A Restatement of the Theory of Monopoly

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Introduction

1. Division of surplus among CS, PS and DWL
2. Cost-to-price pass-through
3. Strategic effects in Bertrand and in Cournot
4. Price responses to demand shocks
5. Curvature of demand and cost

All of these key *typically distinct* ideas in monopoly theory

- Today I want to argue they are all the same thing
  - Some special connections known (linear, cost demand etc.)
    - Bulow-Pfleierer (83), Anderson-Renault (03), Bishop (68)
  - But no previous attempt to unify in maximal generality
  - Unity useful for a few reasons:
    1. Quantitative: link many weak measurements/intuitions
    2. Theoretical: simplify proofs/analysis in general case
    3. Expositional: makes existing results more parsimonious

Plan for talk

Try to convey results and an implication: two papers

1. A Restatement of the Theory of Monopoly
   - General monopoly model $\Longrightarrow$ broad, simple results
   - Some results requiring constant marginal cost
   - Useful analogies to competition
2. Functional Forms and the Theory of Monopoly
   - Most standard assumptions (theory+empirics) restrict
   - Tractable demand functions which avoid this
3. I won't get to my promised merger application

- Wrap up with directions for future research

Model and assumptions

Most classical, general monopoly model

- Weakly increasing, smooth cost $C(q), MC(q) \equiv C'(q)$
- Strictly declining, smooth inverse demand $P(q)$
- Revenue $R(q) \equiv P(q)q, MR(q) \equiv R'(q)$
- Profits $\pi(q) \equiv R(q) - C(q)$
- Assume profits concave: $MR' < MC'$ and optimum exists
- Consumer surplus $CS(q) \equiv \int_0^q P(x) - P(q)dx$ finite
- Let $q^\star \equiv \mathrm{argmax}_q CS(q) + \pi(q)$; then $DWL(q) \equiv \pi\left(q^\star\right) + CS\left(q^\star\right) - \pi(q) - CS(q)$
- Three key interventions
  1. Specific tax tq on product
  2. Increase in inverse demand curve by s
  3. Exogenous introduction of $\tilde{q}$ Cournot competition

Pass-through and demand pass-through

Because most common, link everything to pass-through $\rho \equiv \frac{dP}{dt}$

- First demand pass-through $\rho_d \equiv \frac{dP}{ds}$

Theorem

$$\rho + \rho_d = 1$$

Proof.

s is subsidy, so follows directly from neutrality of taxes. □

- $\rho > 0$, but may be > 1 so ρ_d may be negative
  - Counter-intuitive?
- Tight link to strategic complements v. substitutes in prices
  - Others changing prices shifts willingness to pay
  - Direct with perfect complements
  - Qualitatively in many cases with discrete choice (below)

Pass-through and the division of surplus

Let $\bar{t}$ be such that $q\left(\bar{t}\right) = 0$ or ∞

Theorem

$$\frac{CS}{\pi} = \bar{\rho} \equiv \frac{\int_{t=0}^{\bar{t}} \rho q dt}{\int_{t=0}^{\bar{t}} q dt},$$

Proof.

By envelope $\frac{dCS}{dt} = -\rho q$ and $\frac{d\pi}{dt} = -q$. Integrate up. □

The quantity connection

- When sharply peaked profit function, pass-through low
- Pass-through high when indifferent over range of prices
- With CMC, second-order inverse-elasticity
  - Generalization with non-constant, but messy

$$\rho = \frac{1}{-\frac{d^2\pi}{dm^2}\frac{m^2}{\pi}} = \frac{1}{-\frac{d^2\pi}{dq^2}\frac{q^2}{\pi}}$$

- Thus not surprising applies to quantity behavior as well
- Let $\tilde{q}$ be amount of exogenous quantity competition
- Let $\rho_q \equiv \frac{dq}{d\tilde{q}}$, where q is total then

Quantity pass-through

Theorem

With constant MC, $\rho = \rho_q$.

Proof.

Changes foc to $P'(q - \tilde{q}) + P - c$ so subsidy of $-P'$; price change of $-\rho P'$ and thus quantity change of ρ. □

Note that $\rho_q - 1$ is strategic effect, comp. v. sub

Deadweight loss

Theorem

With CMC, $\frac{DWL}{\pi} = \tilde{\rho} = \frac{\int_{\tilde{q}=0}^{\tilde{q}=q^\star} \rho m d\tilde{q}}{\int_{\tilde{q}=0}^{\tilde{q}=q^\star} m d\tilde{q}}$

Proof.

Harberger: $\frac{dDWL}{d\tilde{q}} = -\rho_q m = -\rho m$ and envelope $\frac{d\pi}{d\tilde{q}} = -m$. □

Both the consumer-producer and private-social

The competitive model and comparative statics

What affects pass-through rates?

- Useful to consider comparison to perfectly competitive
  - Identify supply with (inverse) marginal cost curve

$$\rho = \frac{1}{1+\frac{\epsilon_D}{\epsilon_S}+\frac{\epsilon_{\epsilon_D}}{\epsilon_D}+\frac{1}{\epsilon_S}-\frac{1}{\epsilon_D}}, \ \rho_C = \frac{1}{1+\frac{\epsilon_D}{\epsilon_S}}$$

- Some say: ϵ_D high $\implies$ low pass-through
- Let's assume (vertical stretch) that $\frac{\epsilon_{\epsilon_D}}{\epsilon_D}$ stable with ϵ_D
  - Then key is *relative* elasticity of supply and demand crucial
  - Just like competitive case; when ϵ_S, ϵ_D large, last vanishes
  - If supply perfectly elastic, little effect
  - If supply imperfectly elastic, the intuition right
  - If economies of scale, pass-through explodes!

$\implies$ Cost v. demand curvature depends on competitiveness

Non-concavities and ironing

So far: concavity for differentiability, but all extends

1. Demand pass-through perfectly general
2. Surplus: just add jump integral into surplus
3. Curvature of profits with arc elasticity
4. Quantity pass-through with average slope
5. DWL: integrate jump again
6. Elasticity formula with arc elasticities, as in AP econ

Multiproduct monopoly

Natural multi-dimensional extensions:

1. $\rho + \rho_d = I$
2. $\frac{dCS}{d\mathbf{t}} = -\rho \mathbf{q}$, $\frac{d\pi}{d\mathbf{t}} = -\mathbf{q}$, integrate any way
3. Curvature: $\mathbf{q}^\top \rho \frac{d^2\pi}{d\mathbf{q}^2} \mathbf{q} = \pi$
4. $\rho = \rho_q$ with CMC
5. $\frac{dDWL}{d\tilde{\mathbf{q}}} = -\rho \mathbf{m}$, $\frac{d\pi}{d\tilde{\mathbf{q}}} = -\mathbf{m}$ with CMC
6. Natural extension of elasticities formula, omitted here

Oligopoly

Again, with right partials, results extend to standard oligopoly

- More interesting are additional restrictions
- Strategic complements v. substitutes
  - Bulow-Geanakoplos-Klemperer: determine many things
  - Example: entry, changes in classical industry structure
- Closely tied to pass-through by above logic
  1. Immediate in Cournot (complements and substitutes)
     - Also in sequential extensions: slope of pass-through
  2. Horizontal models, like linear, immediate
     - Also equilibrium ρ v.1 same as individual with CMC
     - And constant 2nd-to-1st-order elasticity ratio
  3. Discrete choice models: scattered qualitative evidence
     - Chen and Riordan (2008): CMC w 2 firms
     - Quint (2010): N firms, CMC, independent values, all signed
- Not clear how far extends, but strong suggestion

Implications of functional forms

This property (pass-through for short) determines a lot

- Evidence mixed: pass-through, sub v. comp, costs etc.
- But standard assumptions severely restrict:

	Log-concave	Log-convex
$MC' > 0$	$\rho < 1$	?
$MC' < 0$	?	$\rho > 1$

- Thus many results driven directly by these
- Particularly common: CMC+demand form
- What do standard demand forms do?
  - Bulow-Pfleiderer (83) identify constant pass-through class
  - Linear $\rho < 1$, constant elasticity $\rho > 1$
  - What about other common forms?

Demand functional forms and pass-through

	$\rho < 1$	$\rho > 1$	Price-dependent
$\rho' > 0$			AIDS
$\rho' < 0$	Normal (Gaussian) Logistic Type I Extreme Value (Gumbel) Double Exponential Type III Extreme Value (Reverse Weibull) Weibull with shape $\alpha > 1$ Gamma with shape $\alpha > 1$		Type II Extreme Value (Fréchet) with shape $\alpha > 1$
Price-dependent			
Does not globally satisfy MUC		Type II Extreme Value (Fréchet) with shape $\alpha < 1$ Weibull with shape $\alpha < 1$ Gamma with shape $\alpha < 1$	

$\Longrightarrow$ Almost everything we use in one box

Apt demand

How can we get flexibility (and tractability)?

- Generalize Bulow-Pfleiderer constant PT demand

$$P(q) = \tilde{p} \pm \left(\frac{\left(\frac{q}{\lambda}\right)^{\frac{1-\overline{\rho}}{2\overline{\rho}}} + 2\overline{\rho}\alpha}{|\overline{\rho}-1|} \right)^2$$

- Apt demand (modulo technicalities); also inverse
- Nice, standard demand function (special forms)
- Flexible on level, elasticity, PT and slope of PT
- Quadratic solution to pricing problems with CMC
  - Generalizes all known tractable (Bulow-Pfleiderer)
- Can be used for...
  1. Any single-product monopoly or oligopoly model
  2. Idiosyncratic terrors in discrete choice model
  3. Monopolistic competition (smooth between discrete points)

Directions for future research

These papers attempt “neat”, unified summary of monopoly

- Aims more at being useful than being “new”
- But suggests a number of directions to go
  1. Effect of curvature on equilibrium PT in discrete choice
  2. Demand functional forms: discrete choice, higher order
  3. Naturally related theory problems
     - Auctions: Mares and Swinkels employ techniques
     - Screening and Mirrlees problem, same things show up
  4. International trade applications: only two extremes solved
  5. Empirical pass-through, theory-guided cross-industry
     - Very little systematic, theory-guided reduced-form
  6. Suggestions for merger guidelines
     - Article just came out in CPI
     - Pass-through, GePP, consumer surplus