B shows that this analysis extends to a significantly larger class of distributions.) We further assume these random variables are independent of each other. They are also independent of the politicians' abilities.

Now we turn to the timing of the game:

Time 1 Nature determines the realizations of each Politician's type and of the random noise (in all periods). These realizations are not observed by the players.

Time 2 Nature sends each politician her own signal. We will consider two cases:

- **Symmetric Uncertainty:** The signals are uninformative; so, in particular, no Politician learns her own (or the other politician's) type.

- **Asymmetric Uncertainty:** Each Politician's signal is perfectly informative of her own type but uninformative of the other Politician's type. So a Politician's type is her private information.

Time 3 Governance period 1 occurs:

3.1 The Incumbent chooses an action a_1. Her choice is not observed by the other players.

3.2 All players observe the level of public goods produced.

Time 4 The Voter makes a choice to reelect the Incumbent or replace her with the Challenger. This choice is observed by all players.

Time 5 Governance period 2 occurs:

5.1 The Politician in office (i.e., the winner of the election) chooses an action a_2. Her choice is not observed by the other players.

5.2 All players observe the level of public goods produced.

Time 6 The game ends.

The Voter values public goods in each period. Write g_t for the level of public goods provided in governance period t. Then, the Voter's payoffs are simply $g_1 + g_2$.

Each Politician's payoffs depend on both a benefit from holding office and the action chosen while in office. The benefit from holding office is given by $B > 0$. The cost of taking an action a is given by the cost function c, with $B > c(\underline{a})$, $c(\overline{a}) > c(\underline{a})$, and $c(\underline{a}) \geq 0$. So, the high action is more costly than the low action. A Politician's payoff in governance period t is 0 if she is not in office and $B - c(a_t)$ if she is in office (where a_t is the action she chose in period t). A Politician's payoffs are given by the sum of her payoffs in each governance period.

2.1 The Approach

Before turning to the results, we preview our approach. Recall, we are interested in determining whether there are multiple sequentially optimal equilibria satisfying points (i) and (ii) from Section 1.5. How will we implement this?

We restrict attention to pure strategy perfect Bayesian equilibrium. Of course, in any perfect Bayesian equilibrium, players (and, in particular, the Voter) play a sequentially optimal strategy.

The approach we will take is to fix the production technology and the beliefs. We then ask: Do there exist benefits of reelection and cost functions so that there are multiple perfect Bayesian equilibria satisfying (i) and (ii) from Section 1.5.

Why is this the question of interest? We view both the Incumbent's benefit from reelection and the cost of effort as fundamentally subjective. To us, the application might suggest certain material benefits of reelection (e.g., salary, prestige, and so on) and material costs of higher actions (e.g., foregone rents, time not devoted to other policy areas, and so on). But the application cannot pin down the Politician's utility from these material outcomes. On the other hand, for a given application, the analyst may have intuitions or empirical knowledge about the nature of the production function or the beliefs. For instance, in a particular application, the analyst may think type and effort are complements in producing public goods. Or, in a particular application, the analyst may think that it is quite likely that the pool of potential politicians includes many high type candidates and so it is quite likely that any given Politician is a high type. Thus, it is of interest to understand conditions on the parameters of the model, so that there is some benefit of reelection and cost function that lead to multiple equilibria in the associated game.

2.2 The Analysis

To analyze the game, begin with the second governance period. In both the cases of symmetric and asymmetric uncertainty, there are no electoral benefits from choosing the high action in this period, as there is no future election. As such, the politician in office will choose the low action ($a_2 = \underline{a}$), independent of whether or not she knows her own type (and, more generally, independent of the history).

First, we focus on the case of symmetric uncertainty. We return to asymmetric uncertainty at the end of this section.

Symmetric Uncertainty

Consider the Voter's electoral decision. Because both candidates will choose the low action in the second governance period, the Voter's electoral decision depends only on his expectation about the politicians' types. The Voter will prefer to reelect the Incumbent if and only if his posterior beliefs, conditional on observing g_1, say that the Incumbent's expected type

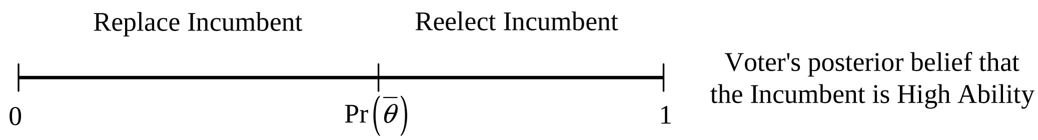

Figure 1: Cutoff rule with respect to posterior beliefs about the Incumbent's type.

is at least as high as the Challenger's. Otherwise, the Voter strictly prefers the Challenger.

Here we see the logic of setting standards for the purpose of selecting good types: The Voter learns information about the Incumbent's type that is relevant for his expected payoffs in the second governance period. Refer to Figure 1. A rational Voter's reelection decision can be described by a cutpoint in the space of beliefs—where the cutpoint is simply the probability that the Challenger is the high type, viz. $\Pr(\overline{\theta})$. If the Voter's posterior belief lies strictly above the cutpoint, then he reelects the Incumbent. If the Voter's posterior belief lies strictly below the cutpoint, then he elects the Challenger. All else equal, high type candidates are more likely to meet the benchmark.

Proposition 2.1 (Select Good Types) *Consider the symmetric uncertainty case. In any pure strategy perfect Bayesian equilibrium the Voter selects good types.*

Definitions B.3 and C.2 formalize the concept of "selecting good types."

Notice, the Voter's posteriors depend both on the level of public goods he observes and his beliefs about the Incumbent's behavior in the first governance period. For a given level of public goods g_1, the Voter's posterior belief will differ based on whether he initially believed the Incumbent chose $\overline{a}$ or $\underline{a}$. In particular, for any given g_1, the Voter's posterior will be higher if he believes the Incumbent took the low action. That is, for any given level of public goods g_1, if the Voter thinks the Incumbent chose $\underline{a}$ rather than $\overline{a}$, the Voter thinks it is more likely that the the Incumbent is the high type. Intuitively, suppose the the Voter observes a relatively low level of public goods. This is a worse signal about the Incumbent's type if the Voter believes the Incumbent took the high action than it is if the Voter believes the Incumbent took the low action.

Figure 2 illustrates a key point. Regardless of his beliefs about the Incumbent's action, the Voter always uses the same cutpoint in belief space. (The cutoff is $\Pr(\overline{\theta})$ on the y-axis. Notice the y-axis of Figure 2 is the same as the x-axis of Figure 1.) However, in order to

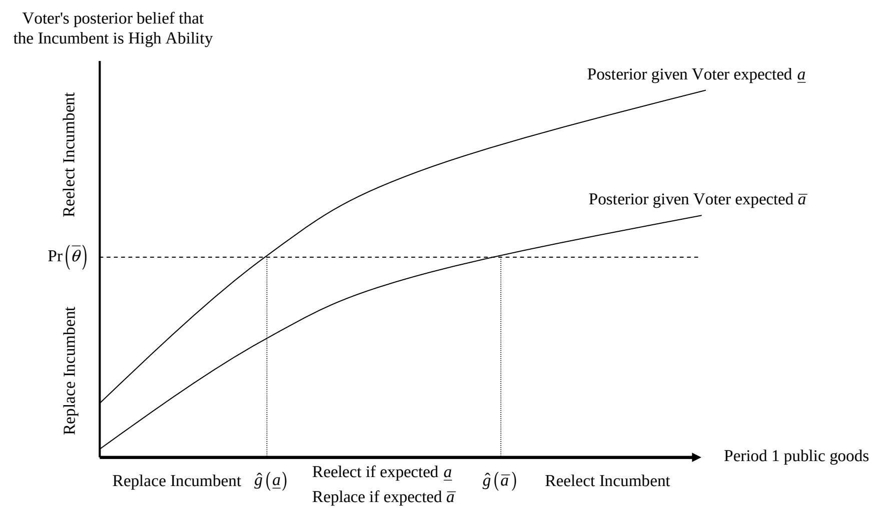

Figure 2: Cutoff rule with respect to the level of first-period public goods provided depends on prior beliefs about Incumbent effort.

do so, he uses different cutpoints with respect to the level of public goods, depending on his beliefs about Incumbent action. If the Voter believes the Incumbent took action $\overline{a}$, he will reelect her if and only if the level public goods rises above $\hat{g}(\overline{a})$. If the Voter instead believes that the Incumbent chooses $\underline{a}$, he will reelect her if and only if the level of public goods rises above $\hat{g}(\underline{a})$. The fact that $\hat{g}(\underline{a}) < \hat{g}(\overline{a})$ reflects the fact that the Voter—selecting for good types—is willing to reelect for a lower level of public goods provision if he believes the Incumbent did not exert effort.

Now turn to the Incumbent's choice of action. The Incumbent benefits from taking the high action because doing so increases the probability that he will be reelected. It does so by increasing the probability that he will produce enough public goods to exceed the Voter's threshold. We will refer to this change in probability as the **incremental increase in probability of reelection** to the Incumbent of taking the high action.

As we saw above, the standard (with respect to level of public goods provision) that the Voter sets depends on what action the Voter believes the Incumbent will choose. This means that how much the probability of reelection increases when the Incumbent takes the high action depends on the Voter's beliefs. As a result, the size of the Incumbent's incremental increase in probability of reelection from taking the high action is sensitive to what action the Voter believes the Incumbent chose. Let us write $\text{IR}(a)$ for the incremental

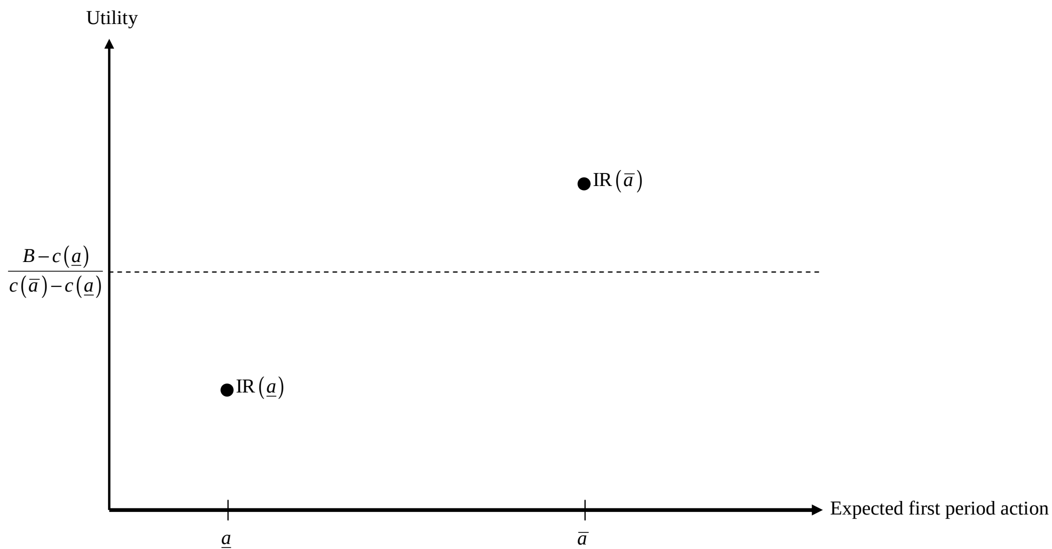

Figure 3: If IR $(\overline{a}) \geq$ IR $(\underline{a})$ then there exists a (B, c) that justifies both $\overline{a}$ and $\underline{a}$.

increase in probability of reelection to the Incumbent of taking the high action, given that the Voter believes the Incumbent took action a.

Note, in the second governance period, the Incumbent's value of holding office is $B - c(\underline{a})$. (This uses the fact that the Incumbent chooses the low action, irrespective of the history.) So, if the Voter expects the Incumbent to take action a, IR $(a)(B - c(\underline{a}))$ is the **incremental benefit** that the Incumbent obtains by choosing the high action over the low action, given that the Voter believes the Incumbent took action a. The Incumbent will choose the high action when the incremental benefit is at least as large as the incremental cost of choosing the high action. So, if the Incumbent believes the Voter expects her to choose the action a, the Incumbent will choose the high action if IR $(a)(B - c(\underline{a})) \geq c(\overline{a}) - c(\underline{a})$.

Given this, when is there an equilibrium in which the Incumbent chooses $\overline{a}$? In such an equilibrium, the Voter must believe the Incumbent chooses $\overline{a}$. And, therefore, for the Incumbent to actually be willing to do so, it must be that IR $(\overline{a})(B - c(\underline{a})) \geq c(\overline{a}) - c(\underline{a})$.

Similarly, when is there an equilibrium in which the Incumbent choose $\underline{a}$? In such an equilibrium, the Voter must believe the Incumbent chooses $\underline{a}$. And, therefore, for the Incumbent to actually be willing to do so, it must be that IR $(\underline{a})(B - c(\underline{a})) \leq c(\overline{a}) - c(\underline{a})$.

Figure 3 shows a case in which the incremental increase in probability of reelection is higher when the Voter believes the Incumbent takes action $\overline{a}$ than when the Incumbent takes the action $\underline{a}$. Here, we can choose benefits and costs so that

$$\mathrm{IR}\left(\overline{a}\right) \geq \frac{c(\overline{a}) - c(\underline{a})}{B - c(\underline{a})} \geq \mathrm{IR}\left(\underline{a}\right).$$

Hence, there are two equilibria.

Notice, it is important that in Figure 3, $\mathrm{IR}\left(\overline{a}\right) \geq \mathrm{IR}\left(\underline{a}\right)$. If this were not the case, we could not choose benefits and costs that simultaneously support both equilibria. (Since $c(\overline{a}) > c(\underline{a})$, doing so would require $c(\underline{a}) > B$. In this case, holding office in the second period is not beneficial and so there is no equilibrium where $\overline{a}$ is chosen in the first period.) Indeed, this condition is key for establishing when there are benefits of reelection and cost functions for which the game has multiple equilibria.

Say a pair (B, c) **justifies** an action a_1 if, when B is the benefit of reelection and c is the cost function, there is an equilibrium in which the politician takes action a_1 in the first period.

Proposition 2.2 (Create Incentives) *Consider the symmetric uncertainty case. There exists a benefit of reelection and a cost function that justifies both $\overline{a}$ and $\underline{a}$ if and only if* $\mathrm{IR}\left(\overline{a}\right) \geq \mathrm{IR}\left(\underline{a}\right)$.

Proposition 2.2 provides a necessary and sufficient condition so that there are two equilibria—one in which the Incumbent takes the high action and another in which the Incumbent takes the low action. In both equilibria, the Voter sets standards for the purpose of selecting good types. But the Voter prefers the first equilibrium to the second. As such, the Voter can simultaneously set standards for the purpose of creating good incentives. Proposition 2.2 provides a necessary and sufficient condition for requirements (i)-(ii) of Section 1.5 to be satisfied.

Note that the incremental increase in probability of reelection, viz. $\mathrm{IR}\left(\cdot\right)$, can be computed from the primitives of the model. But still there is an open question: Do there exist primitives of the model so that the incremental increase in the probability of reelection is higher when the Voter expects the high action? Indeed there do:

Proposition 2.3 *Consider the symmetric uncertainty case.* $\mathrm{IR}\left(\overline{a}\right) \geq \mathrm{IR}\left(\underline{a}\right)$ *if and only if* $\Pr(\overline{\theta}) \geq \frac{1}{2}$.

As a corollary of Propositions 2.2-2.3:

Corollary 2.1 *Consider the symmetric uncertainty case. There exists a benefit of reelection and a cost function that justifies both* $\overline{a}$ *and* $\underline{a}$ *if and only if* $\Pr(\overline{\theta}) \geq \frac{1}{2}$*.*

Corollary 2.1 says that there is a benefit and cost function that justifies both $\underline{a}$ and $\overline{a}$ if and only if the Voter thinks that the probability that the Incumbent is a high type is greater than the probability the Incumbent is the low type.

Consider the special case where the Voter thinks the Incumbent is equally likely to be a high vs. a low type. In this case, $\mathrm{IR}\,(\overline{a}) = \mathrm{IR}\,(\underline{a})$. Here any benefit of reelection and cost function that justifies both $\overline{a}$ and $\underline{a}$ has

$$\mathrm{IR}\,(\overline{a}) = \frac{c(\overline{a}) - c(\underline{a})}{B - c(\underline{a})} = \mathrm{IR}\,(\underline{a}).$$

There are many such benefits of reelection and cost functions. Yet, in a sense, this case is knife-edged. In particular, for a given B and $c(\underline{a})$, there is exactly one $c(\overline{a})$ that justifies both $\overline{a}$ and $\underline{a}$.

Consider instead the case where the Voter thinks the Incumbent is strictly more likely to be a high vs. low type. In this case, $\mathrm{IR}\,(\overline{a}) > \mathrm{IR}\,(\underline{a})$. Now multiplicity of equilibria is no longer knife-edged: For a fixed B and $c(\underline{a})$ we can find infinitely many $c(\overline{a})$ that justify both $\overline{a}$ and $\underline{a}$. The following result formalizes this fact.

Proposition 2.4 *Consider the symmetric uncertainty case. The following are equivalent.*

(i) *There exists a non-empty open set of benefits of reelection and cost functions, so that each element of the set justifies both* $\overline{a}$ *and* $\underline{a}$*.*

(ii) $\mathrm{IR}\,(\overline{a}) > \mathrm{IR}\,(\underline{a})$.

(iii) $\Pr(\overline{a}) > \frac{1}{2}$.

Asymmetric Uncertainty

Now turn to the case of asymmetric uncertainty. Here, the politician in office can choose an action contingent on her type. This leads to three possible **monotone equilibria**, i.e., equilibria where higher types play (weakly) higher actions.[5]

(i) **Pooling on Low**: The Voter believes the Incumbent plays $\underline{a}$ regardless of type.

(ii) **Pooling on High**: The Voter believes the Incumbent plays $\overline{a}$ regardless of type.

[5] See Appendix A on the rationale for focusing on monotone equilibria.

(iii) **Monotonic Separation**: The Voter believes a low type Incumbent would play $\underline{a}$ and a high type Incumbent would play $\overline{a}$.

In each monotone equilibrium, the Voter sets a standard for the purpose of selecting good types. To see why, note that in a monotone equilibrium, the high type plays an action that is at least as high as the action played by the low type. As such, for any given outcome, the Voter's posterior beliefs about the Incumbent are higher when the Incumbent is a high type versus when she is a low type. So, all else equal, high type candidates are more likely to meet the benchmark for reelection than are low type candidates. That is:

Proposition 2.5 (Select Good Types) *Consider the asymmetric uncertainty case. In any monotone pure strategy perfect Bayesian equilibrium, the Voter selects good types.*

Now turn to the question of setting standards for the purpose of creating incentives: Say a pair (B, c), **justifies Pooling on Low** (resp. **justifies Pooling on High**) if, when B is the benefit of reelection and c is the cost function, there is an equilibrium in which the Incumbent takes action $\underline{a}$ (resp. $\overline{a}$) in the first period, independent of her type. Say a pair (B, c), **justifies Monotonic Separation** if, when B is the benefit of reelection and c is the cost function, there is an equilibrium in which a low type Incumbent takes action $\underline{a}$ in the first period and a high type Incumbent takes action $\overline{a}$ in the first period.

It can be shown that if (B, c) justifies Pooling on High, then it does not justify Pooling on Low or Monotonic Separation. As such, we ask whether there is a benefit of reelection and cost function that justifies both Pooling on Low and Monotonic Separation.

Suppose (B, c) justifies Monotonic Separation. This means that a high type Incumbent prefers to take the high action and a low type Incumbent prefers to take the low action, given that the Voter anticipates Monotonic Separation. Let $\mathrm{IR}\,(\theta, \mathrm{MS})$ be type θ's incremental increase in reelection probability (associated with moving from $\underline{a}$ to $\overline{a}$) when the Voter expects Monotonic Separation. A high type Incumbent prefers to take the high action when $\mathrm{IR}\,(\overline{\theta}, \mathrm{MS})(B - c(\underline{a})) \geq c(\overline{a}) - c(\underline{a})$. A low type Incumbent prefers to take the low action when $\mathrm{IR}\,(\underline{\theta}, \mathrm{MS})(B - c(\underline{a})) \leq c(\overline{a}) - c(\underline{a})$. Hence, (B, c) justifies Monotonic Separation if and only if:

$$\mathrm{IR}\,(\overline{\theta}, \mathrm{MS}) \geq \frac{c(\overline{a}) - c(\underline{a})}{B - c(\underline{a})} \geq \mathrm{IR}\,(\underline{\theta}, \mathrm{MS}).$$

Likewise, suppose (B, c) justifies Pooling on Low. This means that each type prefers to take the low action, given that the Voter expects Pooling on Low. Let $\mathrm{IR}\,(\theta, \mathrm{PL})$ be type θ's incremental increase in reelection probability (associated with moving from $\underline{a}$ to $\overline{a}$) when the Voter expects Pooling on Low. Then (B, c) justifies Pooling on Low if and only if:

$$\frac{c(\overline{a}) - c(\underline{a})}{B - c(\underline{a})} \geq \max\{\mathrm{IR}\,(\overline{\theta}, \mathrm{PL}), \mathrm{IR}\,(\underline{\theta}, \mathrm{PL})\}.$$

Putting these two together, we get:

Proposition 2.6 (Create Incentives) *Consider the asymmetric uncertainty case. There exists a benefit of reelection and cost function that justifies both Pooling on Low and Monotonic Separation if and only if*

$$\mathrm{IR}\,(\overline{\theta}, MS) \geq \max\{\mathrm{IR}\,(\overline{\theta}, PL), \mathrm{IR}\,(\underline{\theta}, PL), \mathrm{IR}\,(\underline{\theta}, MS)\}.$$

Recall that if (B, c) justifies Pooling on High, then it cannot justify Pooling on Low or Monotonic Separation. Given this, Proposition 2.6 provides a necessary and sufficient condition so that there are two equilibria. Moreover, the Voter strictly prefers Monotonic Separation to Pooling on Low. Hence, if there exist primitives of the model that satisfy this condition, then the Voter can simultaneously set standards for the purpose of selecting good types and for the purpose of creating incentives. Do such primitives exist?

Proposition 2.7 *Consider the asymmetric uncertainty case.* $\mathrm{IR}\,(\overline{\theta}, MS) \geq \max\{\mathrm{IR}\,(\overline{\theta}, PL),$ $\mathrm{IR}\,(\underline{\theta}, PL), \mathrm{IR}\,(\underline{\theta}, MS)\}$ *if the following conditions are met:*

(i) $f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \geq f(\underline{a}, \overline{\theta}) - f(\underline{a}, \underline{\theta})$

(ii) $f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \leq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \overline{\theta})$

As a corollary of Propositions 2.6 and 2.7:

Corollary 2.2 *Consider the asymmetric uncertainty case. There exists a benefit of reelection and cost function that justifies both Pooling on Low and Monotonic Separation if the following conditions hold:*

(i) $f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \geq f(\underline{a}, \overline{\theta}) - f(\underline{a}, \underline{\theta})$

(ii) $f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \leq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \overline{\theta})$

The first condition is a single-crossing property—i.e., a requirement that type and action are complements in the production function. As is standard, it is necessary and sufficient for monotonic separation to be justifiable. The second condition implies that action has a bigger effect on public goods production than does type. (To see this, note that condition (ii) implies that $f(\overline{a},\overline{\theta}) - f(\overline{a},\underline{\theta}) < f(\overline{a},\underline{\theta}) - f(\underline{a},\underline{\theta})$. Condition (i) says that types and actions are complements. So, the left-hand side of this expression is the largest possible impact of changing type alone and the right-hand side of this expression is the smallest possible impact of changing action alone.)

Is this multiplicity result knife-edged? Much as in the symmetric uncertainty case, the answer is no if the inequalities identified in Proposition 2.7 are strict.

Proposition 2.8 *Consider the asymmetric uncertainty case and the following conditions:*

(i) There is a non-empty open set of benefits of reelections and cost functions, so that each element of the set justifies both Pooling on Low and Monotonic Separation.

(ii) $\mathrm{IR}\,(\overline{\theta}, MS) > \max\{\mathrm{IR}\,(\overline{\theta}, PL), \mathrm{IR}\,(\underline{\theta}, PL), \mathrm{IR}\,(\underline{\theta}, MS)\}$.

(iii) The following hold:

- $f(\overline{a},\overline{\theta}) - f(\overline{a},\underline{\theta}) > f(\underline{a},\overline{\theta}) - f(\underline{a},\underline{\theta})$
- $f(\overline{a},\overline{\theta}) - f(\overline{a},\underline{\theta}) < f(\overline{a},\underline{\theta}) - f(\underline{a},\overline{\theta})$

Condition (iii) implies condition (ii) which is equivalent to condition (i).

Recap

In both the game with symmetric uncertainty and the game with asymmetric uncertainty, we saw that there may be two equilibria. Within any given equilibrium, high type Incumbents are more likely to meet the standard for reelection—that is, within any given equilibrium, Voters set standards for the purpose of selecting good types. Moreover, the Voter's welfare differs across equilibria. In the symmetric uncertainty case, there is one equilibrium where the Incumbent chooses the high action (in the first period) and a second where the Incumbent chooses the low action (in the first period). In the asymmetric uncertainty case, there is one equilibrium where the high type Incumbent chooses the high action (in the first period) and another equilibrium where the Incumbent chooses the low action (in the first period) irrespective of her type. Note, the Incumbent's actual effort decision depends on which of the two "equilibrium standards" she believes the Voter has adopted. The Voter can credibly commit to either of these standards, since both are part of an equilibrium. He has an incentive to commit to the standard associated with the highest level of Voter welfare. As such, the Voter can also set standards for the purpose of creating incentives.

3 A General Model

We have argued that voters can simultaneously set standards for the purpose of selecting good types and for the purpose of creating incentives when there are multiple equilibria that differ in terms of voter welfare. In Section 2, we studied a simple example. Within that example, we provided necessary and sufficient conditions for such equilibria to exist.

The example was meant to clearly explicate why the Voter can set standards both for the purpose of selecting good types and for the purpose of creating incentives. But in many ways the example was special: There were two possible types of the politician, two actions available, a particular technology for production of public goods, and a particular informational environment. Many examples in the literature violate some of these assumptions. As such, the results in Section 2 do not speak directly to those examples—they do not tell us whether the Voter can set standards for the both purposes in a more general class of examples.

Now we consider a model which relaxes these assumptions. As such, it encompasses several important examples from the literature. For the case of symmetric uncertainty, we provide necessary and sufficient conditions for the existence of multiple equilibria that differ in terms of voter welfare. (Appendix C discusses the asymmetric uncertainty case.) To illustrate the value of this approach, we apply these conditions to Persson and Tabellini's (2000) canonical model of electoral accountability and public finance. Doing so shows that Persson and Tabellini's (2000) model is consistent with voters simultaneously setting standards for the purpose of selecting good types and for the purpose of creating incentives.

3.1 The Framework

Here we provide an overview of our general model. Appendix C provides all technical details.

There are three players: an Incumbent, a Challenger, and a Voter. Each player faces uncertainty about the state $\omega \in \Omega$. There is a full-support belief about the set of states and this belief is commonly understood by the players.

The production of public goods depends on actions, types, and shocks. The set of actions A is a closed subset of a (perhaps multidimensional) Euclidean space; the set has a least element $\underline{a}$. Likewise, the set of types Θ and the set of shocks Σ are closed subsets of a (perhaps multidimensional) Euclidean space. Both types and shocks are determined by the state. Type is specific to a Politician. In particular, $\overrightarrow{\theta}^P(\omega) \in \Theta$ represents Politician P's ability at state ω. Shocks are independent of the Politician but dependent on time. In particular, $\overrightarrow{\sigma}_t(\omega) \in \Sigma$ represents the realization of the shock in period t at state ω. The state space is rich in the following sense: if there is a state where the Politician is of type θ and there is a (perhaps distinct) state where the shock is σ, then there is a state where both the Politician is of type θ and the shock is σ.

We denote the **production function** by F. This function maps actions, types, and shocks into a level of public goods $g \in \mathbb{R}$. The production function satisfies three criteria: First, all else equal, higher actions produce higher output. That is, for each (θ, σ), $F(\cdot, \theta, \sigma)$ is strictly increasing in a. Second, all else equal, higher types produce higher output. That is, for each (a, σ), $F(a, \cdot, \sigma)$ is strictly increasing in θ. Third, for any given action a and for any level of public goods g, there exists some (θ, σ) that produce g, i.e., each $F(a, \cdot, \cdot) : \Theta \times \Sigma \to \mathbb{R}$ is onto. In the example of Section 2, $\Sigma = \mathbb{R}$ and F took the form $F(a, \theta, \sigma) = f(a, \theta) + \sigma$, for some **reduced-form production function** $f : A \times \Theta \to \mathbb{R}$.

We make four substantive assumptions about the informational environment and the production technology. First, better realizations of first-period public goods provide no news about the Challenger's type. Second, better realizations of first-period public goods provide good news about the Incumbent's type. Third, ex ante, the Incumbent believes that higher effort is strictly more likely to result in producing enough public goods to exceed any given standard. The fourth assumption prevents multiplicity from arising as a result of "knife-edge" Voter indifference. (That is, it stacks the deck against multiplicity.) The assumption says that the Incumbent assigns zero probability to the following event: after seeing first period production, the Voter's conditional expectation of second-period production by the Incumbent is exactly equal to the Voter's expectation of second-period production by the Challenger.

The timeline is as given in Section 2. Likewise, the payoffs are essentially the same. The payoff to the Voter is the sum of public goods produced in each governance period. In each governance period, the Politician's payoffs depend on both a benefit of reelection (if in office) and a cost of action. The costs are given by a cost function c:

Definition 3.1 *Say $c : A \to \mathbb{R}_+$ is a* ***cost function*** *if it is a strictly increasing function, i.e., $c(a) > c(a')$ whenever $a > a'$.*

So if a Politician is in office in governance period t and chooses action a_t, she gains a payoff $B - c(a_t)$ for that governance period. A Politician's payoffs are given by the sum of her payoffs in each governance period.

3.2 Symmetric Uncertainty Analysis

As in the example, the Politician in office chooses $\underline{a}$ in the second governance period. As such, at the point of reelection, the Voter prefers to reelect the Politician of higher expected type. Thus, the Voter reelects the Incumbent if and only if the level of public goods rises above some threshold. This implies that the Voter sets a standard for the purpose of selecting good types.

Proposition 3.1 (Select Good Types) *Fix two states ω and ω' such that $\overrightarrow{\theta}^I(\omega) > \overrightarrow{\theta}^I(\omega')$ and $\overrightarrow{\sigma}_1(\omega) = \overrightarrow{\sigma}_1(\omega')$. The following holds almost surely: If the Voter reelects the Incumbent at ω', then the Voter reelects the Incumbent at ω.*

Proposition 3.1 says that, holding the Incumbent's effort and the shock constant, the Voter's reelection decision is monotone with respect to the Incumbent's type. That is, the Voter selects good types.

Now ask: which first governance-period actions are justifiable in the sense defined in Section 2? As before, the standard the Voter holds the Incumbent to depends on the action the Voter expects the Incumbent to take. Suppose the Voter believes the Incumbent will take the action a_*. Will the Incumbent actually take this action? Only if her incremental benefit from taking the action a_* over all other actions $a \in A$ is higher than her incremental cost from taking a_* over all other actions, given the Voter's belief.

This argument once again leads us to the idea of the Incumbent's **incremental increase in probability of reelection**. Write $\Pr(a|a_*)$ for the probability that the level of public goods produced exceeds the Voter's threshold, when the Incumbent takes action a and the Voter expects her to take the action a_*. So, if the Voter expects the Incumbent to take the action a_*, then the incremental increase in probability of reelection from choosing a' instead of a is $\mathrm{IR}\,(a', a|a_*) = \Pr(a'|a_*) - \Pr(a|a_*)$.

With this, (B, c) justifies a_* if and only if

$$\mathrm{IR}\,(a_*, a|a_*)(B - c(\underline{a})) \geq c(a_*) - c(a) \qquad \text{for all } a \in A. \tag{1}$$

Likewise, (B, c) justifies a_{**} if and only if

$$\mathrm{IR}\,(a_{**}, a'|a_{**})(B - c(\underline{a})) \geq c(a_{**}) - c(a') \qquad \text{for all } a' \in A. \tag{2}$$

In Equation 1 take $a = a_{**}$ and in Equation 2 take $a' = a_*$. Then, (B, c) justifies both a_* and a_{**} only if

$$\mathrm{IR}\,(a_{**}, a_*|a_{**}) \geq \frac{c(a_{**}) - c(a_*)}{B - c(\underline{a})} \geq -\mathrm{IR}\,(a_*, a_{**}|a_*).$$

Notice that $-\mathrm{IR}\,(a_*, a_{**}|a_*) = \mathrm{IR}\,(a_{**}, a_*|a_*)$. Thus, (B, c) justifies both a_* and a_{**} only if

$$\mathrm{IR}\,(a_{**}, a_*|a_{**}) \geq \frac{c(a_{**}) - c(a_*)}{B - c(\underline{a})} \geq \mathrm{IR}\,(a_{**}, a_*|a_*). \tag{3}$$

This gives a necessary condition for a_* and a_{**} to be justifiable. Namely,

$$\mathrm{IR}\,(a_{**}, a_*|a_{**}) \geq \mathrm{IR}\,(a_{**}, a_*|a_*). \tag{4}$$

Take $a_{**} > a_*$. Equation 3 says that the incremental increase in probability of reelection of moving from the lower action, a_*, to the higher action, a_{**}, must be higher when the Voter expects the Incumbent to take the higher action, a_{**}, than when the Voter expects the Incumbent to take the lower action, a_*.

Equation 4 is the analog of the requirement we saw in Section 2.2. However, there is a key difference. In the example, there were two actions. So, there, if Equation 4 is satisfied, we can find a benefit of reelection and a cost function satisfying Equation 3. Since there are only two actions, this means that Equations 1 and 2 are satisfied. Thus, in the example, Equation 4 is also sufficient to ensure that there are two equilibria.

But Equation 4 is silent about actions distinct from a_* and a_{**}. When there are more than two actions, we may have a benefit of reelection and cost function that satisfies Equation 3 but which fails to justify either a_* or a_{**}. In particular, for a given benefit of reelection and cost function, Equation 1 may fail for some action a, even if it holds for a_{**}. Similarly, Equation 2 may fail for some action a', even if it holds for a_*.

Nonetheless, we will see that in the special case where $a_* = \underline{a}$, Equation 4 is also sufficient to ensure that a_* and a_{**} are justifiable. That is, if Equation 4 holds, then we can always construct some benefit of reelection and some cost function such that Equations 1 and 2 are satisfied.

Proposition 3.2 *Consider the symmetric uncertainty game and actions $a_{**} > a_*$. There exists a benefit of reelect and a cost function that justifies both a_* and a_{**} if and only if the following conditions are met:*

(i) $\mathrm{IR}\,(a_{**}, a_*|a_{**}) \geq \mathrm{IR}\,(a_{**}, a_*|a_*)$, *and*

(ii) $\mathrm{IR}\,(a_{**}, \underline{a}|a_{**}) \geq \mathrm{IR}\,(a_{**}, a_*|a_*)$, *with strict inequality when* $a_* \neq \underline{a}$.

Notice two features of these conditions. First, in the case where the lower action is the lowest action (i.e., $a_* = \underline{a}$), conditions (i) and (ii) coincide. Second, and more importantly, the conditions are local, i.e., they are defined relative to only the actions a_* and a_{**} that we are attempting to justify plus the lowest action $\underline{a}$. So, for instance, the conditions may be satisfied for any two actions—but not for all actions—and, even so, they are sufficient to generate the multiple equilibria result.

It is surprising that these local conditions suffice. To see why they do, consider the case where $a_* = \underline{a}$. The idea will be to fix a benefit of reelection, viz. B, and constants n_* and n_{**}. The constants will turn out to be the costs associated with the high and the lowest actions, i.e., when we later choose a cost function c that justifies both a_* and a_{**}, it will satisfy $c(a_*) = n_*$ and $c(a_{**}) = n_{**}$. As such, we fix $B > n_* > 0$ and (in light of the necessity condition given by Equation 3), we fix $n_{**} > n_*$ so that

$$\mathrm{IR}\,(a_{**}, a_*|a_{**}) \geq \frac{n_{**} - n_*}{B - n_*} \geq \mathrm{IR}\,(a_{**}, a_*|a_*). \tag{5}$$

Suppose, in fact, that c is a cost function with $c(a_*) = n_*$ and $c(a_{**}) = n_{**}$. Notice that, if (B, c) justifies a_*, it must be that $c(\cdot)$ lies above the function $N(\cdot, a_*) : A \to \mathbb{R}$ where

$$N(a, a_*) = \mathrm{IR}\,(a, a_*|a_*)(B - n_*) + n_*.$$

If this condition were not satisfied, the Incumbent would have an incentive to deviate from a_* to an alternate action. Analogously, if (B, c) justifies a_{**}, it must be that $c(\cdot)$ lies above the function $N(\cdot, a_{**}) : A \to \mathbb{R}$ with

$$N(a, a_{**}) = \mathrm{IR}\,(a, a_{**}|a_{**})(B - n_*) + n_{**}.$$

Figure 4 illustrates the functions $N(\cdot, a_*)$ and $N(\cdot, a_{**})$. Each of these function is strictly increasing. Moreover, by Equation 5, $N(a_*, a_*) = n_* \geq N(a_*, a_{**})$ and $N(a_{**}, a_{**}) = n_{**} \geq N(a_{**}, a_*)$. Thus, we can take c to be the upper envelope of the functions and get that (B, c) justifies both a_* and a_{**}.

To sum up: We have seen that Equation 4 is necessary and sufficient to justify both the lowest action $\underline{a}$ and some higher action a_{**}. What if the lower action is not the lowest possible action, i.e., if $a_* \neq \underline{a}$? Now the construction involves choosing four parameters: the benefit of reelection, the cost of $\underline{a}$, the cost of a_*, and the cost of a_{**}. We must

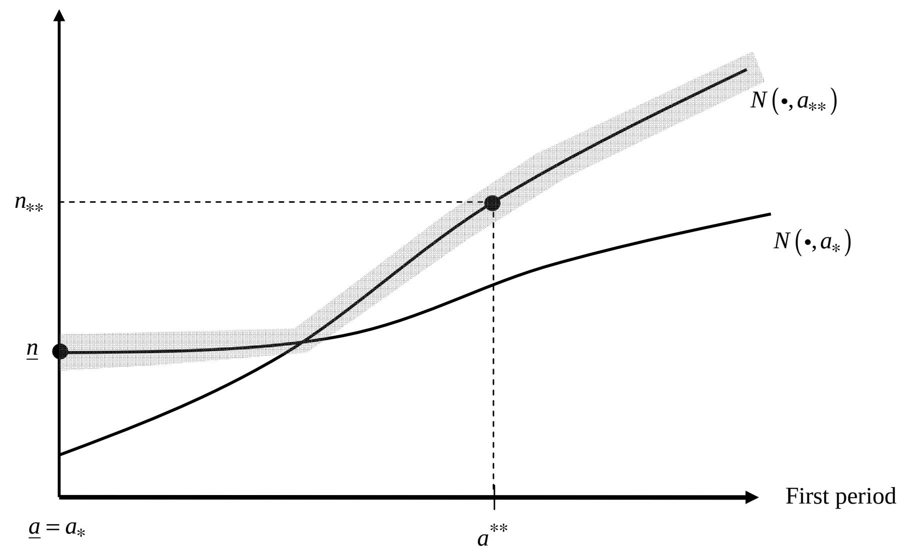

Figure 4: Constructing the cost function.

choose these parameters so that (a) the Incumbent has no incentive to deviate from one "equilibrium action" to the second and (b) the Incumbent has no incentive to deviate from some "equilibrium action" to the lowest action. The second requirement results in an additional constraint—that is, Equation 4 is no longer sufficient.

Finally, we note that when the conditions from Proposition 3.2 hold with strict inequality, there is a significant set of benefits of reelection and cost functions that justify both a_* and a_{**}.

Proposition 3.3 *Consider the symmetric uncertainty case and suppose A is finite. The following are equivalent.*

(i) There exists a non-empty open set of benefits of reelection and cost functions, so that each element of the set justifies both a_* and a_{**}.

(ii) The following hold:

- $\mathrm{IR}(a_{**}, a_* | a_{**}) > \mathrm{IR}(a_{**}, a_* | a_*)$, *and*
- $\mathrm{IR}(a_{**}, \underline{a} | a_{**}) > \mathrm{IR}(a_{**}, a_* | a_*)$.

3.3 Application

Persson and Tabellini (2000, Section 4.5.1) propose a two-period model of electoral accountability and public finance. Here we review a version of their model. On the one hand, we abstract away from parameters that are irrelevant for our analysis (but important for Persson and Tabellini's (2000) comparative statics). On the other hand, we allow for a more general set of payoff parameters for the politician. We do so to address the question: Can we choose payoffs so that there are multiple equilibria satisfying satisfying (i)-(ii) from Section 1.5.

In Persson and Tabellini's (2000) model, the politician uses tax revenues to both produce public goods and extract personal rents. The level of public goods is an increasing function of the politician's competence and the amount of tax revenues devoted to public goods production. There is symmetric uncertainty about the politician's competence, θ. A politician's competence is the realization of a uniformly distributed random variable with support $[1-k, 1+k]$, with $0 < k < 1$

The exogenous tax rate is $\overline{\tau}$ and the exogenous level of wealth is y. Hence, in each period, government revenue is $\overline{\tau}y$.

In each governance period, the politician in office chooses a level of rents $r \in [0, \overline{\tau}y]$. If the politician in office in period t is of type θ and chooses rents r_t, then the level of public goods produced in period t is $f(r_t, \theta) = \theta(\overline{\tau}y - r_t)$. Thus, there are no shocks to production.[6]

The Voter's payoffs in period t are $y(1-\overline{\tau}) + f(r_t, \theta)$. Let $u(\cdot)$ be a strictly increasing function of r. Then the Politician in office in period t receives a payoff of $B + u(r_t)$, where r_t is the level of rents she chooses in period t.

Suppose the Voter believes the Incumbent chose r_* in the first governance period. Persson and Tabellini (2000) show that, upon observing the first period level of public goods g_1, the Voter's posterior belief about the Incumbent's type is: $\theta_* = \frac{g_1}{\overline{\tau}y - r_*}$. The Voter reelects the Incumbent if and only if $\theta_* \geq 1$. Notice, this cutoff rule with respect to posterior beliefs implies a cutoff rule with respect to public goods.

Now consider the case where the Incumbent's actual type is θ and she chooses r_1 in the first governance period. The Voter will reelect her if and only if $g_1 \geq \overline{\tau}y - r_*$, where $g_1 = \theta(\overline{\tau}y - r_1)$. Thus, the Incumbent is reelected if and only if her true type is greater than $\frac{\overline{\tau}y - r_*}{\overline{\tau}y - r_1}$. This implies that the Voter is setting a standard for the purpose of selecting good types. (The implication is a special case of Proposition 3.1.)

[6] To put this into our framework, take the set of shocks Σ to be a singleton.

To determine whether the Voter can also set standards for the purpose of creating incentives, we need to check whether this example satisfies the conditions in Proposition 3.2. To do so, we need to calculate the incremental increase in probability of reelection for the Incumbent. From the Incumbent's perspective, the probability of reelection is the probability that θ is greater than $\frac{\overline{\tau}y-r_*}{\overline{\tau}y-r_1}$. Notice, in this context, choosing high effort is equivalent to foregoing rent seeking. Using this fact, and recalling that the Incumbent believes θ is uniformly distributed on $[1-k, 1+k]$, this implies

$$\Pr(-r_1|-r_*) = 1 - \frac{\frac{\overline{\tau}y-r_*}{\overline{\tau}y-r_1} - (1-k)}{2k}.$$

From this, we have that for any actions r' and r'' and belief r:

$$\text{IR}\left(-r'', -r'|r\right) = \frac{(\overline{\tau}y - r)(r'' - r')}{2k(\overline{\tau}y - r')(\overline{\tau}y - r'')}.$$

It is now straightforward to show that, for any $\overline{\tau}y \geq r_* > r_{**}$, the two conditions from Proposition 3.2 are met. This implies that, for any $\overline{\tau}y \geq r_* > r_{**}$, there exists a (B, u) that justifies both r_* and r_{**}. Thus, Persson and Tabellini's (2000) model is consistent with the Voter setting standards both for the purpose of selecting good types and for the purpose of creating incentives.

4 Conclusion

Our contribution began by clarifying what the existing literature means by "setting standards for the purpose of creating incentives." In particular, canonical analyses of setting standards for the purpose of creating incentives rest on an implicit equilibrium selection argument, one that is based on voter welfare. As such, we argued that, in any agency model of elections, voters can set standards for the purpose of creating incentives—in precisely the sense intended by the pure moral hazard literature—so long as: there are multiple equilibria and voter welfare differs across the equilibria.

Our analysis challenges an important conventional wisdom. The now standard argument is that, when there is candidate heterogeneity, rational voters must set standards for the purpose of selecting good types. As a result, the argument continues, rational voters cannot set standards for the purpose of creating incentives because they cannot use their one vote for two purposes.

We show that this argument is incorrect. In particular, we provide general conditions under which canonical agency models of elections have multiple equilibria that differ according to voter welfare. In each of these equilibria voters set standards for the purpose of selecting good types. But, because voter welfare differs across the equilibria, voters can simultaneously set standards for the purpose of creating incentives. The two rationales are not mutually exclusive.

Our analysis raises two important, but as yet unanswered, questions. The first is empirical and the second is technical.

First: Is the idea that voters set standards for the purpose of creating incentives empirically valid? Now we see that "setting standards for the purpose of creating incentives" is equivalent to: voters commit to a retention rule that (a) induces equilibrium behavior from the incumbent and (b) maximizes voter welfare relative to all other equilibria. As such, the question amounts to: Is it empirically realistic to assume that voters select amongst equilibria based on voter welfare?

We take no stand on whether voters actually behave in this way. To us it seems entirely plausible that in some electoral settings with multiple equilibria voters find ways to commit to such welfare maximizing behavior and that, in other such settings, equilibrium selection occurs for reasons having nothing to do with voter welfare. The question of whether—or in what electoral environments—standard setting for the purpose of creating incentives has explanatory power is an open empirical question.

Second: Do the results here extend to models formalizing other notions of politician action or politician type? The literature on agency models of elections treats a variety of such notions—for instance, type as policy preference (Maskin and Tirole, 2004; Besley, 2006; Meirowitz, 2007; Canes-Wrone and Shotts, 2007), type as responsiveness to sanctioning (Snyder and Ting, 2008; Fox and Shotts, 2009), or type as virtue (Myerson, 2006). It is not obvious whether our results extend to such models because the nature of the agency relationship may be quite different across these models.

To see this, compare models where type is competence and action is effort (such as the ones we study) to models where type is policy preference and action is policy choice. In our competence/effort models, the voter prefers actions that are costly to the politician, irrespective of the politician's type. This feature is lost in policy models. There the type of the politician typically represents the politician's ideal point and the action of the politician represents a policy. So the voter may in fact prefer an action (i.e., a policy) that is beneficial to the politician. Whether this is or is not the case depends on the politician's type (i.e., ideal point).

Can we find politician preferences for such policy games (or games formalizing other notions of type) so that there are multiple equilibria satisfying (i) and (ii) from Section 1.5? We leave this as an open question.

Appendix A Preliminaries

In this section, we will introduce some set-up that will be useful in the remaining appendices.

In the analyses of Sections 2-3, each player faces several sources of uncertainty: uncertainty about each Politician's type and uncertainty about the shock/noise at each time period. This uncertainty will be reflected as uncertainty about a state. Write Ω for the set of states and take Ω to be a Polish space. (This can be $\mathbb{R}^n$ or some compact subset thereof, etc.) Each player has a belief about the state given by a (common) full support probability measure μ on Ω which is commonly understood by the players.

Each Politician (P) is associated with an integrable random variable $\overrightarrow{\theta}^P : \Omega \to \Theta$. Thus, $\overrightarrow{\theta}^P(\omega)$ indicates the type of P at state ω. Likewise, each governance period t is associated with an integrable random variable $\overrightarrow{\sigma}_t : \Omega \to \Sigma$ which indicates the shock at time t for each state.

Note, in the case of symmetric uncertainty, the Incumbent receives a signal Ω at each state. In the case of asymmetric uncertainty, the Incumbent receives a signal $(\overrightarrow{\theta})^{-1}(\{\overrightarrow{\theta}(\omega)\})$ at the state ω, i.e., if the true state ω is such that $\overrightarrow{\theta}(\omega) = \overline{\theta}$ (resp. $\overrightarrow{\theta}(\omega) = \underline{\theta}$), then the Incumbent's signal is $[\overrightarrow{\theta}] = \{\omega : \overline{\theta}(\omega) = \overline{\theta}\}$ (resp. $[\overrightarrow{\theta}] = \{\omega : \underline{\theta}(\omega) = \underline{\theta}\}$). Write Ψ for the set of possible signals for the game. (So the set Ψ differs based on whether we are in the game with symmetric vs. asymmetric uncertainty.)

In this game, a (pure) strategy of the Voter maps each realization of first-period public goods into a reelection decision. Write $\{0, 1\}$ for the set of reelection decisions; $r = 0$ represents the decision to replace the Incumbent and $r = 1$ represents the decision to reelect the Incumbent. A strategy of the Voter s^V is a measurable mapping from the range of F_1^I (a subset of $\mathbb{R}$) to $\{0, 1\}$.

It will be convenient to think of an Incumbent's strategy as reflecting a first- and second-period strategy. Write $s^I = (s_1^I, s_2^I)$, where s_1^I is the Incumbent's first-period strategy and s_2^I is her second-period strategy. We can view s_1^I as a mapping from the set of states Ω to the set of actions A. Each first-period strategy s_1^I is measurable with respect to the Incumbent's signal, i.e., if the Incumbent receives the same signal at ω and ω', then $s_1^I(\omega) = s_1^I(\omega')$. (Since $\overrightarrow{\theta}^I$ is measurable, this implies that each strategy is measurable.) As such, in the case of symmetric uncertainty, s_1^I specifies the same action at each state and, in the case of asymmetric uncertainty, s_1^I specifies one action for each signal.

Likewise, we can view s_2^I as a mapping from $\Omega \times \{r = 1\}$ to A that is measurable with respect to the Incumbent's information. We abuse notation and write $s_2^I(\omega)$ for $s_2^I(\omega, 1)$.

Finally, a Challenger's strategy is a mapping $s^C : \Omega \times \{0\} \to A$ that is measurable with respect to the Challenger's information. (Again, this implies s^C is measurable.) We abuse notation and write $s^C(\omega)$ for $s^C(\omega, 0)$.

A particular type of strategy will be of interest, i.e., a strategy that is monotone. An Incumbent's strategy s^I is **monotone** if $s_1^I(\omega) \geq s_1^I(\omega')$ whenever $\overrightarrow{\theta}^I(\omega) \geq \overrightarrow{\theta}^I(\omega')$. This says that higher types of the Incumbent play at least as high actions (in the first-period). Notice, in the case of symmetric uncertainty, the Incumbent's strategy does not vary by state and so is necessarily monotone.

The Incumbent's payoff function is given by

$$U^I(\omega, s^I, s^V) = \begin{cases} B - c(s_1^I(\omega)) + B - c(s_2^I(\omega)) & \text{if } s^V(F_1^I(\omega, s_1^I(\omega))) = 1 \\ B - c(s_1^I(\omega) & \text{if } s^V(F_1^I(\omega, s_1^I(\omega))) = 0, \end{cases}$$

where $B > 0$. The Voter's payoff function is given by

$$U^V(\omega, s^I, s^V, s^C) = \begin{cases} F_1^I(\omega, s_1^I(\omega)) + F_2^I(\omega, s_2^I(\omega)) & \text{if } s^V(F_1^I(\omega, s_1^I(\omega))) = 1 \\ F_1^I(\omega, s_1^I(\omega)) + F_2^C(\omega, s^C(\omega)) & \text{if } s^V(F_1^I(\omega, s_1^I(\omega))) = 0. \end{cases}$$

The Challenger's payoff function is given by

$$U^C(\omega, s^I, s^V, s^C) = \begin{cases} 0 & \text{if } s^V(F_1^I(\omega, s_1^I(\omega))) = 1 \\ B - c(s^C(\omega)) & \text{if } s^V(F_1^I(\omega, s_1^I(\omega))) = 0. \end{cases}$$

We seek to identify when there are multiple equilibria where: the Voter selects good types in each equilibria and prefers one equilibrium over the second. We take the notion of "equilibrium" to be perfect Bayesian equilibrium, in the spirit of Fudenberg and Tirole (1991).[7] We impose two restrictions on the concept. First, we only consider pure strategy equilibria. Second, we only consider equilibria in monotone strategies, i.e., perfect Bayesian equilibria, viz. $(s_{1,*}^I, s_{2,*}^I, s_*^V, s_*^C)$, where s_1^I is monotone. (Recall, this is the requirement that higher types play weakly higher actions.)

Let us briefly comment on these restrictions: If we find multiple monotone pure-strategy

[7]As is standard in the applied game theory literature, we take the amendment of perfect Bayesian equilibria that is designed for games that have unobservable actions. The standard definition does not include the "don't signal what you don't know" condition.

perfect Bayesian equilibria satisfying (i)-(ii) of Section 1.4, then certainly we have found multiple perfect Bayesian equilibria satisfying these conditions. The particular restriction to monotone equilibrium is intended so that we can focus on equilibria where Voter's select good types (i.e., equilibria satisfying condition (i)). In a non-monotone equilibrium, the Voter need not select good types: There, a lower type Politician plays higher actions and, so, good outcomes need not be good news about the politician's type.

In light of the above, we are interested in the following question: Do there exist benefit of reelection B and a cost function $c : A \to \mathbb{R}$ so that there are multiple monotone pure-strategy perfect Bayesian equilibria of the associated game? As such, the concept of justifiability again arises.

Definition A.1 *Fix a first-period strategy of the Incumbent, viz.* s_1^I*. Say the pair* $(\mathbf{B}, \mathbf{c})$ ***justifies*** $\mathbf{s_1^I}$ *if there exists a profile* (s_*^I, s_*^V, s_*^C) *so that:*

(i) $s_{1,*}^I = s_1^I$*, and*

(ii) (s_*^I, s_*^V, s_*^C) *is a monotone pure-strategy perfect Bayesian equilibrium of the game when the benefit of reelection is* B *and the cost function is* c*.*

Definition A.2 *Fix a subset of first-period strategies for the Incumbent, viz.* $\mathcal{S}_1^I$*. The set* $\mathcal{S}_1^I$ *is* ***justifiable*** *if there exists a pair* (B, c) *that justifies each* $s_1^I \in \mathcal{S}_1^I$*.*

Definition A.2 says that a set of first-period strategies $\mathcal{S}_1^I$ is justifiable if there exists a benefit of reelection B and a cost function c so that each first-period strategy in $\mathcal{S}_1^I$ induces an equilibrium in the game induced by (B, c).

We will also be interested in the case where there exists a "significant set" of pairs (B, c) that justify a first-period strategy. For this, we need to define the notion of "significance." To abstract away from technical concerns, we define this concept when there are a finite number of actions. In this case: Each pair (B, c) can be viewed as an element of $\mathbb{R}^{|A|+1}$. (Here, we identify a cost function c with a vector $(c(\underline{a}), \ldots,) \in \mathbb{R}^{|A|}$.) As such, the set of all pairs (B, c) is a strict subset, viz. $\mathcal{C}$, of $\mathbb{R}^{|A|+1}$. We endow $\mathcal{C}$ with the relative topology.

Definition A.3 *Fix a subset of first periods strategies of the incumbent, viz.* $\mathcal{S}_1^I$*. The set* $\mathcal{S}_1^I$ *is* ***strictly justifiable*** *if there exists a non-empty open set* U *in* $\mathcal{C}$ *so that each* $(B, c) \in U$ *justifies each* $s_1^I \in \mathcal{S}_1^I$*.*

To show strict justifiability of certain sets, it will be useful to have the following: Let $\mathcal{C}^+$ be the set of all (B, c) with $B > c(\underline{a})$ and endow $\mathcal{C}^+$ with the relative topology.

Lemma A.1 *The set $\mathcal{C}^+$ is open in $\mathcal{C}$*

Proof. Notice that $\mathcal{C}\backslash\mathcal{C}^+$ is closed in $\mathcal{C}$: Fix a sequence $(B^n, c^n) \in \mathcal{C}\backslash\mathcal{C}^+$ that converges to (B, c). Note, for each n, $B^n \leq c^n(\underline{a})$. It follows that $B \leq c(\underline{a})$. Thus, $\mathcal{C}\backslash\mathcal{C}^+$ is closed and $\mathcal{C}^+$ is open. ■

Appendix B The Example

In this appendix, we provide proofs for Section 2. There, we made a number of special distributional assumptions. Here, we relax these particular assumptions, proving a more general result.

Let us recall some background: In the example, the set of actions is $A = \{\overline{a}, \underline{a}\} \subseteq \mathbb{R}$ and the set of types is $\Theta = \{\overline{\theta}, \underline{\theta}\} \subseteq \mathbb{R}$. There is a set of shocks $\Sigma = \mathbb{R}$. The production function $F : A \times \Theta \times \Sigma \to \mathbb{R}$ is additive in shocks, i.e., there is a reduced-form production function $f : A \times \Theta \to \mathbb{R}$ so that $F(a, \theta, \sigma) = f(a, \theta) + \sigma$.

Here, the set of states is $\Omega = \mathbb{R}^2$. The first coordinate of the state tracks the Politicians' types and the second coordinate of the state tracks the shock in both periods. That is, if $\omega = (x_1, \cdot)$ and $\omega' = (x_1, \cdot)$ (resp. $\omega = (\cdot, x_1)$ and $\omega' = (\cdot, x_1)$), then $\overrightarrow{\theta}^P(\omega) = \overrightarrow{\theta}^P(\omega')$ (resp. $\overrightarrow{\sigma}_t(\omega) = \overrightarrow{\sigma}_t(\omega')$) for each Politician P (resp. governance period t).

The random variables $\overrightarrow{\theta}^I$ and $\overrightarrow{\theta}^C$ are identically distributed. So, *ex ante*, the Voter thinks the Incumbent and the Challenger have the same probability of being high ability. Write $\Pr(\overline{\theta})$ for the probability the Voter assigns to I (resp. C) being of type $\overline{\theta}$. Likewise, the random variables σ_1 and σ_2 are identically distributed. Write $\Phi : \mathbb{R} \to [0, 1]$ for their cumulative density function (CDF). The CDF Φ is (by assumption) absolutely continuous.

It is assumed that the random variables $\overrightarrow{\theta}^I$, $\overrightarrow{\theta}^C$, σ_1, and σ_2 are jointly independent. Notice, then, that here the shocks are noise: Since the random variables $\overrightarrow{\sigma}_1$ and $\overrightarrow{\sigma}_2$ are independent, the first-period shock does not provide information about the second-period shock. Since the random variables $\overrightarrow{\sigma}_t$ and each $\overrightarrow{\theta}^P$ are independent, the shock in period t does not provide information about the ability of politician P.

The prior belief μ and the random variables $\overrightarrow{\sigma}_t$ satisfy a number of joint conditions:

Assumption B.1 *The belief μ has* ***full support*** *with respect to $\overrightarrow{\sigma}_1$ and $\overrightarrow{\sigma}_2$, i.e., for each $\sigma, \sigma' \in \mathbb{R}$ with $\sigma > \sigma'$, $\Phi(\sigma) > \Phi(\sigma')$.*

Assumption B.2 *The belief μ is* ***atomless*** *with respect to $\overrightarrow{\sigma}_1$ and $\overrightarrow{\sigma}_2$, i.e., for each $\sigma \in \mathbb{R}$ and each t, $\mu((\overrightarrow{\sigma}_t)^{-1}(\sigma)) = 0$.*

Since the CDF Φ is absolutely continuous, it admits a probability density function (PDF), viz. $\phi : \mathbb{R} \to \mathbb{R}$. By assumption, there is some PDF ϕ that satisfy the following four criteria:

Assumption B.3 *The PDF ϕ is* ***continuous****.*

Assumption B.4 *The PDF ϕ is* ***symmetric*** *about zero, i.e., $\phi(\sigma) = \phi(-\sigma)$ for all $\sigma \in \mathbb{R}$.*

Assumption B.5 *The PDF ϕ is* ***single-peaked****, i.e., if either $0 \geq \sigma > \sigma'$ or $\sigma' > \sigma \geq 0$ then $\phi(\sigma) > \phi(\sigma')$.*

Order $A \times \Theta$ so that $(a, \theta) > (a', \theta')$ if either (i) $a \geq a'$ and $\theta > \theta'$ or (ii) $a > a'$ and $\theta \geq \theta'$

Assumption B.6 *The PDF ϕ satisfies the* ***(strict) monotone likelihood ratio property (MLRP) relative to*** $\mathbf{A} \times \mathbf{\Theta}$*. That is, if $(a, \theta) > (a', \theta')$ and $g_1 > g_1'$,*

$$\frac{\phi\left(g_1 - f(a, \theta)\right)}{\phi\left(g_1 - f\left(a', \theta'\right)\right)} > \frac{\phi\left(g_1' - f\left(a, \theta\right)\right)}{\phi\left(g_1' - f\left(a', \theta'\right)\right)}$$

Recall, the Voter observes a realization of first-period public goods and uses this to form a belief about the type of the Politician. The type of the Politician and action the Politician chooses determine the distribution from which the realization of first-period public goods production is drawn. The MLRP gives a relationship between the observed variable (first-period public goods) and the distribution from which it was drawn. Specifically, it says that when the Voter sees higher realizations of first-period public goods, he believes that it is more likely this realization was drawn from a distribution associated with a high ability Politician (resp. Politician who chose a high action) than a low ability ability Politician (resp. Politician who chose a low action). That is, high levels of public goods are good news about the incumbent's ability (resp. actions).

In Section 2, we assumed that σ_1 and σ_2 were normally distributed. It is readily verified that the assumption satisfies Assumptions B.1-B.6.

B.1 Analysis: Selecting Good Types

Begin with the second governance period.

Remark B.1 *Fix a pure strategy Perfect Bayesian Equilibrium, viz. (s_*^I, s_*^V, s_*^C). At each (second-period) information set, $s_{2,*}^I$ (viz. s^C) chooses $\underline{a}$.*

Now turn to the Voter. In a perfect Bayesian equilibrium, each politician chooses $\underline{a}$ irrespective of the history. As such, it is a best response for the Voter to reelect the Incumbent if and only if his expectation about the Incumbent's type (conditional upon observing a first-period public goods level g_1) is at least as high as his expectation of the Challenger's type (conditional upon observing a first-period public goods level g_1). Because the Challenger's type is independent of the Incumbent's type and first-period action, the Voter's expectation of the Challenger's type simply corresponds to his prior expectation.

The Voter's conditional expectation about the Incumbent's type depends on the strategy the Voter expects the Incumbent to employ in the first-period. In particular, suppose the Voter expects the Incumbent to choose a first-period strategy $s^I_{1,*}$, where $s^I_{1,*}(\omega) = a_1$ if $\overrightarrow{\theta}^I(\omega) = \overline{\theta}$ and $s^I_{1,*}(\omega) = b_1$ if $\overrightarrow{\theta}^I(\omega) = \underline{\theta}$. Then, if the Voter observes a first-period level of public goods g_1, his expectation that the Incumbent is type $\overline{\theta}$ is

$$\Pr(\overline{\theta} \,|\, [g_1]) = \frac{\Pr(\overline{\theta})\phi(g_1 - f(a_1, \overline{\theta}))}{\Pr(\overline{\theta})\phi(g_1 - f(a_1, \overline{\theta})) + \Pr(\underline{\theta})\phi(g_1 - f(b_1, \underline{\theta}))}, \tag{6}$$

and his expectation that the Incumbent is type $\underline{\theta}$ is

$$\Pr(\underline{\theta} \,|\, [g_1]) = \frac{(1 - \Pr(\underline{\theta}))\phi(g_1 - f(b_1, \underline{\theta}))}{\Pr(\overline{\theta})\phi(g_1 - f(a_1, \overline{\theta})) + \Pr(\underline{\theta})\phi(g_1 - f(b_1, \underline{\theta}))}. \tag{7}$$

So, $\Pr(\underline{\theta} \,|\, [g_1]) = 1 - \Pr(\overline{\theta} \,|\, [g_1])$.

Lemma B.1 *Fix a pure-strategy perfect Bayesian equilibrium, viz.* (s^I_*, s^V_*, s^C_*).

(i) If $\Pr(\overline{\theta} \,|\, [g_1]) > \Pr(\overline{\theta})$, *it is a strict best response for the Voter to reelect the Incumbent.*

(ii) If $\Pr(\overline{\theta}) > \Pr(\overline{\theta} \,|\, [g_1])$, *it is a strict best response for the Voter to replace the Incumbent.*

(iii) If $\Pr(\overline{\theta} \,|\, [g_1]) = \Pr(\overline{\theta})$, *the Voter is indifferent between reelecting vs. replacing the Incumbent.*

Proof. Fix a level of first-period public goods g_1. Using Remark B.1, the Voter's expected payoff from reelecting the Incumbent (conditional upon g_1) is

$$\Pr(\overline{\theta} \,|\, [g_1]) f(\underline{a}, \overline{\theta}) + (1 - \Pr(\overline{\theta} \,|\, [g_1])) f(\underline{a}, \underline{\theta}),$$

and the Voter's expected payoff from electing the Challenger is

$$\Pr(\overline{\theta}) f(\underline{a}, \overline{\theta}) + (1 - \Pr(\overline{\theta})) f(\underline{a}, \underline{\theta}).$$

It follows that it is a strict best response for the Voter to reelect (resp. replace) the Incumbent if and only if $\Pr(\overline{\theta}\,|\,[g_1]) > \Pr(\overline{\theta})$ (resp. $\Pr(\overline{\theta}) > \Pr(\overline{\theta}\,|\,[g_1])$). The Voter is indifferent between reelection and replacement if and only if $\Pr(\overline{\theta}\,|\,[g_1]) = \Pr(\overline{\theta})$. ■

Lemma B.1 says that, in equilibrium, the Voter uses a cut-off rule in the space of posterior beliefs: The Voter reelects the Incumbent if, conditional upon realizing a first-period level of public goods g_1, he assigns a higher probability to the Incumbent being a high type than he initially did (i.e., unconditionally).

We will see that the MLRP implies that, in a monotone equilibrium, the Voter uses a cut-off rule in the space of public goods: The Voter reelects the Incumbent if and only if the realization of first-period public goods meets a certain standard. Under the MLRP, if the Voter sees a better signal then he interprets it as having been drawn from a distribution associated with a higher type. So, if he was willing to reelect the Incumbent conditional upon seeing g_1, then certainly a realization of $g_1' > g_1$ is even better-news about the Incumbent's type and so he should also reelect the Incumbent when he sees g_1'.

Definition B.1 *Say s^V is a **cut-off strategy** if there exists a realization of first-period public goods, viz. $g_* \in [-\infty, \infty]$, so that $s^V(g_1) = 0$ if $g_* > g_1$ and $s^V(g_1) = 1$ if $g_1 > g_*$. In this case, say g_* is the **threshold** of s^V.*

Lemma B.2 *Fix a monotone pure-strategy perfect Bayesian equilibrium, viz. $\left(s_*^I, s_*^V, s_*^C\right)$. Write $s_{1,*}^I(\omega) = a_1$ if $\overrightarrow{\theta}^I(\omega) = \overline{\theta}$ and $s_{1,*}^I(\omega) = b_1$ if $\overrightarrow{\theta}^I(\omega) = \underline{\theta}$. Also, write*

$$g_* = \frac{f(a_1, \overline{\theta}) + f(b_1, \underline{\theta})}{2}. \tag{8}$$

(i) If $g_1 > g_*$, it is a strict best response for the Voter to reelect the Incumbent.

(ii) If $g_* > g_1$, it is a strict best response for the Voter to replace the Incumbent.

(iii) If $g_1 = g_*$,, the Voter is indifferent between reelecting vs. replacing the Incumbent.

Proof. We make use of Lemma B.1. Note, it is a strict best response for the Voter to reelect the Incumbent if and only if $\Pr(\overline{\theta}\,|\,[g_1]) > \Pr(\overline{\theta})$. Thus, we will show that $\Pr(\overline{\theta}\,|\,[g_1]) > \Pr(\overline{\theta})$ if and only if $g_1 > g_*$. An analogous argument establishes that $\Pr(\overline{\theta}) > \Pr(\overline{\theta}\,|\,[g_1])$ if and only if $g_* > g_1$. And similarly that $\Pr(\overline{\theta}\,|\,[g_1]) = \Pr(\overline{\theta})$ if and only if $g_* = g_1$.

Using Equation 6, $\Pr(\overline{\theta}\,|\,[g_1]) > \Pr(\overline{\theta})$ if and only if $\phi(g_1 - f(a_1, \overline{\theta})) > \phi(g_1 - f(b_1, \underline{\theta}))$. We will show that $\phi(g_1 - f(a_1, \overline{\theta})) > \phi(g_1 - f(b_1, \underline{\theta}))$ if and only if $g_1 > g_*$.

To see this, note that

$$g_* - f(a_1, \overline{\theta}) = \frac{f(b_1, \underline{\theta}) - f(a_1, \overline{\theta})}{2} = -(g_* - f(b_1, \underline{\theta})).$$

So, by symmetry of ϕ, $\phi(g_* - f(a_1, \overline{\theta})) = \phi(g_* - f(b_1, \underline{\theta}))$. Using the fact that $s^I_{1,*}$ is monotone, $a_1 \geq b_1$. As such, the MLRP implies that $g_1 > g_*$ if and only if $\phi(g_* - f(a_1, \overline{\theta})) > \phi(g_* - f(b_1, \underline{\theta}))$, as desired. ■

Lemma B.2 says that, in each monotone pure-strategy perfect Bayesian equilibrium, the Voter makes use of a cut-off strategy. A consequence is that the Voter selects good types.

Definition B.2 *Fix a pure-strategy monotone perfect Bayesian equilibrium, viz.* (s^I_*, s^V_*, s^C_*). *Say the* ***Voter reelects the Incumbent at*** ω *if* $s^V_*(f(s^I_{1,*}(\omega), \overrightarrow{\theta}^I(\omega))) = 1$.

Definition B.3 *Fix a perfect Bayesian equilibrium, viz.* (s^I_*, s^V_*, s^C_*). *Say, in this equilibrium, the* ***Voter selects good types*** *if the following holds: For each pair of states* ω *and* ω' *with* $\overrightarrow{\theta}^I(\omega) > \overrightarrow{\theta}^I(\omega')$ *and* $\overrightarrow{\sigma}_1(\omega) = \overrightarrow{\sigma}_2(\omega)$, *if the Voter reelects the Incumbent at* ω' *then the Voter reelects the Incumbent at* ω.

Proposition B.1 (Selecting Good Types) *In any monotone pure-strategy perfect Bayesian equilibrium, the Voter selects good types.*

The proof is immediate from Lemma B.2.

B.2 Analysis: Creating Incentives

We first consider the case of symmetric uncertainty and then turn to the case of asymmetric uncertainty.

Symmetric Uncertainty

Here, a first-period strategy $s^I_{1,*}$ is constant across all states, i.e., there is some action a_1 so that $s^I_{1_*}(\Omega) = \{a_1\}$. As such, $s^I_{1,*}$ can take on (at most) one of two possible forms:

Play High Here, $s^I_{1,*}(\omega) = \overline{a}$, for all $\omega \in \Omega$.

Play Low Here, $s^I_{1,*}(\omega) = \underline{a}$, for all $\omega \in \Omega$.

We refer to a profile, viz. (s^I_*, s^V_*, s^C_*), as a **high equilibrium** (resp. **low equilibrium**) if it is a perfect Bayesian equilibrium with $s^I_{1,*}(\omega) = \overline{a}$ (resp. $s^I_{1,*}(\omega) = \underline{a}$), for all $\omega \in \Omega$. Write

$$\overline{g} = \tfrac{f(\overline{a},\overline{\theta})+f(\overline{a},\underline{\theta})}{2} \quad \text{and} \quad \underline{g} = \tfrac{f(\underline{a},\overline{\theta})+f(\underline{a},\underline{\theta})}{2}$$

for the associated equilibrium thresholds, i.e., as given in Lemma B.2.

Consider a high equilibrium and suppose the Incumbent actually plays the action a (whether or not $a = \overline{a}$). Then, at a state ω, the Incumbent gets reelected if $\overrightarrow{\sigma}_1(\omega) > \overline{g} - f(a, \overrightarrow{\theta}^I(\omega))$ and gets replaced if $\overrightarrow{\sigma}_1(\omega) < \overline{g} - f(a, \overrightarrow{\theta}^I(\omega))$. Using the fact that $\overrightarrow{\theta}^I$ and $\overrightarrow{\sigma}_1$ are independent, the probability that the Incumbent assigns to getting reelected is

$$\Pr(a|\overline{a}) = 1 - \Phi(\overline{g} - \mathbb{E}(f(a, \cdot))), \tag{9}$$

where we write $\mathbb{E}(f(a, \cdot))$ for the Incumbent's expectation of $f(a, \cdot)$, i.e., for $\Pr(\overline{\theta})f(a, \overline{\theta}) + (1 - \Pr(\overline{\theta}))f(a, \underline{\theta})$. (Notice, here, we use the fact that $\overrightarrow{\sigma}_1$ is atomless, so that $s^V_*(\overline{g})$ does not affect the Incumbent's expectation of the likelihood of success.[8])

Likewise, looking at a low equilibrium where the Incumbent actually plays the action a, we have that the Incumbent assesses the probability that she will get reelected as

$$\Pr(a|\underline{a}) = 1 - \Phi(\underline{g} - \mathbb{E}(f(a, \cdot))). \tag{10}$$

A consequence of these last two paragraphs:

Lemma B.3 *Let (s^I_*, s^V_*, s^C_*) and (r^I_*, r^V_*, r^C_*) be two high (resp. low) equilibria. The Incumbent assigns the same probability to being reelected under each of these equilibria. This probability is given by Equation 9 (resp. Equation 10) in the case where (s^I_*, s^V_*, s^C_*) and (r^I_*, r^V_*, r^C_*) are two high (resp. low) equilibria.*

Now, notice that we can write the incremental returns in reelection probabilities (from choosing $\overline{a}$ over $\underline{a}$) as:

$$\mathrm{IR}\,(\overline{a}) = \Phi(\overline{g} - \mathbb{E}(f(\underline{a}, \cdot))) - \Phi(\overline{g} - \mathbb{E}(f(\overline{a}, \cdot)))$$

and

$$\mathrm{IR}\,(\underline{a}) = \Phi(\underline{g} - \mathbb{E}(f(\underline{a}, \cdot))) - \Phi(\underline{g} - \mathbb{E}(f(\overline{a}, \cdot))).$$

[8]The reason to make this assumption is to ensure that the multiplicity result does not hinge on one particular realization of public goods—and how we break the Voter's indifference in that case.

Note, these definitions are in terms of primitives of the model.

Lemma B.4

(i) The pair (B, c) *justifies the play high strategy if and only if* $\mathrm{IR}\,(\overline{a})(B - c(\underline{a})) \geq c(\overline{a}) - c(\underline{a})$.

(ii) The pair (B, c) *justifies the play low strategy if and only if* $\mathrm{IR}\,(\underline{a})(B - c(\underline{a})) \leq c(\overline{a}) - c(\underline{a})$.

Proof. Begin by showing the "only if" part of (i)-(ii): Suppose that (B, c) justifies the strategy s_1^I. Then there is an equilibrium (s_*^I, s_*^V, s_*^C) of the associated game with $s_{1,*}^I = s_1^I$. There are two cases.

Case I: If this is a high equilibrium, then the Incumbent's expected payoffs from choosing $\overline{a}$ must be at least as high as the Incumbent's expected payoffs from choosing $\underline{a}$. Applying Lemma B.3, this requirement is:

$$\Pr(\overline{a}|\overline{a})(B - c(\underline{a})) + (B - c(\overline{a})) \geq \Pr(\underline{a}|\overline{a})(B - c(\underline{a})) + (B - c(\underline{a})),$$

from which it follows that $\mathrm{IR}\,(\overline{a})(B - c(\underline{a})) \geq c(\overline{a}) - c(\underline{a})$.

Case II: If this is a low equilibrium, then the Incumbent's expected payoffs from choosing $\underline{a}$ must be at least as high as the Incumbent's expected payoffs from choosing $\overline{a}$. Applying Lemma B.3, this requirement is:

$$\Pr(\underline{a}|\underline{a})(B - c(\underline{a})) + (B - c(\underline{a})) \geq \Pr(\overline{a}|\underline{a})(B - c(\underline{a})) + (B - c(\overline{a})),$$

from which it follows that $c(\overline{a}) - c(\underline{a}) \geq \mathrm{IR}\,(\underline{a})(B - c(\underline{a}))$.

Now turn to the "if" part of (i): Construct a strategy profile (s^I, s^V, s^C) as follows. Set $s_1^I(\Omega) = \{\overline{a}\}$, $s_2^I(\Omega) = \{\underline{a}\}$, and $s^C(\Omega) = \{\underline{a}\}$. Let s^V be a cut-off strategy with a threshold of $\overline{g}$. It is readily verified that, if $\mathrm{IR}\,(\overline{a})(B - c(\underline{a})) \geq c(\overline{a}) - c(\underline{a})$, then (s^I, s^V, s^C) is a high equilibrium.

The "if" part of (ii) follows a similar argument: Construct a strategy profile (s^I, s^V, s^C) as follows. Set $s_1^I(\Omega) = \{\underline{a}\}$, $s_2^I(\Omega) = \{\underline{a}\}$, and $s^C(\Omega) = \{\underline{a}\}$. Let s^V be a cut-off strategy with a threshold of $\underline{g}$. It is readily verified that, if $c(\overline{a}) - c(\underline{a}) \geq \mathrm{IR}\,(\underline{a})(B - c(\underline{a}))$, then (s^I, s^V, s^C) is a low equilibrium. ∎

Proposition B.2 *Let* $s_1^I(\Omega) = \{\overline{a}\}$ *and* $r_1^I(\Omega) = \{\underline{a}\}$.

(i) The set $\{s_1^I, r_1^I\}$ *is justifiable if and only if* $\mathrm{IR}\,(\overline{a}) \geq \mathrm{IR}\,(\underline{a})$.

(ii) The set $\{s_1^I, r_1^I\}$ is strictly justifiable if and only if $\mathrm{IR}\,(\overline{a}) > \mathrm{IR}\,(\underline{a})$.

Proof. We divide the proof into parts (i) and (ii).

Part (i): Begin with the "only if." Suppose (B, c) justifies both the play high and the play low strategy. Since it justifies the play high strategy and (by definition of a cost function) $c(\overline{a}) > c(\underline{a})$, it follows from Lemma B.4 that $\mathrm{IR}\,(\overline{a})(B - c(\underline{a})) > 0$. Now note that, by Assumption B.1, $\mathrm{IR}\,(\overline{a}) > 0$. It follows that $B > c(\underline{a})$. With this, Lemma B.4 gives that $\mathrm{IR}\,(\overline{a}) \geq \mathrm{IR}\,(\underline{a})$.

Now turn to the "if." Suppose $\mathrm{IR}\,(\overline{a}) \geq \mathrm{IR}\,(\underline{a})$. Then there exists some (B, c) so that $B > c(\underline{a})$ and $\mathrm{IR}\,(\overline{a})(B - c(\underline{a})) \geq c(\overline{a}) - c(\underline{a}) \geq \mathrm{IR}\,(\underline{a})(B - c(\underline{a}))$. It follows from Lemma B.4 that $\{s_1^I, r_1^I\}$ is justifiable.

Part (ii): Begin with the "only if." Suppose that $\mathrm{IR}\,(\overline{a}) \leq \mathrm{IR}\,(\underline{a})$. We will show that $\{s_1^I, r_1^I\}$ is not strictly justifiable. Certainly this conclusion follows (from part (i)) if $\mathrm{IR}\,(\overline{a}) < \mathrm{IR}\,(\underline{a})$. Suppose $\mathrm{IR}\,(\overline{a}) = \mathrm{IR}\,(\underline{a})$. Then, using part (i), for each (B, c) that justifies $\{s_1^I, r_1^I\}$, $c(\overline{a}) = \mathrm{IR}\,(\overline{a})(B - c(\underline{a})) + c(\underline{a})$. A consequence is that, for each pair B and $c(\underline{a})$ there is (at most) a unique number $c(\overline{a})$ so that the pair (B, c) justifies $\{s_1^I, r_1^I\}$. As such, there is no non-empty open set of benefits of reelections and cost functions that justify $\{s_1^I, r_1^I\}$.

Now turn to the "if." Suppose that $\mathrm{IR}\,(\overline{a}) > \mathrm{IR}\,(\underline{a})$. Then there exists a non-empty open set $\mathcal{U} \subseteq \mathbb{R}$ so that, for each $x \in \mathcal{U}$, $\mathrm{IR}\,(\overline{a}) > x > \mathrm{IR}\,(\underline{a})$. Note, there exists a continuous surjective function $g : \mathcal{C}^+ \to (0, \infty)$ so that

$$g(B, c(\underline{a}), c(\overline{a})) = \frac{c(\overline{a}) - c(\underline{a})}{B - c(\underline{a})}.$$

The set $(g)^{-1}(\mathcal{U})$ is a non-empty open set in $\mathcal{C}^+$. So, using Lemma A.1, $(g)^{-1}(\mathcal{U})$ is a non-empty open set in $\mathcal{C}$. Moreover, each $(B, c) \in (g)^{-1}(\mathcal{U})$, $\mathrm{IR}\,(\overline{a})(B - c(\underline{a})) > c(\overline{a}) - c(\underline{a}) > \mathrm{IR}\,(\underline{a})(B - c(\underline{a}))$. Now, it follows from Lemma B.4 that $\{s_1^I, r_1^I\}$ is strictly justifiable. ■

Proposition B.2 is simply a restatement of Proposition 2.2 and Proposition 2.4(i)-(ii). We now turn to show Proposition 2.3 and Proposition 2.4(ii)-(iii).

It will be useful to track $\mathrm{IR}\,(\overline{a})$ and $\mathrm{IR}\,(\underline{a})$ through an auxiliary function $\mathrm{R} : \mathbb{R} \to [-1, 1]$. Set

$$\mathrm{R}\,(g) = \Phi(x + r) - \Phi(x),$$

where

$$r = \Pr(\overline{\theta})[f(\overline{a}, \overline{\theta}) - f(\underline{a}, \overline{\theta})] + (1 - \Pr(\overline{\theta}))[f(\overline{a}, \underline{\theta}) - f(\underline{a}, \underline{\theta})].$$

The next Lemma is immediate.

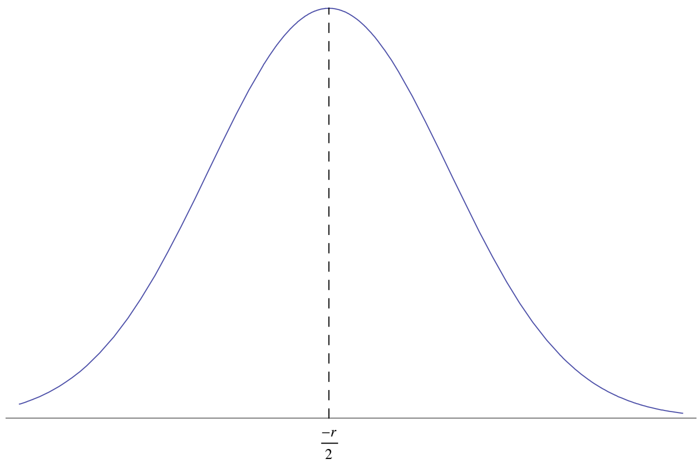

Figure 5: The function R is single peaked and symmetric around $-\frac{r}{2}$.

Lemma B.5

(i) $\mathrm{IR}\,(\overline{a}) = \mathrm{R}\,(\overline{g} - \mathbb{E}(f(\overline{a}, \cdot)))$*;*

(ii) $\mathrm{IR}\,(\underline{a}) = \mathrm{R}\,(\underline{g} - \mathbb{E}(f(\overline{a}, \cdot)))$.

Figure 5 depicts the function R. There are three properties that will be useful, for our purposes:

Lemma B.6

(i) For each $x \in \mathbb{R}$, $\mathrm{R}\,(x) > 0$.

(ii) The function R *is strictly increasing at each* $x < -\frac{r}{2}$ *and strictly decreasing at each* $x > -\frac{r}{2}$.

(iii) The function R *is symmetric around* $-\frac{r}{2}$.

Condition (i) is immediate from the full support assumption (Assumption B.1). Conditions (ii)-(iii) are shown as Lemmata D.1-D.2 in the mathematical appendix.

Lemma B.7

(i) $\Pr(\overline{\theta}) \geq \frac{1}{2}$ *if and only if* $\mathrm{R}\,(\overline{g} - \mathbb{E}(f(\overline{a}, \cdot))) \geq \mathrm{R}\,(\underline{g} - \mathbb{E}(f(\overline{a}, \cdot)))$.

(ii) $\Pr(\overline{\theta}) > \frac{1}{2}$ *if and only if* $\mathrm{R}\,(\overline{g} - \mathbb{E}(f(\overline{a}, \cdot))) > \mathrm{R}\,(\underline{g} - \mathbb{E}(f(\overline{a}, \cdot)))$.

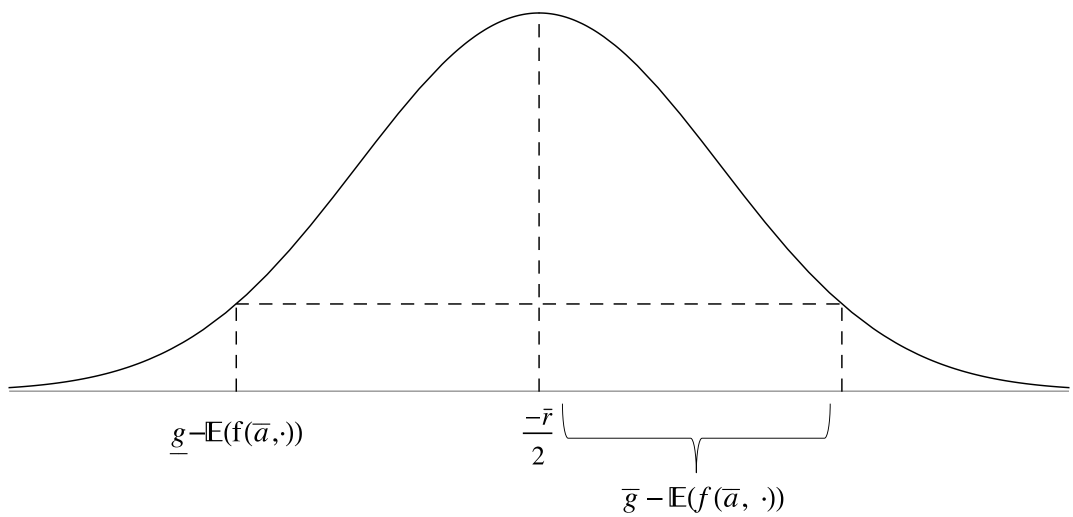

Figure 6: There is an upper bound on $\overline{g} - \mathbb{E}(f(\overline{a},\cdot))$ consistent with the condition $R(\overline{g} - \mathbb{E}(f(\overline{a},\cdot))) > R(\underline{g} - \mathbb{E}(f(\overline{a},\cdot)))$.

Proof. First notice that $\mathrm{R}\,(\overline{g} - \mathbb{E}(f(\overline{a},\cdot))) \geq \mathrm{R}\,(\underline{g} - \mathbb{E}(f(\overline{a},\cdot)))$ if and only if

(a) $-\frac{r}{2} > \underline{g} - \mathbb{E}(f(\overline{a},\cdot))$ and

(b) $-\frac{r}{2} + (-\frac{r}{2} - \underline{g} + \mathbb{E}(f(\overline{a},\cdot)) \geq \overline{g} - \mathbb{E}(f(\overline{a},\cdot)$.

Moreover, $\mathrm{R}\,(\overline{g} - \mathbb{E}(f(\overline{a},\cdot))) > \mathrm{R}\,(\underline{g} - \mathbb{E}(f(\overline{a},\cdot)))$ if and only if both (a) and (b) hold with strict inequality. Figure 6 demonstrates why these conditions are necessary and sufficient for $\mathrm{R}\,(\overline{g} - \mathbb{E}(f(\overline{a},\cdot))) \geq \mathrm{R}\,(\underline{g} - \mathbb{E}(f(\overline{a},\cdot)))$ (resp. $\mathrm{R}\,(\overline{g} - \mathbb{E}(f(\overline{a},\cdot))) > \mathrm{R}\,(\underline{g} - \mathbb{E}(f(\overline{a},\cdot)))$). In particular, use the fact that $\overline{g} - \mathbb{E}(f(\overline{a},\cdot)) > \underline{g} - \mathbb{E}(f(\overline{a},\cdot))$ and conditions (ii)-(iii) of Lemma B.6.)

Condition (a) is equivalent to

$$\Pr(\overline{\theta})[f(\overline{a},\overline{\theta}) - f(\underline{a},\underline{\theta})] \geq (1 - \Pr(\overline{\theta}))[f(\underline{a},\overline{\theta}) - f(\overline{a},\underline{\theta})].$$

Thus, (a) is clearly satisfied if $\Pr(\overline{\theta}) \geq \frac{1}{2}$. As such, we will show that (b) holds (resp. (b) holds with strict inequality) if and only if $\Pr(\overline{\theta}) \geq \frac{1}{2}$ (resp. $\Pr(\overline{\theta}) > \frac{1}{2}$).

Condition (b) holds is equivalent to

$$(1 - 2\Pr(\overline{\theta}))[f(\underline{a},\underline{\theta}) - f(\underline{a},\overline{\theta})] \geq (1 - 2\Pr(\overline{\theta}))[f(\overline{a},\overline{\theta}) - f(\overline{a},\underline{\theta})]. \tag{11}$$

(The inequality is strict if and only if the inequality in (b) is strict.) Notice $[f(\underline{a},\underline{\theta}) - f(\underline{a},\overline{\theta})] < 0$ and $[f(\overline{a},\overline{\theta}) - f(\overline{a},\underline{\theta})] > 0$. So, Equation 11 holds if and only if $(1-2\Pr(\overline{\theta})) \leq 0$ or $\Pr(\overline{\theta}) \geq \frac{1}{2}$. Likewise, the inequality in Equation 11 is strict if and only if $\Pr(\overline{\theta}) > \frac{1}{2}$. ■

Asymmetric Uncertainty

Here, a first-period strategy $s^I_{1,*}$ can differ at distinct states if and only if the Incumbent's type, at those states, are different. As such, a monotone pure-strategy $s^I_{1,*}$ can take on (at most) one of three possible forms:

Pool on High Effort The Incumbent plays $\overline{a}$ independent of the signal she sees, i.e., $s^I_{1,*}(\omega) = \overline{a}$, for all $\omega \in \Omega$.

Pool on Low Effort The Incumbent plays $\underline{a}$ independent of the signal she sees, i.e., $s^I_{1,*}(\omega) = \underline{a}$, for all $\omega \in \Omega$.

Monotonic Separation The Incumbent plays $\overline{a}$ if she sees the signal $\overline{\theta}$ and $\underline{a}$ if she sees the signal $\underline{\theta}$, i.e., $s^I_{1,*}(\omega) = \overline{a}$ if $\overrightarrow{\theta}^I(\omega) = \overline{\theta}$ and $s^I_{1,*}(\omega) = \underline{a}$ if $\overrightarrow{\theta}^I(\omega) = \underline{\theta}$.

We refer to a profile, viz. (s^I_*, s^V_*, s^C_*), as a **pool on high equilibrium** (resp. **pool on low equilibrium**) if it is a perfect Bayesian equilibrium with $s^I_{1,*}(\omega) = \overline{a}$ (resp. $s^I_{1,*}(\omega) = \underline{a}$), for all $\omega \in \Omega$. Likewise, we refer to a profile, viz. (s^I_*, s^V_*, s^C_*), as a **monotonic separating equilibrium** if it is a perfect Bayesian equilibrium with $s^I_{1,*}(\omega) = \overline{a}$ if $\overrightarrow{\theta}^I(\omega) = \overline{\theta}$ and $s^I_{1,*}(\omega) = \underline{a}$ if $\overrightarrow{\theta}^I(\omega) = \underline{\theta}$

If (s^I_*, s^V_*, s^C_*) is a pool on high (resp. low) equilibrium, then s^V_* is a cut-off strategy with threshold $\overline{g}$ (resp. $\underline{g}$). Likewise, if (s^I_*, s^V_*, s^C_*) is a monotonic separating equilibrium profile, s^V_* is a cut-off strategy with threshold

$$g^m = \frac{f(\overline{a},\overline{\theta}) + f(\underline{a},\underline{\theta})}{2}$$

Note, $\overline{g} > g^m > \underline{g}$.

Consider a pool on high equilibrium and suppose the Incumbent actually plays the action a (whether or not $a = \overline{a}$). Then, at a state ω, the Incumbent gets reelected if $\overrightarrow{\sigma}_1(\omega) > \overline{g} - f(a, \overrightarrow{\theta}^I(\omega))$ and gets replaced if $\overrightarrow{\sigma}_1(\omega) < \overline{g} - f(a, \overrightarrow{\theta}^I(\omega))$. If the Incumbent learns that she is of type θ, the probability that the Incumbent (of type θ) assigns to getting reelected is

$$\Pr(a|\mathrm{PH}, \theta) = 1 - \Phi(\overline{g} - f(a,\theta)), \tag{12}$$

Here we use the fact that an Incumbent of type θ receives the signal θ. (We again also use the fact that $\overrightarrow{\sigma}_1$ is atomless, so that $s_*^V(\overline{g})$ does not affect the Incumbent's expectation of the likelihood of success.)

Likewise, looking at a pool on low equilibrium where the Incumbent is of type θ and actually plays the action a, we have that the Incumbent assesses the probability that she will get reelected as

$$\Pr(a|\mathrm{PL}\,,\theta) = 1 - \Phi(\underline{g} - f(a,\theta)). \tag{13}$$

And, looking a monotonic separating equilibrium where the Incumbent is of type θ and actually plays a, we have that the Incumbent assesses the probability that she will get reelected as

$$\Pr(a|\mathrm{MS}\,,\theta) = 1 - \Phi(g^m - f(a,\theta)). \tag{14}$$

A consequence of these last two paragraphs:

Lemma B.8 *Let (s_*^I, s_*^V, s_*^C) and (r_*^I, r_*^V, r_*^C) be two pool on high (resp. pool on low; monotonic seperating) equilibria. The Incumbent assigns the same probability to being reelected under each of these equilibria.*

- *In the case where (s_*^I, s_*^V, s_*^C) and (r_*^I, r_*^V, r_*^C) are pool on high equilibria, this probability is given by Equation 12.*
- *In the case where (s_*^I, s_*^V, s_*^C) and (r_*^I, r_*^V, r_*^C) are pool on low equilibria, this probability is given by Equation 13.*
- *In the case where (s_*^I, s_*^V, s_*^C) and (r_*^I, r_*^V, r_*^C) are monotonic separating equilibria, this probability is given by Equation 14.*

Now, notice that we can write the incremental returns in reelection probabilities (from choosing $\overline{a}$ over $\underline{a}$) as:

$$\begin{aligned}
\mathrm{IR}\,(\mathrm{PH}\,,\theta) &= \Phi(\overline{g} - f(\underline{a},\theta)) - \Phi(\overline{g} - f(\overline{a},\theta)) \\
\mathrm{IR}\,(\mathrm{PL}\,,\theta) &= \Phi(\underline{g} - f(\underline{a},\theta)) - \Phi(\underline{g} - f(\overline{a},\theta)) \\
\mathrm{IR}\,(\mathrm{MS}\,,\theta) &= \Phi(g^m - f(\underline{a},\theta)) - \Phi(g^m - f(\overline{a},\theta)).
\end{aligned}$$

Note, these incremental returns can be computed from primitives of the model.

Lemma B.9

(i) The pair (B, c) justifies the pool on high strategy s_1^I if and only if, for each $\theta \in \{\overline{\theta}, \underline{\theta}\}$, $\mathrm{IR}\,(\mathrm{PH}\,, \theta)(B - c(\underline{a})) \geq (c(\overline{a}) - c(\underline{a}))$.

(ii) The pair (B, c) justifies the pool on low strategy s_1^I if and only if, for each $\theta \in \{\overline{\theta}, \underline{\theta}\}$, $\mathrm{IR}\,(\mathrm{PL}\,, \theta)(B - c(\underline{a})) \leq (c(\overline{a}) - c(\underline{a}))$.

(iii) The pair (B, c) justifies the monotonic separation strategy s_1^I if and only if both $\mathrm{IR}\,(\mathrm{MS}\,, \overline{\theta})(B - c(\underline{a})) \geq (c(\overline{a}) - c(\underline{a}))$ and $\mathrm{IR}\,(\mathrm{MS}\,, \underline{\theta})(B - c(\underline{a})) \leq (c(\overline{a}) - c(\underline{a}))$.

Proof. Begin by showing the "only if" part of (i)-(iii): Suppose that (B, c) justifies the strategy s_1^I. Then there is an equilibrium (s_*^I, s_*^V, s_*^C) of the associated game with $s_{1,*}^I = s_1^I$. There are three cases.

Case I: Suppose this is a pool on high equilibrium. Then an Incumbent of type θ's payoffs from choosing $\overline{a}$ must be at least as high as her payoffs from choosing $\underline{a}$. Applying Lemma B.8, this requirement is:

$$\Pr(\overline{a}|\mathrm{PH}\,, \theta)(B - c(\underline{a})) + (B - c(\overline{a})) \geq \Pr(\underline{a}|\mathrm{PH}\,, \theta)(B - c(\underline{a})) + (B - c(\underline{a})),$$

from which it follows that $\mathrm{IR}\,(\mathrm{PH}\,, \theta)(B - c(\underline{a})) \geq (c(\overline{a}) - c(\underline{a}))$.

Case II: Suppose this is a pool on low equilibrium. Then an Incumbent of type θ's payoffs from choosing $\underline{a}$ must be at least as high as her payoffs from choosing $\overline{a}$. Applying Lemma B.8, this requirement is:

$$\Pr(\underline{a}|\mathrm{PL}\,, \theta)(B - c(\underline{a})) + (B - c(\underline{a})) \geq \Pr(\overline{a}|\mathrm{PL}\,, \theta)(B - c(\underline{a})) + (B - c(\overline{a})),$$

from which it follows that $(c(\overline{a}) - c(\underline{a})) \geq \mathrm{IR}\,(\mathrm{PL}\,, \theta)(B - c(\underline{a}))$.

Case III: Suppose this is a monotonic separating equilibrium. Then an Incumbent of type $\overline{\theta}$'s payoffs from choosing $\overline{a}$ must be higher than her payoffs from choosing $\underline{a}$; and, an Incumbent of type $\underline{\theta}$'s payoffs from choosing $\underline{a}$ must be higher than her payoffs from choosing $\overline{a}$. Applying Lemma B.8, these requirements are:

$$\Pr(\overline{a}|\mathrm{MS}\,, \overline{\theta})(B - c(\underline{a})) + (B - c(\overline{a})) \geq \Pr(\underline{a}|\mathrm{MS}\,, \overline{\theta})(B - c(\underline{a})) + (B - c(\underline{a})),$$

and

$$\Pr(\underline{a}|\mathrm{MS}\,, \underline{\theta})(B - c(\underline{a})) + (B - c(\underline{a})) \geq \Pr(\overline{a}|\mathrm{MS}\,, \underline{\theta})(B - c(\underline{a})) + (B - c(\overline{a})).$$

From this it follows that $\mathrm{IR}(\mathrm{MS}, \overline{\theta})(B-c(\underline{a})) \geq (c(\overline{a})-c(\underline{a}))$ and $(c(\overline{a})-c(\underline{a})) \geq \mathrm{IR}(\mathrm{MS}, \underline{\theta})(B-c(\underline{a}))$.

Now turn to the "if" part of (i): Construct a strategy profile (s^I, s^V, s^C) as follows. Set $s_1^I(\Omega) = \{\overline{a}\}$, $s_2^I(\Omega) = \{\underline{a}\}$, and $s^C(\Omega) = \{\underline{a}\}$. Let s^V be a cut-off strategy with a threshold of $\overline{g}$. Suppose, for each θ, $\mathrm{IR}(\mathrm{PH}, \theta)(B - c(\underline{a})) \geq (c(\overline{a}) - c(\underline{a}))$. It is readily verified that, in this case, (s^I, s^V, s^C) is a pool on high equilibrium.

The "if" part of (ii) follows a similar argument: Construct a strategy profile (s^I, s^V, s^C) as follows. Set $s_1^I(\Omega) = \{\underline{a}\}$, $s_2^I(\Omega) = \{\underline{a}\}$, and $s^C(\Omega) = \{\underline{a}\}$. Let s^V be a cut-off strategy with a threshold of $\underline{g}$. Suppose, for each θ, $\mathrm{IR}(\mathrm{PL}, \theta)(B-c(\underline{a})) \leq (c(\overline{a})-c(\underline{a}))$. It is readily verified that, in this case, (s^I, s^V, s^C) is a pool on low equilibrium.

Finally, turn to the "if" part of (iii): Construct a strategy profile (s^I, s^V, s^C) as follows. Set $s_1^I(\omega) = \overline{a}$ if $\overrightarrow{\theta}^I(\omega) = \overline{\theta}$ and set $s_1^I(\omega) = \underline{a}$ if $\overrightarrow{\theta}^I(\omega) = \underline{\theta}$. Suppose that $\mathrm{IR}(\mathrm{MS}, \overline{\theta})(B - c(\underline{a})) \geq (c(\overline{a}) - c(\underline{a}))$ and $\mathrm{IR}(\mathrm{MS}, \underline{\theta})(B - c(\underline{a})) \leq (c(\overline{a}) - c(\underline{a}))$. It is readily verified that, in this case, (s^I, s^V, s^C) is a monotonic separating equilibrium. ■

At times it will be useful to track each $\mathrm{IR}(\cdot, \theta)$ through an auxiliary function $\mathrm{R} : \Theta \times \mathbb{R} \to [-1, 1]$. Set

$$\mathrm{R}(\theta, x) = \Phi(x + r(\theta)) - \Phi(x),$$

where

$$r(\theta) = f(\overline{a}, \theta) - f(\underline{a}, \theta).$$

The next Lemma is immediate.

Lemma B.10

(i) $\mathrm{IR}(\mathrm{PH}, \theta) = \mathrm{R}(\theta, \overline{g} - f(\overline{a}, \theta))$;

(ii) $\mathrm{IR}(\mathrm{PL}, \theta) = \mathrm{R}(\theta, \underline{g} - f(\overline{a}, \theta))$;

(iii) $\mathrm{IR}(\mathrm{MS}, \theta) = \mathrm{R}(\theta, g^m - f(\overline{a}, \theta))$.

We note properties of the function R:

Lemma B.11

(i) For each $(\theta, x) \in \Theta \times \mathbb{R}$, $\mathrm{R}(\theta, x) > 0$.

(ii) For each θ the function $\mathrm{R}(\theta, \cdot)$ is strictly increasing at each $x < -\frac{r(\theta)}{2}$ and strictly decreasing at each $x > -\frac{r(\theta)}{2}$.

(iii) For each θ the function $\mathrm{R}(\theta, \cdot)$ is symmetric around $-\frac{r(\theta)}{2}$.

Condition (i) is immediate from the full support assumption (Assumption B.1). Conditions (ii)-(iii) are shown as Lemmata D.1-D.2 in the mathematical appendix.

We begin by showing that there is no (B, c) that justifies both the pool on high strategy and the pool on low strategy (resp. the pool on high strategy and monotonic separation).

Proposition B.3 *Let $\{s_1^I, r_1^I\} \subseteq \mathcal{E}$, where s_1^I is the pool on high strategy.*

(i) If r_1^I is the pool on low strategy, then $\mathcal{E}$ is not justifiable.

(ii) If r_1^I is the monotonic separation strategy, then $\mathcal{E}$ is not justifiable.

To prove Proposition B.3, we make use of the following auxiliary Lemma.

Lemma B.12 *Let s_1^I be the pool on high strategy.*

(i) If r_1^I is the pool on low strategy and $\{s_1^I, r_1^I\}$ is justifiable, then $\mathrm{IR}\,(\mathrm{PH}\,, \underline{\theta}) \geq \mathrm{IR}\,(\mathrm{PL}\,, \underline{\theta})$.

(ii) If r_1^I is the monotonic separation strategy and $\{s_1^I, r_1^I\}$ is justifiable, then $\mathrm{IR}\,(\mathrm{PH}\,, \underline{\theta}) \geq \mathrm{IR}\,(\mathrm{MS}\,, \underline{\theta})$.

Proof. Suppose (B, c) justifies $\{s_1^I, r_1^I\}$, where s_1^I is the pool on high strategy and r_1^I is either the pool on low strategy or the monotonic separation strategy. It suffices to show that $(B - c(\underline{a})) > 0$. If so, the result follows from Lemma B.9.

Use the fact that (B, c) justifies the pool on high strategy. Then, by Lemma B.9(i), $\mathrm{IR}\,(\mathrm{PH}\,, \underline{\theta})(B - c(\underline{a})) > 0$. By Lemma B.11(i), $\mathrm{IR}\,(\mathrm{PH}\,, \underline{\theta}) > 0$. It follows that $B > c(\underline{a})$, as required. ■

Proof of Proposition B.3. Suppose $\{s_1^I, r_1^I\}$ is justifiable, where s_1^I is the pool on high strategy.

Begin with part (i), where r_1^I is the pool on low strategy. By Lemmata B.10-B.12, $\mathrm{R}\,(\underline{\theta}, \overline{g} - f(\overline{a}, \underline{\theta})) \geq \mathrm{R}\,(\underline{\theta}, \underline{g} - f(\overline{a}, \underline{\theta}))$. Note,

$$\overline{g} - f(\overline{a}, \underline{\theta}) = \frac{f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2} = -\frac{r(\underline{\theta})}{2} + \frac{f(\overline{a}, \overline{\theta}) - f(\underline{a}, \underline{\theta})}{2} > -\frac{r(\underline{\theta})}{2}$$

Since $\overline{g} - f(\overline{a}, \underline{\theta}) > \underline{g} - f(\overline{a}, \underline{\theta})$, Lemma B.11 implies that a necessary condition for $\mathrm{R}\,(\underline{\theta}, \overline{g} - f(\overline{a}, \underline{\theta})) \geq \mathrm{R}\,(\underline{\theta}, \underline{g} - f(\overline{a}, \underline{\theta}))$ is that

$$\underline{g} - f(\overline{a}, \underline{\theta}) \leq -\frac{r(\underline{\theta})}{2} - \frac{f(\overline{a}, \overline{\theta}) - f(\underline{a}, \underline{\theta})}{2}.$$

Using the fact that

$$\underline{g} - f(\overline{a}, \underline{\theta}) = -\frac{r(\underline{\theta})}{2} + \frac{f(\underline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2},$$

a necessary condition for $\mathrm{R}\left(\underline{\theta}, \overline{g} - f(\overline{a}, \underline{\theta})\right) \geq \mathrm{R}\left(\underline{\theta}, \underline{g} - f(\overline{a}, \underline{\theta})\right)$ is that

$$f(\underline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \leq f(\underline{a}, \underline{\theta}) - f(\overline{a}, \overline{\theta}).$$

But this implies that

$$f(\underline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \leq f(\underline{a}, \overline{\theta}) - f(\overline{a}, \overline{\theta}).$$

and so $f(\overline{a}, \overline{\theta}) \leq f(\overline{a}, \underline{\theta})$, a contradiction.

Now turn to part (ii), where r_1^I is the monotonic separation strategy. By Lemmata B.10-B.12, $\mathrm{R}\left(\underline{\theta}, \overline{g} - f(\overline{a}, \underline{\theta})\right) \geq \mathrm{R}\left(\underline{\theta}, g^m - f(\overline{a}, \underline{\theta})\right)$. But,

$$g^m - f(\overline{a}, \underline{\theta}) = -\frac{r(\underline{\theta})}{2} + \frac{f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2} > -\frac{r(\underline{\theta})}{2}.$$

But now, using Lemma B.11 and the fact that $\overline{g} - f(\overline{a}, \underline{\theta}) > g^m - f(\overline{a}, \underline{\theta})$, we have that $\mathrm{R}\left(\underline{\theta}, g^m - f(\overline{a}, \underline{\theta})\right) > \mathrm{R}\left(\underline{\theta}, \overline{g} - f(\overline{a}, \underline{\theta})\right)$, a contradiction. ■

Proposition B.4 *Let s_1^I be the pool on low strategy and r_1^I be the monotonic separation strategy.*

(i) The set $\{s_1^I, r_1^I\}$ is justifiable if and only if $\mathrm{IR}\left(\mathrm{MS}, \overline{\theta}\right) \geq \max\{\mathrm{IR}\left(\mathrm{MS}, \underline{\theta}\right), \mathrm{IR}\left(\mathrm{PL}, \underline{\theta}\right)\}$.

(ii) The set $\{s_1^I, r_1^I\}$ is strictly justifiable if and only if $\mathrm{IR}\left(\mathrm{MS}, \overline{\theta}\right) > \max\{\mathrm{IR}\left(\mathrm{MS}, \underline{\theta}\right), \mathrm{IR}\left(\mathrm{PL}, \underline{\theta}\right)\}$.

Lemma B.13 *The following inequality holds:* $\mathrm{IR}\left(\mathrm{MS}, \overline{\theta}\right) > \mathrm{IR}\left(\mathrm{PL}, \overline{\theta}\right)$.

Proof. Note, by Lemma B.10, $\mathrm{IR}\left(\mathrm{MS}, \overline{\theta}\right) > \mathrm{IR}\left(\mathrm{PL}, \overline{\theta}\right)$ if and only if $\mathrm{R}\left(\overline{\theta}, g^m - f(\overline{a}, \theta)\right) > \mathrm{R}\left(\overline{\theta}, \underline{g} - f(\overline{a}, \theta)\right)$. We show this inequality.

Observe that

$$g^m - f(\overline{a}, \overline{\theta}) = \frac{f(\underline{a}, \underline{\theta}) - f(\overline{a}, \overline{\theta})}{2} < \frac{f(\underline{a}, \overline{\theta}) - f(\overline{a}, \overline{\theta})}{2} = -\frac{r(\overline{\theta})}{2}.$$

So, using Lemma B.11(ii) and the fact that $g^m - f(\overline{a}, \overline{\theta}) > \underline{g} - f(\overline{a}, \overline{\theta})$, $\mathrm{IR}\left(\mathrm{MS}, \overline{\theta}\right) > \mathrm{IR}\left(\mathrm{PL}, \overline{\theta}\right)$. ■

Proof of Proposition B.4. Part (i): Begin with the "only if." Suppose (B, c) justifies both the pool on low and monotonic separation strategies. Since it justifies the monotonic separation strategy and (by definition of a cost function) $c(\overline{a}) > c(\underline{a})$, it follows from Lemma B.9 that $\text{IR}\,(\text{MS}\,,\overline{\theta})(B - c(\underline{a})) > 0$. Now note that, by Lemma B.11(i), $\text{IR}\,(\text{MS}\,,\overline{\theta}) > 0$. It follows that $B > c(\underline{a})$. With this, Lemma B.9 gives that $\text{IR}\,(\text{MS}\,,\overline{\theta}) \geq \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$.

Now turn to the "if." Suppose $\text{IR}\,(\text{MS}\,,\overline{\theta}) \geq \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$. It follows from Lemma B.13 that $\text{IR}\,(\text{MS}\,,\overline{\theta}) \geq \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\overline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$. As such, then there exists some (B, c) so that $B > c(\underline{a})$ and

$$\text{IR}\,(\text{MS}\,,\overline{\theta}) \geq \frac{c(\overline{a}) - c(\underline{a})}{B - c(\underline{a})} \geq \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\overline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}.$$

It follows from Lemma B.9 that $\{s_1^I, r_1^I\}$ is justifiable.

Part (ii): Begin with the "only if." Suppose that $\text{IR}\,(\text{MS}\,,\overline{\theta}) \leq \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$. Certainly this conclusion follows (from part (ii)) if $\text{IR}\,(\text{MS}\,,\overline{\theta}) < \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$. Suppose $\text{IR}\,(\text{MS}\,,\overline{\theta}) = \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$. Then, using Lemma B.9, for each (B, c) that justifies the monotonic separation strategy r_1^I,

$$c(\overline{a}) = \text{IR}\,(\text{MS}\,,\overline{\theta})(B - c(\underline{a})) + c(\underline{a}).$$

A consequence is that, for each pair B and $c(\underline{a})$ there is (at most) a unique number $c(\overline{a})$ so that the pair (B, c) justifies $\{s_1^I, r_1^I\}$. As such, there is no non-empty open set of benefits of reelections and cost functions that justify $\{s_1^I, r_1^I\}$.

Now turn to the "if." Suppose that $\text{IR}\,(\text{MS}\,,\overline{\theta}) > \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$. Then, using Lemma B.13, there is a non-empty open set $\mathcal{U} \subseteq \mathbb{R}$ so that, for each $x \in \mathcal{U}$, $\text{IR}\,(\text{MS}\,,\overline{\theta}) > x > \max\{\text{IR}\,(\text{MS}\,,\underline{\theta}), \text{IR}\,(\text{PL}\,,\overline{\theta}), \text{IR}\,(\text{PL}\,,\underline{\theta})\}$. Note, there exists a continuous onto function $g : \mathcal{C}^+ \to (0, \infty)$ so that

$$g(B, c(\underline{a}), c(\overline{a})) = \frac{c(\overline{a}) - c(\underline{a})}{B - c(\underline{a})}.$$

So, $(g)^{-1}(\mathcal{U})$ is a non-empty open set in $\mathcal{C}^+$. So, using Lemma A.1, $(g)^{-1}(\mathcal{U})$ is a non-empty open set in $\mathcal{C}$. Moreover, for each $(B, c) \in (g)^{-1}(\mathcal{U})$,

- $\text{IR}\,(\text{MS}\,,\overline{\theta})(B - c(\underline{a})) > c(\overline{a}) - c(\underline{a})$;
- $c(\overline{a}) - c(\underline{a}) > \text{IR}\,(\text{MS}\,,\underline{\theta})(B - c(\underline{a}))$;
- $c(\overline{a}) - c(\underline{a}) > \text{IR}\,(\text{PL}\,,\overline{\theta})(B - c(\underline{a}))$;
- $c(\overline{a}) - c(\underline{a}) > \text{IR}\,(\text{PL}\,,\underline{\theta})(B - c(\underline{a}))$.

It follows from Lemma B.9 that $\{s_1^I, r_1^I\}$ is strictly justifiable. ■

Lemma B.14

(i) $\mathrm{IR}(\mathrm{MS}, \overline{\theta}) \geq \max\{\mathrm{IR}(\mathrm{MS}, \underline{\theta}), \mathrm{IR}(\mathrm{PL}, \underline{\theta})\}$ *if the following conditions are met:*

- $f(\overline{a}, \overline{\theta}) - f(\underline{a}, \overline{\theta}) \geq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \underline{\theta})$.
- $f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \leq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \overline{\theta})$.

(ii) $\mathrm{IR}(\mathrm{MS}, \overline{\theta}) > \max\{\mathrm{IR}(\mathrm{MS}, \underline{\theta}), \mathrm{IR}(\mathrm{PL}, \underline{\theta})\}$ *if the following conditions are met:*

- $f(\overline{a}, \overline{\theta}) - f(\underline{a}, \overline{\theta}) > f(\overline{a}, \underline{\theta}) - f(\underline{a}, \underline{\theta})$.
- $f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) < f(\overline{a}, \underline{\theta}) - f(\underline{a}, \overline{\theta})$.

Proof. We show part (i). (Part (ii) is analogous.) Here is the structure of the proof: We first show that $\mathrm{IR}(\mathrm{MS}, \overline{\theta}) \geq \mathrm{IR}(\mathrm{MS}, \underline{\theta})$ if and only if $f(\overline{a}, \overline{\theta}) - f(\underline{a}, \overline{\theta}) \geq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \underline{\theta})$. Then, we show that $\mathrm{IR}(\mathrm{MS}, \underline{\theta}) \geq \mathrm{IR}(\mathrm{PL}, \underline{\theta})$ if and only if $f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \leq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \overline{\theta})$. Taken together, we get the desired result.

Observe that $\mathrm{IR}(\mathrm{MS}, \overline{\theta}) \geq \mathrm{IR}(\mathrm{MS}, \underline{\theta})$ holds if and only if

$$\Phi\left(g^m - f(\underline{a}, \overline{\theta})\right) - \Phi\left(g^m - f(\overline{a}, \overline{\theta})\right) \geq \Phi\left(g^m - f(\underline{a}, \underline{\theta})\right) - \Phi\left(g^m - f(\overline{a}, \underline{\theta})\right)$$

or, if and only if

$$\Phi\left(\frac{f(\overline{a}, \overline{\theta}) + f(\underline{a}, \underline{\theta}) - 2f(\underline{a}, \overline{\theta})}{2}\right) - \Phi\left(\frac{f(\underline{a}, \underline{\theta}) - f(\overline{a}, \overline{\theta})}{2}\right) \geq \Phi\left(\frac{-f(\underline{a}, \underline{\theta}) + f(\overline{a}, \overline{\theta})}{2}\right) - \Phi\left(\frac{f(\overline{a}, \overline{\theta}) + f(\underline{a}, \underline{\theta}) - 2f(\overline{a}, \underline{\theta})}{2}\right).$$

Using symmetry, each $\Phi(x) = 1 - \Phi(-x)$. So $\mathrm{IR}(\mathrm{MS}, \overline{\theta}) \geq \mathrm{IR}(\mathrm{MS}, \underline{\theta})$ holds if and only if

$$\Phi\left(\frac{f(\overline{a}, \overline{\theta}) + f(\underline{a}, \underline{\theta}) - 2f(\underline{a}, \overline{\theta})}{2}\right) - \Phi\left(\frac{2f(\overline{a}, \underline{\theta}) - f(\overline{a}, \overline{\theta}) - f(\underline{a}, \underline{\theta})}{2}\right) \geq 0.$$

So, by monotonicity of the CDF Φ, $\mathrm{IR}(\mathrm{MS}, \overline{\theta}) \geq \mathrm{IR}(\mathrm{MS}, \underline{\theta})$ holds if and only if

$$\frac{f(\overline{a}, \overline{\theta}) + f(\underline{a}, \underline{\theta}) - 2f(\underline{a}, \overline{\theta})}{2} \geq \frac{2f(\overline{a}, \underline{\theta}) - f(\overline{a}, \overline{\theta}) - f(\underline{a}, \underline{\theta})}{2}$$

or

$$f(\overline{a}, \overline{\theta}) - f(\underline{a}, \overline{\theta}) \geq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \underline{\theta}),$$

as required.

Next note that, by Lemma B.10, $\mathrm{IR}(\mathrm{MS}, \underline{\theta}) \geq \mathrm{IR}(\mathrm{PL}, \underline{\theta})$ if and only if $\mathrm{R}(\underline{\theta}, g^m - f(\overline{a}, \underline{\theta})) \geq \mathrm{R}(\underline{\theta}, \underline{g} - f(\overline{a}, \underline{\theta}))$. Note that

$$g^m - f(\overline{a}, \underline{\theta}) = \frac{f(\overline{a}, \overline{\theta}) + f(\underline{a}, \underline{\theta}) - 2f(\overline{a}, \underline{\theta})}{2} = -\frac{r(\underline{\theta})}{2} + \frac{f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2} > -\frac{r(\underline{\theta})}{2}.$$

Since $g^m - f(\overline{a}, \underline{\theta}) > \underline{g} - f(\overline{a}, \underline{\theta})$, it follows from Lemma B.10(ii)-(iii) that $\mathrm{R}(\underline{\theta}, g^m - f(\overline{a}, \underline{\theta})) \geq \mathrm{R}(\underline{\theta}, \underline{g} - f(\overline{a}, \underline{\theta}))$ if and only if

$$\begin{aligned} \underline{g} - f(\overline{a}, \underline{\theta}) &\leq -\frac{r(\underline{\theta})}{2} - [g^m - f(\overline{a}, \underline{\theta})] \\ &= -\frac{r(\underline{\theta})}{2} - \frac{f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2}. \end{aligned}$$

Note

$$\begin{aligned} \underline{g} - f(\overline{a}, \underline{\theta}) &= \frac{f(\underline{a}, \overline{\theta}) + f(\underline{a}, \underline{\theta}) - 2f(\overline{a}, \underline{\theta})}{2} \\ &= -\frac{r(\underline{\theta})}{2} + \frac{f(\underline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2} \end{aligned}$$

So

$$\underline{g} - f(\overline{a}, \underline{\theta}) \leq -\frac{r(\underline{\theta})}{2} - \frac{f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2}$$

if and only if

$$-\frac{r(\underline{\theta})}{2} + \frac{f(\underline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2} \leq -\frac{r(\underline{\theta})}{2} - \frac{f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta})}{2}$$

or, equivalently, if and only if

$$f(\overline{a}, \overline{\theta}) - f(\overline{a}, \underline{\theta}) \leq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \overline{\theta}),$$

as required. ■

Corollary B.1 *Let s_1^I be the pool on low strategy and r_1^I be the monotonic separation strategy.*

(i) The set $\{s_1^I, r_1^I\}$ is justifiable if

- $f(\overline{a}, \overline{\theta}) - f(\underline{a}, \overline{\theta}) \geq f(\overline{a}, \underline{\theta}) - f(\underline{a}, \underline{\theta})$.

- $f(\overline{a},\overline{\theta}) - f(\overline{a},\underline{\theta}) \leq f(\overline{a},\underline{\theta}) - f(\underline{a},\overline{\theta})$.

(ii) The set $\{s_1^I, r_1^I\}$ is strictly justifiable if

- $f(\overline{a},\overline{\theta}) - f(\underline{a},\overline{\theta}) > f(\overline{a},\underline{\theta}) - f(\underline{a},\underline{\theta})$.
- $f(\overline{a},\overline{\theta}) - f(\overline{a},\underline{\theta}) < f(\overline{a},\underline{\theta}) - f(\underline{a},\overline{\theta})$.

Appendix C The General Analysis

It will be useful to fix mathematical conventions that we will use throughout: Fix metrizable spaces X and Y. Let $\overrightarrow{y} : X \to Y$ be measurable. Write $[y]$ for $\overrightarrow{y}^{-1}(y) = \{x : \overrightarrow{y}(x) = y\}$, which is again a measurable set. Given a probability measure μ on X, write $\mu[\overrightarrow{y}]$ for the image measure of μ under $\overrightarrow{y}$, i.e., the measure ν on Y obtained by setting each $\nu(E) = \mu(\overrightarrow{y}^{-1}(E))$. Write μ^2 for a probability measure on $X \times X$ that is the product measure obtained by $\mu \times \mu$.

Let Z be a metrizable space and let $\overrightarrow{z} : X \times Y \to Z$ be a measurable mapping. Write $(\overrightarrow{z}, \overrightarrow{y}) : X \to Z$ for the mapping with $(\overrightarrow{z}, \overrightarrow{y})(y, z) = (\overrightarrow{z}(z, \overrightarrow{y}(y))$ If $\overrightarrow{y}$ is a constant mapping (i.e., for each x, $\overrightarrow{y}(x) = y_*$), we abuse notation and write $(\overrightarrow{z}, y_*)$ for the mapping $(\overrightarrow{z}, \overrightarrow{y})$.

C.1 Production and Informational Environment

Recall, the production function is a function $F : A \times \Theta \times \Sigma \to \mathbb{R}$. This is assumed to be a measurable integrable function. Fix a Politician P and a period t. Then, the production of public goods is determined by the state and the action chosen. Write $F_t^P : A \times \Omega \to \mathbb{R}$ for the **(P,t)-induced production function**, where $F_t^P(a, \omega) = F(a, \overrightarrow{\theta}^P(\omega), \overrightarrow{\sigma}_t(\omega))$. Note, the induced production function is Politician specific, since the mapping from states to production depends on the mapping from states into types. The induced production function is also time specific, since the mapping from states to production depends on the mapping from states into shocks. It is readily verified that F_t^P is an integrable random variable.

We impose certain primitive restrictions on the informational environment and the production technology. These assumptions should be seen as joint assumptions on the prior μ and the production function F.

To preview: The first two assumptions concern the Voter. Taken together they imply that better realizations of first-period public goods provide no news about the Challenger but good news about the Incumbent. Thus, in a sense, they ensure that the Voter selects for good types in "all equilibria." (See Proposition C.1 below.) The latter two assumptions concern the Incumbent. The key assumption here is that, ex ante, the Incumbent thinks higher effort is (strictly) more likely to result in meeting a certain benchmark.

Begin with the assumption that realizations about the first-period public goods provide no information about the Challenger. Formally:

Assumption C.1 *For each monotone first-period strategy s_1^I, $F_1^I(s_1^I(\cdot),\cdot)$ and $F_2^C(\underline{a},\cdot)$ are μ-independent.*

Assumption C.1 says that the level of first-period public goods provided by the Incumbent does not provide information about the level of public goods that the Challenger provides in the second period, provided the Challenger chooses the lowest action.

Now turn to the condition that better realizations of first-period public goods provide good news about the Incumbent: To express this assumption, we must first specify how the Voter updates given information about the level of public goods provided. Note that the $(I,1)$-induced production function and a given first-period strategy s_1^I uniquely determine how states map into the provision of public goods, i.e., there is a measurable mapping $(F_1^I, s_1^I) : \Omega \to \mathbb{R}$ so that $(F_1^I, s_{1,}^I)(\omega) = F_1^I(s_1^I(\omega),\omega)$. So, if the Voter sees a first-period level of public goods g_1 and expects that the Incumbent's first period strategy is s_1^I, the Voter thinks that the set of states that are possible are those with $(F_1^I, s_1^I)(\omega) = g_1$. Thus, the Voter conditions on the subalgebra generated by (F_1^I, s_1^I).

Fix a **monotone first period strategy**, viz., s_1^I, i.e., a strategy with $s_1^I(\omega) \geq s_1^I(\omega')$ whenever $\overrightarrow{\theta}^I(\omega) \geq \overrightarrow{\theta}^I(\omega')$. (Note, in the case of symmetric uncertainty, all strategies are monotone.) Write $\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_1^I)] : \Omega \to \mathbb{R}$ for a version of conditional expected value of $F_2^I(\underline{a},\cdot)$ given the subalgebra generated by (F_1^I, s_1^I) where s_1^I is monotone.[9] Note, by definition of a version of conditional expected value, $\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_1^I)](\omega) = \mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_1^I)](\omega')$, whenever $(F_1^I, s_1^I)(\omega) = (F_1^I, s_1^I)(\omega')$. So, we can abuse notation and, for a given level of public goods g_1, set

$$\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_1^I)]([g_1]) = \mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_1^I)](\omega),$$

for some $\omega \in ((F_1^I, s_1^I)^{-1}(g_1))$. Note, $[g_1]$ should be viewed as the event $((F_1^I, s_1^I)^{-1}(\{g_1\}))$ and so depends on the subalgebra generated by (F_1^I, s_1^I).

[9] Appendix A explained why we restrict attention to monotonicity.

Assumption C.2 *Fix a monotone strategy, viz.* s_1^I*. Each version of conditional expected value, viz.* $\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I,s_1^I)]$*, satisfies the following: If* $g_1 > g_1'$*, then*

$$\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I,s_1^I)]([g_1]) > \mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I,s_1^I)]([g_1']), \tag{15}$$

μ^2*-almost surely.*

Assumption C.2 says that, the conditional expected value of Incumbent's second-period production, when she chooses the low action, is almost surely increasing in the level of public goods the Voter observes.

Next, turn to the Incumbent. Write $[s_1^I, r_1^I; \tau] \subset \Omega$ for the set of states at which the Voter's conditional expectation meets a certain threshold τ, when the Voter expects the Incumbent to choose s_1^I and she actually chooses r_1^I. That is,

$$[s_1^I, r_1^I; \tau] = (\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I,s_1^I)])^{-1}([\tau,\infty)) \cap (F_1^I, r_1^I)^{-1}([\tau,\infty)).$$

(Note, the first argument of $[s_1^I, r_1^I; \tau]$ indicates the Voter's expectation and the second indicates the actual strategy chosen.)

Notice an implication of Assumption C.2: If $r_1^I > q_1^I$ and choosing q_1^I meets a threshold in terms of conditional expectation, then it is almost surely the case that choosing r_1^I also meets that threshold. From this it follows that, if $r_1^I > q_1^I$, then $\mu([s_1^I, r_1^I;\tau]) \geq \mu([s_1^I, q_1^I;\tau])$ for each $\tau \in \mathbb{R}$. This says that, from an ex ante perspective, a threshold (in terms of the Voter's conditional expectation) is weakly more likely to be met when the Incumbent chooses a higher level of effort. Likewise, a threshold (in terms of the Voter's conditional expectation) is weakly more likely to be met when the Incumbent chooses a higher level of effort and knows he is of a particular type, i.e., if $r_1^I > q_1^I$, then $\mu([s_1^I, r_1^I;\tau] \cap [\theta]) \geq \mu([s_1^I, q_1^I;\tau] \cap [\theta])$ for each $\theta \in \Theta$ and $\tau \in \mathbb{R}$.

The next assumption strengthens these implications. It asks that, from an ex ante perspective, a threshold (in terms of the Voter's conditional expectation) is strictly more likely to be met when the Incumbent chooses a higher level of effort and learns a particular realization of the signal. Recall, Ψ is the set of Politician signals.

Assumption C.3 *Fix a monotone strategy* s_1^I*. If* $r_1^I > q_1^I$*, then* $\mu([s_1^I, r_1^I;\tau] \cap \psi) > \mu([s_1^I, q_1^I;\tau] \cap \psi)$ *for each* $\tau \in \mathbb{R}$ *and each* $\psi \in \Psi$*.*

Assumption C.3 should be viewed as an analogue of a full support condition. (Since μ has full support, it is satisfied when $\psi \cap [s_1^I, r_1^I;\tau] \backslash [s_1^I, q_1^I;\tau]$ contains a non-empty open set.)

Notice, the assumption differs based on whether the game is the symmetric vs. asymmetric uncertainty game, since in the two cases the signals differ.

One final assumption

Assumption C.4 *For each monotone first-period strategy s_1^I and each first-period strategy r_1^I, the set*

$$(\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I,s_1^I)])^{-1}(\{\mathbb{E}(F_2^C(\cdot,\underline{a}))\}) \cap (F_1^I,r_1^I)^{-1}(\{\mathbb{E}(F_2^C(\cdot,\underline{a}))\})$$

is μ-null.

Suppose the Voter expects the Incumbent to choose s_1^I and the Incumbent actually chooses r_1^I. Assumption C.4 says that, ex ante, there is zero probability that the Voter's conditional expectation of the Incumbent coincides with his ex ante expectation of the Challenger. Under this assumption, the Incumbent's best response does not depend on the Voter's reelection decision, in the case where he is indifferent between the Incumbent and the Challenger. Thus, if there are indeed multiple equilibria satisfying conditions (i)-(ii) of Section 1.5, it cannot be viewed as a result of how the Voter behaves in (what would ordinarily be viewed as) a "degenerate case."

C.2 Analysis: Selecting Good Types

Begin with the second governance period. Again:

Remark C.1 *Fix a pure strategy perfect Bayesian equilibrium, viz. (s_*^I, s_*^V, s_*^C). At each (second-period) information set, $s_{2,*}^I$ (viz. s^C) chooses $\underline{a}$.*

Now, turn to the Voter. In a perfect Bayesian equilibrium, each politician chooses $\underline{a}$ irrespective of the history. As such, it is a best response for the Voter to reelect the Incumbent if and only if his conditional expectation of $F_2^I(\cdot,\underline{a})$ is greater than his expectation of $F_2^C(\cdot,\underline{a})$. Notice, these expectations are conditional on the realization of first period public goods provision. Moreover, they depend on the Voter's beliefs about the Incumbent's first period action. Specifically, if the Voter sees a first-period level of public goods g_1 and expects that the Incumbent's first period strategy is $s_{1,*}^I$, the Voter thinks that the set of states that are possible are those with $(F_1^I, s_{1,*}^I)(\omega) = g_1$. Thus, the Voter conditions on the subalgebra generated by $(F_1^I, s_{1,*}^I)$. Since we restrict attention to monotonic equilibria, here $s_{1,*}^I$ is monotonic.

Thus, in a perfect Bayesian equilibrium, after observing a first period level of public goods g_1, the Voter reelects the Incumbent if and only if

$$\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_{1,*}^I)]([g_1]) \geq \mathbb{E}[F_2^C(\underline{a},\cdot)|(F_1^I, s_{1,*}^I)]([g_1]).$$

Note, $\mathbb{E}[F_2^C(\underline{a},\cdot)|(F_1^I, s_{1,*}^I)]([g_1])$ is equal to the unconditional expectation of $F_2^C(\cdot,\underline{a})$, viz. $\mathbb{E}[F_2^C(\underline{a},\cdot)]$, almost surely. (This follows from Assumption C.1.) So, it is almost surely the case that the Voter reelects the Incumbent if and only if

$$\mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_{1,*}^I)]([g_1]) \geq \mathbb{E}[F_2^C(\underline{a},\cdot)].$$

This condition says that (it is almost surely the case that) the Voter reelects the Incumbent if and only if his conditional expectation of the public goods the Incumbent will provide (in the second-period) surpasses a threshold in the space of conditional expectations (namely, $\mathbb{E}[F_2^C(\underline{a},\cdot)]$). Assumption C.2 gives that increasing the level of public goods provides a higher conditional expectation, almost surely. As such, it implies that the Voter adopts a reelection strategy that is equivalent to one where he reelects the Incumbent if and only if she provides a level of first-period public goods that meets some benchmark in public goods space. That is, the Voter sets a standard, viz. g_*, in the space of public goods.

In what follows, we write G for the range of F_1^I, which is also the domain of the Voter's strategy.

Definition C.1 *Say a profile (s^I, s^V, s^C) is a **cut-off profile** if there exists a realization of public goods, viz. $g_* \in [-\infty,\infty]$, so that*

(i) the set $\{g \in G : g > g_* \text{ and } s^V(g) = 0\}$ is $((F_1^I, s_1^I)(\mu))$-null;

(ii) the set $\{g \in G : g < g_* \text{ and } s^V(g) = 1\}$ is $((F_1^I, s_1^I)(\mu))$-null.

A cut-off profile, viz. (s^I, s^V, s^C), is one in which there is some (perhaps infinite) level of public goods, viz. g_*, so that the Voter almost surely reelects the Incumbent if the level of public goods provided is above g_* and the Voter almost surely elects the Challenger if the level of public goods provided is below g_*. Notice, if s^V is a cut-off strategy (per Definition B.1), then (s^I, s^V, s^C) is necessarily a cut-off profile (per Definition C.1). But, (s^I, s^V, s^C) may be a cut-off profile even if s^V is not a cut-off strategy.

Lemma C.1 *Any monotone pure-strategy perfect Bayesian equilibrium, viz. (s_*^I, s_*^V, s_*^C), is a cut-off profile.*

Proof. Begin by defining g_*. For this consider the set

$$G^+ = \{g \in G : \mathbb{E}[F_2^I(\underline{a},\cdot)|(F_1^I, s_{1,*}^I)]([g]) \geq \mathbb{E}[F_2^C(\underline{a},\cdot)],\ \mu\text{-almost surely}\}.$$

If $G^+ \neq \emptyset$, set $g_* = \inf G^+$. If $G^+ = \emptyset$, set $g_* = \infty$.

First, consider the set $\{g \in G : g > g_* \text{ and } s_*^V(g) = 0\}$. If $g_* = \infty$, then the set is empty and so $((F_1^I, s_{1,*}^I)(\mu))$-null. If $g_* \neq \infty$, then using the fact that the Voter optimizes conditional upon each level of public goods g and Assumption C.2, the inverse image of $\{g \in G : g > g_* \text{ and } s_*^V(g) = 0\}$ under $(F_1^I, s_{1,*}^I)$ is μ-null. It follows that the set $\{g \in G : g > g_* \text{ and } s_*^V(g) = 0\}$ is $(F_1^I, s_{1,*}^I)(\mu)$-null.

Next, consider the set $\{g \in G : g < g_* \text{ and } s_*^V(g) = 1\}$. Using the fact that the Voter optimizes conditional upon each level of public goods g and Assumption C.2, the inverse image of $\{g \in G : g < g_* \text{ and } s_*^V(g) = 1\}$ under $(F_1^I, s_{1,*}^I)$ is μ-null. It follows that the set $\{g \in G : g < g_* \text{ and } s_*^V(g) = 1\}$ is $(F_1^I, s_{1,*}^I)(\mu)$-null. ■

Now consider states $\omega, \omega' \in \Omega$ with $(\overrightarrow{\theta}^I(\omega), \overrightarrow{\sigma}_1(\omega)) = (\theta, \sigma)$, $(\overrightarrow{\theta}^I(\omega'), \overrightarrow{\sigma}_1(\omega')) = (\theta', \sigma)$ and $\theta > \theta'$. Because the two states have the same shocks and $s_{1,*}^I$ is monotone, it follows that the level of public goods produced at ω is strictly greater than the level of public goods produced at ω'. So, by Lemma C.1, it is almost surely the case that, if the Voter reelects the Incumbent when the state is ω' then the Voter also reelects the Incumbent when the states is ω.

Definition C.2 *Fix a perfect Bayesian equilibrium, viz.* (s_*^I, s_*^V, s_*^C). *Say, in this equilibrium, the* ***Voter almost surely selects good types*** *if the following holds the following holds: If* ω *and* ω' *are states with* $\overrightarrow{\theta}^I(\omega) > \overrightarrow{\theta}^I(\omega')$, $\overrightarrow{\sigma}_1(\omega) = \overrightarrow{\sigma}_1(\omega')$, *and* $s^V(F_1^I(\omega', s_{1,*}^I(\omega'))) = 1$, *then* $s^V(F_1^I(\omega, s_{1,*}^I(\omega))) = 1$ μ*-almost surely.*

Proposition C.1 (Selecting Good Types) *In any monotone pure-strategy perfect Bayesian equilibrium, the Voter almost surely selects good types.*

The proof is immediate from Lemma C.1.

C.3 Creating Incentives

We first consider the case of symmetric uncertainty and then turn to the case of asymmetric uncertainty.

Symmetric Uncertainty

This section is devoted to proving Proposition 3.2.

Fix a monotone pure-strategy perfect Bayesian equilibrium (s^I_*, s^V_*, s^C_*) with $s^I_{1,*}(\Omega) = \{a_*\}$. Suppose the Incumbent actually chooses r^I_1 with $r^I_1(\Omega) = \{a\}$. Then, the Incumbent assesses the likelihood of reelection as $\mu([s^I_{1,*}, r^I_1, \mathbb{E}(F^C_1(\underline{a}, \cdot))])$. As in the main text, we write

$$\Pr(a|a_*) := \mu([s^I_{1,*}, r^I_1, \mathbb{E}(F^C_1(\underline{a}, \cdot))]).$$

Note, $[s^I_{1,*}, r^I_1, \mathbb{E}(F^C_1(\underline{a}, \cdot))]$ can be computed based on primitives alone and so $\Pr(a|a_*)$ can be computed based on primitives alone. In turn, from this, we can compute the incremental increase in probability of reelection from choosing a over a', when the Voter expects $s^I_{1,*}(\Omega) = \{a_*\}$, namely $\mathrm{IR}\,(a, a'|a_*) = \Pr(a|a_*) - \Pr(a'|a_*)$.

Lemma C.2 *Fix a monotone pure-strategy perfect Bayesian equilibrium, viz.* (s^I_*, s^V_*, s^C_*), *with* $s^I_{1,*}(\Omega) = \{a_*\}$. *If* $a > a'$ *then*

(i) $\Pr(a|a_*) > \Pr(a'|a_*)$

(ii) $\mathrm{IR}\,(a, a'|a_*) > 0$.

The lemma is immediate from Assumption C.3.

Lemma C.3 *Fix a first-period strategy* s^I_1 *with* $s^I_1(\Omega) = \{a_*\}$. *The pair* (B, c) *justifies* $\{s^I_1\}$ *if and only if* $c(a) \geq \mathrm{IR}\,(a, a_*|a_*)(B - c(\underline{a})) + c(a_*)$ *for each* $a \in A$.

Proof. First suppose that (B, c) justifies s^I_1. Then, for each action $a \in A$,

$$\Pr(a_*|a_*)(B - c(\underline{a})) + (B - c(a_*)) \geq \Pr(a|a_*)(B - c(\underline{a})) + (B - c(a)),$$

and so $c(a) \geq [\Pr(a|a_*) - \Pr(a_*|a_*)](B - c(\underline{a})) + c(a_*)$ for each $a \in A$.

Conversely, suppose that $c(a) \geq \mathrm{IR}\,(a, a_*|a_*)(B - c(\underline{a})) + c(a_*)$ for each $a \in A$. Construct $s^I_2(\Omega) = s^C(\Omega) = \{\underline{a}\}$ and $s^V(g) = 1$ if and only if $\mathbb{E}[F^I_2(\cdot, \underline{a})]([g]) \geq \mathbb{E}[F^C_2]$. It is readily verified that (s^I, s^V, s^C) is indeed a perfect Bayesian equilibrium of the game. ■

Lemma C.4 *Suppose* (B, c) *justifies* s^I_1 *with* $s^I_1(\Omega) = \{a_*\}$ *and* $a_* > \underline{a}$. *Then,* $B > c(\underline{a})$.

Proof. Suppose, contra hypothesis, (B, c) justifies s^I_1 with $s^I_1(\Omega) = \{a_*\}$, $a_* > \underline{a}$ and $c(\underline{a}) \geq B$. Then, by Lemma C.3,

$$c(\underline{a}) \geq \mathrm{IR}\,(\underline{a}, a_*|a_*)(B - c(\underline{a})) + c(a_*).$$

Using Lemma C.2 and the fact that $c(\underline{a}) \geq B$, $\text{IR}\,(\underline{a}, a_*|a_*)(B - c(\underline{a})) \geq 0$. As such,

$$c(\underline{a}) \geq \text{IR}\,(\underline{a}, a_*|a_*)(B - c(\underline{a})) + c(a_*) \geq c(a_*),$$

a contradiction. ■

Proof of Proposition 3.2. Fix strategies s_1^I and r_1^I with $s_1^I(\Omega) = \{a_{**}\}$, $r_1^I(\Omega) = \{a_*\}$, and $a_{**} > a_*$.

Begin with the "only if." Suppose (B, c) justifies $\{s_1^I, r_1^I\}$. Applying Lemma C.3 to the strategy s_1^I and actions a_* and $\underline{a}$ gives

I. $c(a_*) \geq \text{IR}\,(a_*, a_{**}|a_{**})(B - c(\underline{a})) + c(a_{**})$, and

II. $c(\underline{a}) \geq \text{IR}\,(\underline{a}, a_{**}|a_{**})(B - c(\underline{a})) + c(a_{**})$.

Applying Lemma C.3 to the strategy r_1^I and actions a_{**} and $\underline{a}$ gives

III. $c(a_{**}) \geq \text{IR}\,(a_{**}, a_*|a_*)(B - c(\underline{a})) + c(a_*)$, and

IV. $c(\underline{a}) \geq \text{IR}\,(\underline{a}, a_*|a_*)(B - c(\underline{a})) + c(a_*)$.

Put I and III together. This gives

$$c(a_*) + \text{IR}\,(a_{**}, a_*|a_{**})(B - c(\underline{a})) \geq c(a_{**}) \geq \text{IR}\,(a_{**}, a_*|a_*)(B - c(\underline{a})) + c(a_*),$$

where we use the fact that $\text{IR}\,(a_{**}, a_*|a_{**}) = -\text{IR}\,(a_*, a_{**}|a_{**})$. By Lemma C.4, $B > c(\underline{a})$ and so

$$\text{IR}\,(a_{**}, a_*|a_{**}) \geq \text{IR}\,(a_{**}, a_*|a_*),$$

as required.

Put II and III together. This gives

$$c(\underline{a}) + \text{IR}\,(a_{**}, \underline{a}|a_{**})(B - c(\underline{a})) \geq c(a_{**}) \geq \text{IR}\,(a_{**}, a_*|a_*)(B - c(\underline{a})) + c(a_*),$$

where we use the fact that $\text{IR}\,(a_{**}, \underline{a}|a_{**}) = -\text{IR}\,(\underline{a}, a_{**}|a_{**})$. Using the fact that $c(a_*) \geq c(\underline{a})$, we have

$$\begin{aligned} c(\underline{a}) + \text{IR}\,(a_{**}, \underline{a}|a_{**})(B - c(\underline{a})) &\geq \text{IR}\,(a_{**}, a_*|a_*)(B - c(\underline{a})) + c(a_*) \\ &\geq \text{IR}\,(a_{**}, a_*|a_*)(B - c(\underline{a})) + c(\underline{a}), \end{aligned}$$

where the second inequality is strict when $a_* \neq \underline{a}$. By Lemma C.4, $B > c(\underline{a})$ and so

$$\text{IR}\,(a_{**}, \underline{a}|a_{**}) \geq \text{IR}\,(a_{**}, a_*|a_*),$$

where the inequality is strict when $a_* \neq \underline{a}$.

Now we turn to the "if" part. We suppose that conditions (i)-(ii) of the Proposition hold and we will show that we can construct a part (B, c) that justifies $\{s_1^I, r_1^I\}$.

To do so, it will be useful to fix certain constants: First choose B and $\underline{n}$ so that $B > \underline{n} > 0$. If $a_* = \underline{a}$, fix $n_* = \underline{n}$. If $a_* \neq \underline{a}$, fix n_* so that:

i$_*$ $n_* > \underline{n}$;

ii$_*$ $\underline{n} + \text{IR}\,(a_*, \underline{a}|a_*)(B - \underline{n}) > n_*$; and

iii$_*$ $\underline{n} + [\text{IR}\,(a_{**}, \underline{a}|a_{**}) - \text{IR}\,(a_{**}, a_*|a_*)](B - \underline{n}) > n_*$.

To see that requirements i$_*$–ii$_*$ can be satisfied simultaneously, use Lemma C.2. To see that requirements i$_*$–iii$_*$ can be satisfied simultaneously, use condition (ii) of the Proposition and the fact that, in this case, $a_* \neq \underline{a}$.

Now fix n_{**}

i$_{**}$ $n_{**} \geq n_* + \text{IR}\,(a_{**}, a_*|a_*)(B - \underline{n})$;

ii$_{**}$ $n_* + \text{IR}\,(a_{**}, a_*|a_{**})(B - \underline{n}) \geq n_{**}$; and

iii$_{**}$ $\underline{n} + \text{IR}\,(a_{**}, \underline{a}|a_{**})(B - \underline{n}) \geq n_{**}$ with strict inequality if $a_* \neq \underline{a}$.

To see that requirements i$_{**}$–ii$_{**}$ can be satisfied simultaneously, use condition (i) of the Proposition. Condition iii$_{**}$ follows from condition ii$_{**}$, if $a_* = \underline{a}$. To see that requirements i$_{**}$–iii$_{**}$ can be satisfied simultaneously when $a_* > \underline{a}$, use condition iii$_*$ above. Note, it follows from Lemma C.2 and i$_{**}$ that $n_{**} > n_*$.

Construct a function $N : A \times \{a_*, a_{**}\} \to \mathbb{R}$ so that

$$N(a, a_*) = \text{IR}\,(a, a_*|a_*)(B - \underline{n}) + n_*$$

and

$$N(a, a_{**}) = \text{IR}\,(a, a_{**}|a_{**})(B - \underline{n}) + n_{**}.$$

It follows from Lemma C.2 that $N(\cdot, a_*)$ and $N(\cdot, a_{**})$ are strictly increasing in a. Moreover,

- $\underline{n} \geq \max\{N(\underline{a}, a_*), N(\underline{a}, a_{**})\}$;

- $n_* = N(a_*, a_*) \geq N(a_*, a_{**})$;

- $n_{**} = N(a_{**}, a_{**}) \geq N(a_{**}, a_*)$.

The first of these follows from requirement ii$_*$ on n_* and requirement iii$_{**}$ on n_{**}.[10] The second of these follows from requirement ii$_{**}$ on n_{**}. The third of these follows from requirement i$_{**}$ on n_{**}.

Now let $\hat{N} : A \to \mathbb{R}$ be the upper envelope of $N(\cdot, a_*)$ and $N(\cdot, a_{**})$, i.e., $\hat{N}(a) = \max\{N(a, a_*), N(a, a_{**})\}$ for each $a \in A$. It is strictly increasing. Moreover, it satisfies

- $\hat{N}(\underline{a}) \leq \underline{n}$,

- $\hat{N}(a_*) = n_*$, and

- $\hat{N}(a_{**}) = n_{**}$.

It follows that we can construct a strictly increasing function $c : A \to \mathbb{R}$ that lies everywhere above $\hat{N}$, i.e., for each $a \in A$, $c(a) \geq \hat{N}(a)$, with

- $c(\underline{a}) = \underline{n}$,

- $c(a_*) = n_*$, and

- $c(a_{**}) = n_{**}$.

Applying Lemma C.3 we get that the (B, c) constructed justifies both s_1^I and r_1^I. ■

We now turn to the proof of Proposition 3.3. For this, we will need a preliminary result.

Lemma C.5 *Let A be finite. Fix first-period strategies s_1^I and r_1^I with $s_1^I(\Omega) = \{a_*\}$, $r_1^I(\Omega) = \{a_{**}\}$ and $a_{**} > a_*$. Suppose, there exists some $(\hat{B}, \hat{c})$ with*

- $\frac{\hat{c}(a) - \hat{c}(a_*)}{\hat{B} - \hat{c}(\underline{a})} > \mathrm{IR}\,(a, a_* | a_*)$ *for all $a \in A \backslash \{a_*\}$ and*

- $\frac{\hat{c}(a) - \hat{c}(a_{**})}{\hat{B} - \hat{c}(\underline{a})} > \mathrm{IR}\,(a, a_{**} | a_{**})$ *for all $a \in A \backslash \{a_{**}\}$.*

Then, $\{s_1^I, r_1^I\}$ is strictly justifiable.

Proof. For each $a' \in \{a_*, a_{**}\}$ and $a \in A \backslash \{a'\}$, construct a function $g[a, a'] : \mathcal{C}^+ \to \mathbb{R}$ with $g[a, a'](B, c) = \frac{c(a) - c(a')}{B - c(\underline{a})}$. This function is continuous. It follows that the sets $\mathcal{U}[a, a'] = (g[a, a'])^{-1}((\mathrm{IR}\,(a, a' | a'), \infty))$ are each open in $\mathcal{C}^+$. As such, the sets $\mathcal{U}[a'] = \cap_{a \in A \backslash \{a'\}} \mathcal{U}[a, a']$ are open in $\mathcal{C}^+$ and so $\mathcal{U} = \mathcal{U}[a_*] \cap \mathcal{U}[a_{**}]$ is open in $\mathcal{C}^+$.

[10] Of course, $\underline{n} = N(\underline{a}, a_*)$ if $a_* = \underline{a}$.

Consider the interior of $\mathcal{U}$ in $\mathcal{C}$, viz. $\text{int}\,\mathcal{U}$. We will show that this set is non-empty. This suffices to show that $\{s_1^I, r_1^I\}$ is strictly justifiable, as required.

Since $\mathcal{U}$ is open in $\mathcal{C}^+$, there exists a $\mathcal{V}$ open in $\mathcal{C}$ with $\mathcal{U} = V \cap \mathcal{C}^+$. It follows that the interior of $\mathcal{U}$ in $\mathcal{C}$, viz. $\text{int}\,\mathcal{U}$, can be written as

$$\text{int}\,\mathcal{U} = \text{int}\,(\mathcal{V} \cap \mathcal{C}^+) = \mathcal{V} \cap \mathcal{C}^+,$$

where the second equality uses Lemma A.1.

Now, consider the pair $(\hat{B}, \hat{c})$ as given by the statement of the result. By construction, this pair is contained in $\mathcal{U}$ and so contained in $\mathcal{V}$. Moreover,

$$\frac{\hat{c}(a_{**}) - \hat{c}(a_*)}{\hat{B} - \hat{c}(\underline{a})} > \text{IR}\,(a_{**}, a_*|a_*) > 0$$

so $\hat{B} > \hat{c}(\underline{a})$ and $(\hat{B}, \hat{c})$ is contained in $\mathcal{C}^+$. As such, $\text{int}\,\mathcal{U} \neq \emptyset$. $\blacksquare$

Proof of Proposition 3.3. Begin by showing that part (i) implies part (ii): To do so, fix strategies s_1^I and r_1^I with $s_1^I(\Omega) = \{a_{**}\}$, $r_1^I(\Omega) = \{a_*\}$, and $a_{**} > a_*$. Suppose, contra hypothesis, that $\{s_1^I, r_1^I\}$ is strictly justifiable but either $\text{IR}\,(a_{**}, a_*|a_{**}) \leq \text{IR}\,(a_{**}, a_*|a_*)$ or $\text{IR}\,(a_{**}, \underline{a}|a_{**}) \leq \text{IR}\,(a_{**}, a_*|a_*)$. It follows from Proposition 3.2 that either $\text{IR}\,(a_{**}, a_*|a_{**}) = \text{IR}\,(a_{**}, a_*|a_*)$ or $\text{IR}\,(a_{**}, \underline{a}|a_{**}) = \text{IR}\,(a_{**}, a_*|a_*)$.

If $\text{IR}\,(a_{**}, a_*|a_{**}) = \text{IR}\,(a_{**}, a_*|a_*)$: Apply Lemma C.3, if (B, c) justifies $\{s_1^I, r_1^I\}$, then $c(a_{**}) = \text{IR}\,(a_{**}, a_*|a_{**})(B - c(\underline{a})) + c(a_*)$. A consequence is that, for each triple B, $c(\underline{a})$, $c(a_*)$, there is a unique number $c(a_{**})$ so that the pair (B, c) justifies $\{s_1^I, r_1^I\}$. As such, there is no non-empty open set of benefits of reelections and cost functions that justify $\{s_1^I, r_1^I\}$.

If $\text{IR}\,(a_{**}, \underline{a}|a_{**}) = \text{IR}\,(a_{**}, a_*|a_*)$: Note, in the case where $a_* = \underline{a}$, this is equivalent to the condition that $\text{IR}\,(a_{**}, a_*|a_{**}) = \text{IR}\,(a_{**}, a_*|a_*)$, which we have ruled out. So, we will suppose that $a_* \neq \underline{a}$. Apply Lemma C.3 to get that if (B, c) justifies s_1^I then

$$c(\underline{a}) - \text{IR}\,(\underline{a}, a_{**}|a_{**})(B - c(\underline{a})) \geq c(a_{**}).$$

Since $-\text{IR}\,(\underline{a}, a_{**}|a_{**}) = \text{IR}\,(a_{**}, \underline{a}|a_{**}) = \text{IR}\,(a_{**}, a_*|a_*)$, this gives that

$$c(\underline{a}) + \text{IR}\,(a_{**}, a_*|a_*)(B - c(\underline{a})) \geq c(a_{**}). \tag{16}$$

But now apply Lemma C.3 to get that if (B, c) justifies r_1^I then

$$c(a_{**}) \geq c(a_*) + \text{IR}\,(a_{**}, a_*|a_*)(B - c(\underline{a})). \tag{17}$$

Putting equations 16-17 together, we get that $c(\underline{a}) = c(a_*)$, contradicting the fact that $a_* \neq \underline{a}$.

Now we show that part (ii) implies part (i): To do so, fix strategies s_1^I and r_1^I with $s_1^I(\Omega) = \{a_{**}\}$, $r_1^I(\Omega) = \{a_*\}$, and $a_{**} > a_*$. Also, assume that $\text{IR}\,(a_{**}, a_*|a_{**}) > \text{IR}\,(a_{**}, a_*|a_*)$ and $\text{IR}\,(a_{**}, \underline{a}|a_{**}) > \text{IR}\,(a_{**}, a_*|a_*)$.

We repeat the proof of Proposition 3.2: Choose B, $\underline{n}$, and n_* as before. But now, choose n_{**}

i$_{**}$ $n_{**} > n_* + \text{IR}\,(a_{**}, a_*|a_*)(B - \underline{n})$;

ii$_{**}$ $n_* + \text{IR}\,(a_{**}, a_*|a_{**})(B - \underline{n}) > n_{**}$; and

iii$_{**}$ $\underline{n} + \text{IR}\,(a_{**}, \underline{a}|a_{**})(B - \underline{n}) > n_{**}$.

To see that requirements i$_{**}$–ii$_{**}$ can be satisfied simultaneously, use the fact that $\text{IR}\,(a_{**}, a_*|a_{**}) > \text{IR}\,(a_{**}, a_*|a_*)$. Condition iii$_{**}$ holds trivially if $a_* = \underline{a}$. To see that requirements i$_{**}$–iii$_{**}$ can be satisfied simultaneously when $a_* > \underline{a}$, use condition iii$_*$ in the definition of n_*. As before, it follows from Lemma C.2 and i$_{**}$ that $n_{**} > n_*$.

Use this new definition of n_{**} to construct the function $N : A \times \{a_*, a_{**}\} \to \mathbb{R}$. The construction is the same relative to this new choice of n_{**}. With this, once again, $\hat{N}(a) = \max\{N(a, a_*), N(a, a_{**})\}$ for each $a \in A$. The function exhibits the same properties as it previously did. It follows that we can construct a strictly increasing cost function $c : A \to \mathbb{R}$ with that lies everywhere strictly above $\hat{N}$, i.e., for each $a \in A$, $c(a) \geq \hat{N}(a)$, with

- $c(\underline{a}) = \underline{n}$,
- $c(a_*) = n_*$,
- $c(a_{**}) = n_{**}$, and
- $c(a) > \hat{N}(a)$ for each $a \in A \backslash \{\underline{a}, a_*, a_{**}\}$.

It now follows from Lemma C.5 that $\{s_1^I, r_1^I\}$ is strictly justifiable. ■

Asymmetric Uncertainty

Now we turn to the case of asymmetric uncertainty. Here, we will stop short of providing necessary and sufficient conditions for $\{s_1^I, r_1^I\}$ to be justifiable. Instead, we explain the difficulty in doing so.

Fix a monotone pure-strategy perfect Bayesian equilibrium (s_*^I, s_*^V, s_*^C). In this case, the Incumbent learns she is of some type θ and assesses the likelihood of reelection as $\mu([s_{1,*}^I, r_1^I, \mathbb{E}(F_1^C(\underline{a}, \cdot))] \cap [\theta])$. We write

$$\Pr(\theta, r_1^I | s_{1,*}^I) := \mu([s_{1,*}^I, r_1^I, \mathbb{E}(F_1^C(\underline{a}, \cdot))] \cap [\theta]).$$

Note, if $\overrightarrow{\theta}^I(\omega) = \theta$ and $r_1^I(\omega) = q_1^I(\omega) = a$, then $\Pr(\theta, r_1^I | s_{1,*}^I) = \Pr(\theta, q_1^I | s_{1,*}^I)$. With this in mind, we write $\Pr(\theta, a | s_{1,*}^I)$.

We next compute the incremental increase in probability of reelection from an Incumbent of type θ choosing action a over a', when the Voter expects her to play $s_{1,*}^I$, namely

$$\mathrm{IR}\,(\theta, a, a' | s_{1,*}^I) = \Pr(\theta, a | s_{1,*}^I) - \Pr(\theta, a' | s_{1,*}^I).$$

Note:

Lemma C.6 *Suppose (B, c) justifies $\{s_1^I\}$. For each ω,*

$$c(a) \geq \mathrm{IR}\,(\overrightarrow{\theta}^I(\omega), a, s_1^I(\omega) | s_1^I)(B - c(\underline{a})) + c(s_1^I(\omega))$$

for all $a \in A$. Moreover, if there exists some ω with for all $s_1^I(\omega) > \underline{a}$, then $B > c(\underline{a})$.

The proof is analogous to the proof of earlier results and so it omitted.

In what follows below, we fix $\{s_1^I, r_1^I\}$ with $s_1^I \neq r_1^I$ and ask if this set is justifiable. Take Θ to be finite. Then, we can write $s_1^I(\Omega) \cup r_1^I(\Omega) \cup \{\underline{a}\} = \{a_1, \ldots, a_K\}$, where $a_1 = \underline{a}$ and $a_k < a_{k+1}$ for each $k = 1, \ldots, K-1$. Applying Lemma C.6 we have:

Lemma C.7 *If (B, c) justifies $\{s_1^I, r_1^I\}$, then $B > c(a_1)$ and, for each $q_1^I \in \{s_1^I, r_1^I\}$ the following holds:*

(i) If $\overrightarrow{\theta}(\omega) = \theta$ and $q_1^I(\omega) = a_{k+1}$, then $c(a_j) + \mathrm{IR}\,(\theta, a_{k+1}, a_j | q_1^I)(B - c(a_1)) \geq c(a_{k+1})$ for all $j \leq k$; and

(ii) If $\overrightarrow{\theta}(\omega) = \theta$ and $q_1^I(\omega) = a_j$ for some $j \leq k$, then $c(a_{k+1}) \geq c(a_j) + \mathrm{IR}\,(\theta, a_{k+1}, a_j | q_1^I)(B - c(a_1))$.

Lemma C.7 is a necessary condition for justifiability. A pair (B, c) can satisfy these conditions and yet still fail to justify $\{s_1^I, r_1^I\}$.

That said, suppose we can find constants $B, n_1, \ldots, n_K$ that satisfy the following conditions:

(i) $B > n_1$;

(ii) for each $k = 1, \ldots, K-1$ and $q_1^I \in \{s_1^I, r_1^I\}$,

  (a) if $\overrightarrow{\theta}(\omega) = \theta$ and $q_1^I(\omega) = a_{k+1}$, then $n_j + \mathrm{IR}\,(\theta, a_{k+1}, a_j|q_1^I)(B - n_1) \geq n_{k+1}$ for all $j \leq k$; and

  (b) if $\overrightarrow{\theta}(\omega) = \theta$ and $q_1^I(\omega) = a_j$ for some $j \leq k$, then $n_{k+1} \geq n_j + \mathrm{IR}\,(\theta, a_{k+1}, a_j|q_1^I)(B - n_1)$.

In this case, we can borrow the technique used in the symmetric uncertainty case (i.e., the proof of Proposition 3.2) to construct a cost function c so that (B, c) justifies $\{s_1^I, r_1^I\}$.

Sketch of Argument. Fix some strategy $q_1^I \in \{s_1^I, r_1^I\}$ and some state ω with $q_1^I(\omega) = a_k$ and $\overrightarrow{\theta}^I(\omega) = \theta$. In this case, define $N(\theta, \cdot, a_k|q_1^I) : A \to \mathbb{R}$ so that $N(\theta, a, a_k|q_1^I) = \mathrm{IR}\,(\theta, a, a_k|q_1^I)(B - n_1) + n_k$. This is an increasing function, since $\mathrm{IR}\,(\theta, \cdot, a_k|q_1^I)$ must be increasing. Moreover, given some strategy $\hat{q}_1^I \in \{s_1^I, r_1^I\}$ and some state $\hat{\omega}$ with $\hat{q}_1^I(\omega) = a_j$ and $\overrightarrow{\theta}^I(\hat{\omega}) = \hat{\theta}$,

$$N(\theta, a_k, a_k|q_1^I) = n_k \geq \mathrm{IR}\,(\hat{\theta}, a_k, a_j|\hat{q}_1^I)(B - n_1) + n_j = N(\hat{\theta}, a_k, a_j|\hat{q}_1^I)$$

(Note, the case where $j = k$ is immediate. For the case of $j < k$ (resp. $j > k$) apply the fact that $B > n_1$ and part (ii)b (resp. (ii)a) of the conditions for $n_1, \ldots, n_K$. Also notice that here we allow $\hat{q}_1^I = q_1^I$, $\hat{\theta} = \theta$, and $a_k = a_j$.) Also, applying the fact that $B > n_1$ and part (ii)a of the conditions for $n_1, \ldots, n_K$, we have that, $n_1 \geq N(\theta, a_1, a_k|q_1^I)$ for each such map.

Now, we can define a function $\hat{N} : A \to \mathbb{R}$ that is the upper envelope of the functions $N(\theta, a_k, a_k|q_1^I)$. This function has the property that $\hat{N}(a_1) \leq n_1$ with equality if $n_1 \in s_1^I(\Omega) \cup r_1^I(\Omega)$ and $\hat{N}(a_k) = n_k$ for $k = 2, \ldots, K$. It follows that we can construct a cost function c that lies above $\hat{N}$ with $c(a_k) = n_k$ for each $k = 1, \ldots, K$. Then, it can be shown that (B, c) justifies $\{s_1^I, r_1^I\}$. ∎

So the question is: Can we find restrictions on the primitives of the model, so that we can ensure that we can choose $B, n_1, \ldots, n_K$ with $B > n_1$ and conditions (ii)a-(ii)b are satisfied. One possibility is the following:

Condition C.1 *For each $\theta, \theta' \in \Theta$ and all $q_1^I, v_1^I \in \{s_1^I, r_1^I\}$, $\mathrm{IR}\,(\theta, a_k, a_j|q_1^I) \geq \mathrm{IR}\,(\theta', a_k, a_j|v_1^I)$.*

This is a sufficient (but not necessary) condition. To us, it seems particularly taxing on the informational environment and production technology.

Appendix D Mathematical Appendix

Let Φ be the CDF fixed in Appendix B. Choose $y_* > 0$ and define a function $X : \mathbb{R} \to [0, 1]$ so that $X(x) = \Phi(x + y_*) - \Phi(x)$. We point to two properties of X.

Lemma D.1 *The function X is differentiable, with*

$$\begin{array}{ll} X'(x) > 0 & \textit{if } -\frac{y_*}{2} > x \\ X'(x) = 0 & \textit{if } x = -\frac{y_*}{2} \\ X'(x) < 0 & \textit{if } x > -\frac{y_*}{2}. \end{array}$$

Proof. Differentiability follows from the fact that Φ is absolutely continuous. Note then that $X'(x) = \phi(x + y_*) - \phi(x)$. So, it suffices to show that $\phi(x + y_*) > \phi(x)$ if $-\frac{y_*}{2} > x$, $\phi(x) > \phi(x + y_*)$ if $x > -\frac{y_*}{2}$, and $\phi(x + y_*) = \phi(x)$ if $x = -\frac{y_*}{2}$.

Case I. If $\mathbf{x > -\frac{y_*}{2}}$, then $\boldsymbol{\phi}(\mathbf{x}) > (\mathbf{x} + \mathbf{y}_*)$. If $x \geq 0$, then single-peakedness implies $\phi(x) > \phi(x + y_*)$. Suppose then that $0 > x > -\frac{y_*}{2}$. By single-peakedness and symmetry, $\phi(x) > \phi(-\frac{y_*}{2}) = \phi(\frac{y_*}{2})$. Notice that $x + y_* > \frac{y_*}{2}$, since $x > -\frac{y_*}{2}$. As such, $\phi(\frac{y_*}{2}) > \phi(x + y_*)$. So, putting the above facts together, we have that $\phi(x) > \phi(\frac{y_*}{2}) > \phi(x + y_*)$.

Case II. If $\mathbf{x < -\frac{y_*}{2}}$, then $\boldsymbol{\phi}(\mathbf{x} + \mathbf{y}_*) > \boldsymbol{\phi}(\mathbf{x})$. If $x < -\frac{y_*}{2}$ then $x < 0$. So, if $x + y_* \leq 0$ then it follows from single-peakedness that $\phi(x + y_*) > \phi(x)$. Suppose then that $x + y_* > 0$. Note, there is an upper bound on $x + y_*$, namely $x + y_* < \frac{y_*}{2}$. (Here we use the fact that $x < -\frac{y_*}{2}$.) So, applying single-peakedness and symmetry, $\phi(x + y_*) > \phi(-\frac{y_*}{2}) > \phi(x)$.

Case III. If $\mathbf{x = -\frac{y_*}{2}}$, then $\boldsymbol{\phi}(\mathbf{x} + \mathbf{y}_*) = \boldsymbol{\phi}(\mathbf{x})$. Here, $-x = x + y_*$, so the result follows from symmetry. ■

Lemma D.2 *The function X is symmetric about $-\frac{y_*}{2}$, i.e., $X(-\frac{y_*}{2} + x) = X(-\frac{y_*}{2} - x)$, for each $x \in \mathbb{R}$.*

Proof. Fix x and note that

$$\begin{aligned} X(-\frac{y_*}{2} + x) &= \Phi(x + \frac{y_*}{2}) - \Phi(x - \frac{y_*}{2}) \\ &= 1 - \Phi(-x - \frac{y_*}{2}) - 1 + \Phi(-x + \frac{y_*}{2}) \\ &= X(-\frac{y_*}{2} - x), \end{aligned}$$

as required. ■

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