Monopoly, Ramsey and Lindahl in Rochet and Tirole (2003)*

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Abstract

Rochet and Tirole (2003) consider the consumer Ramsey problem in a model of two-sided markets. I extend their analysis to the social Ramsey and Lindahl problems, comparing the solutions to all three to monopoly pricing.

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Rochet and Tirole (2003) (RT2003) consider a model of two-sided markets in which the total volume of transactions in the market are $D^A(p^A)D^B(p^B)$, where D^I are demands derived from primitives of consumer demand and p^I is the per-transaction price charged to side I. Firm profits are $(p^A + p^B - c)D^A(p^A)D^B(p^B)$. RT2003 show that the normative properties of their model depend only on prices. Consumer surplus on side I is the product of gross surplus $V^I(p^I) \equiv \frac{\int_{p^I}^{\infty} D^I(p)dp}{D^I(p^I)}$ and the demand on the opposite side of the market $D^J(p^J)$ embodying the basic two-sided externality. Constrained to a given price level $\overline{p} \equiv p^A + p^B$, RT2003 show that the necessary first-order condition for consumer-optimal price balance (division of this price level), the consumer Ramsey problem, is

$$\mu^A(p^A)\overline{V}^A(p^A) = \mu^B(p^B)\overline{V}^B(p^B)$$

where the *market power* (Weyl, 2008a) $\mu^I \equiv -\frac{D^I}{D^{I\prime}}$ and the average surplus $\overline{V}^I \equiv \frac{V^I}{D^I}$. Fabinger and Weyl (2008) show that $\overline{V}^I(p^I) = \mu^I(p^I)\overline{\rho}^I(p^I)$ where the *average pass-through rate* $\overline{\rho}(p)$ is a weighted average of pass-through rates at prices above p^I. That is $\overline{\rho}(p) = \int_p^{\infty} \lambda(q;p)\rho(q)dq$ where $\rho(q) \equiv \frac{1}{1-\mu'(q)}$ is the monopolist's *pass-through rate* at price q and $\int_p^{\infty} \lambda(q;p)dq = 1$; see Fabinger and Weyl (2008) for more details. Therefore the RT2003 Ramsey conditions become

$$\mu^{A^2}(p^A)\overline{\rho}^A(p^A) = \mu^{B^2}(p^B)\overline{\rho}^B(p^B) \tag{1}$$

Weyl (2008b) showed that this is sufficient for consumer surplus maximizing price balance if the following second-order condition holds

$$\sum_{I=A,B} \frac{\mu^I(p^I)\left(2\rho^I[p^I]\overline{\rho}^I[p^I] - \overline{\rho}^I[p^I] - \rho^I[p^I]\right)}{\rho^I(p^I)} < 0 \tag{2}$$

If instead we consider the social surplus (even weighting on consumer surplus and firm profits) version of this Ramsey problem the same mathematics[1] yields a weighted average of

[1]A proof is available on request.

condition (1) and the monopolist's optimal price balance $\mu^A(p^A) = \mu^B(p^B)$:

$$\left(\overline{p} - c + \overline{\rho}^A[p^A]\mu^A[p^A]\right)\mu^A(p^A) = \left(\overline{p} - c + \overline{\rho}^B[p^B]\mu^B[p^B]\right)\mu^B(p^B) \tag{3}$$

so long as $\overline{p} > c$. Assuming both the monopolist's and the consumer optimality problems are strictly concave, the socially optimal balance therefore lies between the two when the price level is above cost. However, when the price level is below cost, the socially optimal balance will be further away from the equating $\mu^A = \mu^B$ than is the consumer optimal balance, as volume is now harmful as net subsidies must be paid per-transaction.

Given these, a natural way to approach the Lindahl (unconstrained optimal) pricing problem is to consider the optimal price level, assuming that balance is determined in a socially optimal way. However, this turns out to fail because, as I will show, $c - \overline{p} = \overline{V}^I$ for both I at the socially optimal prices.

Starting instead from primitives, the derivative of social welfare with respect to price on side I is

$$D^{I'}V^J + (\overline{p} - c)D^{I'}D^J$$

or

$$c - \overline{p} = \overline{V}^A(p^A) = \overline{V}^B(p^B) \tag{4}$$

is the first-order condition for the Lindahl problem. This an intuitive condition closely related to monopoly pricing[2]. The per-transacton externality one side of the market causes the other is the average value that consumers on the other side of the market take from that transaction, their average surplus. In classic Pigouvian fashion this formula requires that the

[2]The comparative statics of Lindahl prices are therefore closely related to those of monopoly discussed in Weyl (2008b). However $\overline{V}^A = \overline{V}^B$ does not continue to govern the socially optimal price balance away from the socially optimal price level, whereas in the monopoly problem $\mu^A = \mu^B$ always gives the optimal balance constrained to any price level weakly above cost.

social planner subsidize transactions on each side I by the amount of the external benefit of that transaction, including both the amount captured by the platform p^J and the amount captured by the consumers $\overline{V}^J$. Because, just as in the monopoly problem (Weyl, 2008b), the subsidy below the platform's total marginal cost on side A, $(c - p^B) - p^A$, is equal to the subsidy on side B, $(c - p^A) - p^B$, the social planner should equate the negative of the total mark-up to the externality on both sides.

The second-order condition for the Lindahl problem is also closely related to that for the monopoly problem. The second-own derivative condition is just that $\overline{V}^{I'} > -1$ which is trivially satisfied as $\overline{V}^{I'} = -\frac{V^I D^{I'}}{(D^I)^2} - 1 > -1$ as $D^{I'} < 0$ by assumption. The cross-derivative condition $\frac{\partial p^{A^\star}}{\partial p^B}\frac{\partial p^{B^\star}}{\partial p^A} < 1$ is, by some simple algebra

$$\overline{\rho}^A \overline{\rho}^B < 1 \tag{5}$$

This is a clear analog of the monopolist's second-order condition, called cross-subsidy contraction (CSC) (Weyl, 2008b), that $\rho^A \rho^B < 1$, except that average pass-through rates replace (local) pass-through rates because the social planner takes into account (infra-marginal) consumer surplus as well as marginal consumer valuations.

Finally, compare the balance formulae under each of the four criteria:

1. Monopoly: $\mu^A = \mu^B$
2. Consumer Ramsey: $\overline{\rho}^A {\mu^A}^2 = \overline{\rho}^B {\mu^B}^2$
3. Social Ramsey: $\left(\overline{p} - c + \overline{\rho}^A \mu^A\right) \mu^A = \left(\overline{p} - c + \overline{\rho}^B \mu^B\right) \mu^B$
4. Lindahl: $\overline{\rho}^A \mu^A = \overline{\rho}^B \mu^B$

Thus clearly these balance conditions will given similar answers to the extent that average pass-through rates are similar across the two sides of the market and at least some will tend to diverge from others when average pass-through rates differ across sides. This generalizes

RT2003's observation that the Monopoly and Consumer Ramsey conditions coincide when demand on both sides of the market is linear; it also shows the limits of their result.

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