Multidimensional Heterogeneity and Product Design

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Core idea and motivation

Firms often design products to attract most valuable customers

- Insurers reduce coverage to drive away the sick
- "Soap" operas attract women who control purse
- Credit cards offer points, travel to attract big spenders

Salient, but theory literature not equipped to handle

1. Rothschild-Stiglitz (76), etc.: non-existence
   - Not very useful in applications...
2. Platforms literature largely ignores
3. Multidimensional screening opaque, special cases
   - Again hard to connect to applications
4. More recent literature enriches heterogeneity but...

We follow Einav and co-authors with rich heterogeneity

- But enrich instruments, allows to nest wide range of issues
- Each "effect" is intuitive, empirically measurable quantity

Outline

1. Simple example develops core idea, key quantity
   - $\text{Cov}\left(\text{preferences}, \text{contribution} \middle| \text{marginal}\right)$
   - Insurance: marginal utility for coverage, cost of covering
   - In platforms, infinite series multiplier from sorting
2. General model and connection to literature
   - Formula classifying/quantifying effects, when they emerge
   - Taxonomy of several literatures using this
3. Series of examples illustrating applications, effects
   1. Competition in insurance and Rothschild-Stiglitz
   2. Broadcast media and soaps: non-transferable utility
   3. Credit cards and non-linear pricing
4. Technical issues and conclusion
   - Multiplicity, implementation and insulating tariffs
   - Conclusion and directions for future research

A classic insurance example

Original Rotschild-Stiglitz context was insurance, so start there

- Monopoly (or planner) insurer chooses price P, coverage ρ
- Key is to *add rich heterogeneity for smooth derivatives*
  - Potential users' types $\boldsymbol{\theta} \in \mathbb{R}^{\mathcal{T}} \sim f(\boldsymbol{\theta})$ smooth, $\mathcal{T}$ large
    - Income, sophistication, health, risk-preferences, etc.
  - Determines (quasi-linear) preferences $u(\rho; \boldsymbol{\theta}) - P$
  - But also *cost of coverage* $c(\rho; \boldsymbol{\theta})$
- Participating users $\Theta(P, \rho) \equiv \{\boldsymbol{\theta} : u(\rho; \boldsymbol{\theta}) \geq P\}$
  - Integrate number $N \equiv \int_{\Theta} f$, cost $C \equiv \int_{\Theta} cf$, $AC \equiv \frac{C}{N}$
- Marginal users $\partial\Theta \equiv \{\boldsymbol{\theta} : u(\rho; \boldsymbol{\theta}) = P\}$
  - Integrate for # of marginals $M \equiv \int_{\partial\Theta} f$, $MC \equiv \frac{\int_{\partial\Theta} cf}{M} \equiv \widetilde{c}$
- Also *average marginal utility* $\overline{u'} \equiv \frac{\int_{\Theta} u' f}{N}$ and
  - *Average marginal utility among marginals* $\widetilde{u'} \equiv \frac{\int_{\partial\Theta} u' f}{M}$
  - Key because of Spence (1975) distinction

Derivation

Useful to think of increasing ρ while holding N fixed

- By Leibnitz, $\frac{\partial N}{\partial \rho} = M\widetilde{u'}$, $\frac{\partial N}{\partial P} = -M$

$\Longrightarrow$ To hold fixed N, P must rise with ρ by $\widetilde{u'}$

- Then what is $\left.\frac{\partial C}{\partial \rho}\right|_{N\text{fixed}}$?
  - By Leibnitz, rises by $N\overline{c'} + M\widetilde{u'c}$, but falls by $M \cdot MC \cdot \widetilde{u'}$
  - Total effect $N\overline{c_\rho} + M\left(\widetilde{u'c} - MC \cdot \widetilde{u'}\right)$; but MC just $\widetilde{c}$
    - Recall that $E[xy] - E[x]E[y] = Cov(x, y)$

$\Longrightarrow$ $\widetilde{u'c} - \widetilde{c} \cdot \widetilde{u'} = Cov\left(u', c|\partial\Theta\right) \equiv \sigma$, crucial quantity!

- Social welfare is $\int_\theta u - C$, optimal quantity just $P = MC$
  - Derivative wrt ρ is $N\overline{u'} - N\overline{c'} - M\sigma$
- Profits are $NP - C$, optimal quantity is $MR = P - \frac{N}{M} = MC$
  - *Cournot distortion* $\mu \equiv \frac{N}{M}$; $\frac{\partial \pi}{\partial \rho} = N\widetilde{u'} - N\overline{c'} - M\sigma$

Privately and social optimal pricing

Simple/intuitive social optimality, distortions on coverage

$$\underbrace{N\left(\overline{u'}-\overline{c'}\right)}_{\text{classic optimality}} = \underbrace{M\sigma}_{\text{optimal ``cream-skimming'' (Bundorf et al., Forthcoming)}}$$

Private optimum similar, but incorporates *Spence distortion*:

$$N\left(\underbrace{\widetilde{u'}}_{\text{Spence distortion}} - \overline{c'}\right) = M\sigma$$

- Classic Spence: cater to marginal not average consumer
  - Can only extract willingness of marginal to pay
  - Can go either way
- Then you were a tourist...

A platform example

Platforms: “users” value and generate characteristics:

1. Payments: credit, debit, PayPal, etc.
2. Advertising: newspapers, websites, TV, etc.
3. Operating systems: smart phones, video games, etc.

Most common context for this covariance to matter:

- Also generates some nice (slightly) different economics
- Key now: users combine role generating ρ and consuming
  - User utility $u(K;\boldsymbol{\theta}) - P$ where K comes from...
  - Each user generating $k(\boldsymbol{\theta})$, $K = \int_{\Theta} kf$
- Increasing N reduces price by $-\frac{1}{M}$ but also raises by $\frac{\partial P}{\partial K}\frac{dK}{dN}$
  - From earlier, $\frac{\partial P}{\partial K} = \widetilde{u}'$ and $\frac{dK}{dN} = \widetilde{k} + M\sigma\frac{dK}{dN}$
  - Infinite series: attracting sorts more in (out), compounds

$\Longrightarrow$ $\frac{dK}{dN} = \frac{\widetilde{k}}{1-M\sigma} \Longrightarrow \frac{dP}{dN} = -\frac{1}{M} + \frac{\widetilde{k}}{1-M\sigma}$

Pricing platforms

Combining with earlier yields simple rules:

1. Social:

$$S \equiv C' - P = \tilde{k} \underbrace{\frac{\overbrace{\overline{u'}N}^{\text{direct externality}}}{1 - M\sigma}}_{\text{infinite series formula}}$$

2. Private:

$$D \equiv C' + \mu - P = \tilde{k} \frac{\widetilde{u'}N}{1 - M\sigma}$$

- Spence distortion can be magnified or mitigated
- Uncorrelated collapses to simple model with average
- Key feature is not contribution heterogeneity alone...
  - Key is covariance! Sorting effect of characteristic

General model

Paper shows this is crucial in far broader model allowing for

1. Any number of *instruments* (e.g. non-linear pricing) $\boldsymbol{\rho}$
    - Need not include price so “non-transferable utility”
    - Also arbitrarily many endogenous characteristics **K** f
2. Third-degree price discrimination (many “sides” to market)
3. (Symmetrically differentiated Bertrand) Competition
    - Fairly easy to relax, but maintains continuity with monopoly

Nests most contract theory, platforms, matching, static IO

1. Platform j profits are integral $\Pi = \int_{\Theta^j} \pi\left(\boldsymbol{\rho}^j, \mathbf{K}^j\right) f$
2. User i’s utility on platform j is $u^j\left(\boldsymbol{\rho}^j, \mathbf{K}^j; \boldsymbol{\theta}_i\right)$
3. *Characteristic l* generated by users $K_l^j = \int_{\Theta^j} k_l\left(\boldsymbol{\theta}, \boldsymbol{\rho}^j, \boldsymbol{K}^j\right) f$
4. $f, \mathbf{u}, \boldsymbol{\theta}$ symmetric in exchanging firms

$\Longrightarrow$ Natural to look for a symmetric equilibrium in $\boldsymbol{\rho}^j$

New quantities in general formula

Now many instruments, need not include price; new concepts

- *Focal* instrument $\rho^\star$; like price adjusts to hold N fixed
- "Non-transferable utility" $\iff \rho^\star \neq P, P \notin \boldsymbol{\rho}$
- Then $\widetilde{u_\rho}$ not well-defined: like income effects
  - Need to hold fixed number using $\rho^\star$: $\phi_\rho = \frac{\widetilde{u_\rho}}{\widetilde{u_{\rho^\star}}}$
- For $\rho \neq \rho^\star$, $\sigma_{u_\rho - \phi_\rho u_{\rho^\star}, x} \equiv \mathrm{Cov}\left(u_\rho - \phi_\rho u_{\rho^\star}, x \middle| \partial\Theta \right) \equiv \sigma(\rho, x)$
- h may be π (profits) or $\pi + u$ (welfare): spatial economy
  - "Intensive elasticity" just case when $\frac{\partial \pi}{\partial \rho} \neq 0$, like u
  - $\overline{x_{\rho|\rho^\star}} \equiv \overline{x_\rho - \phi_\rho x_{\rho^\star}}$
- Finally, shadow value of characteristic λ_k
  - Also for instrument, but always 0 because can create freely
  - Key difference between instruments, characteristics
- *Exiting* X, *switching* S; $H = X$ for planner, $X + S$ for eq.
- Bolded things indicate vector

General formula and effects

With all this (a bit daunting) very simple formula; focal:

$$\underbrace{\widetilde{\pi}^H}_{\text{Akerlof-Einav-Finkelstein selection}} = \underbrace{\frac{N}{M^H}\frac{\overline{h_{\rho^\star}} + \mathbf{k}_{\rho^\star}^\top \boldsymbol{\lambda}}{-\widetilde{u_{\rho^\star}}}}_{\text{Cournot}/\mu} + \underbrace{M^H \frac{\sigma_{\rho^\star,\pi+\mathbf{k}^\top\boldsymbol{\lambda}}}{-\widetilde{u_{\rho^\star}}}}_{\text{if } \rho^\star \neq P}$$

Thus all standard quantities show up generally

$$\underbrace{\boldsymbol{\lambda}}_{\text{Recursive platform}} = \underbrace{M^H}_{\text{RS}} \overbrace{\boldsymbol{\sigma}\left(\boldsymbol{\rho}, \pi + \mathbf{k}^\top \boldsymbol{\lambda}\right)}^{\text{sorting}} + N \overbrace{\underbrace{h_{\rho|\rho^\star}}_{\text{Spence}} + \mathbf{k}_{\rho|\rho^\star}^\top \boldsymbol{\lambda}}^{\text{Intensive}}$$

There is a lot here in full generality, but organizes many things

- Illustrate by accounting, connecting to literature, examples
- Some also have 3rd-degree discrimination (multiple sides)

Table of effects

	Hetero. prefs.	Hetero. contrib.	Platform	$\rho \neq P$	$P \notin \rho$	2nd° disc.	Market power	Comp.
μ							X	
Spence	X		O	O		O	X	
Death spiral		X					–	X
Smooth selection	X	X						X
σ	X	$X1$	$X1O$	$X1O$		$X2$	X	
RS blow-up		X	O	O			–	X
Finite cream-skimming	X	$X1$	$X1O$	$X1O$		$X2$	X	X
ϕ					X			
Recursive			X				X	
Intensive effect						X	X	

Related literatures

Two key contributions of paper are:

1. Novel σ, quantifies general sorting effect
    - Natural question: why didn't this show up before?
    - Different answers for different literature strands
        1. Platforms (empirics + theory), most IO ignored, thought hard
        2. Contract theory focused on one dimension, so bang-bang
        3. Recent (Einav, Levin...) empirical/computational
        4. Multi-D screening only examples, derivatives hard
2. General framework uniting literatures with price theory
    - Key idea: add *more* realistic heterogeneity to smooth
    - Makes possible to take derivatives, get σ
    - But also easy to add in all other effects price theoretically

To illustrate, series of examples showing all effects

- Table shows previous literature, what examples will develop

Table of literatures

A= analytical, B = bang-bang extreme, C = closed-form examples, E= computational/empirical

	Eivan+ related	Empirical IO/platforms	Platform theory	Classical contract theory	Multi-D screen	Insurance	Broadcast	Credit cards
μ		A	A	A	C	A	A	A
Spence		E	A	A	C	A	A	A
Death spiral	A	B				A		
Smooth selection	A	E			C	A		
σ	E			B	C	A	A	A
RS blow-up				B		A		
Finite cream-skimming					C	A		
ϕ							A	
Recursive		E	A				A	
Intensive effect	E			A	C			A

Setting up competition in the insurance model

Classic result is Rothschild-Stiglitz: no equilibrium

- This is because of *perfect* competition
- If we go to symmetrically differentiated, can quantify
- Start with two insurers; symmetry here means:
  - Cost of covering same on both plans given ρ
  - Distribution of utilities symmetric
  - Sym. eq. in P, ρ: sufficient differentiation
- Two margins: *switching S* between, each *expansion X*
  - By symmetry, same for each, size M^S and M^X
- By symmetry, social optimum just change uniformly:

$$P = \tilde{c}^X$$
$$N\left(\overline{u'} - \overline{c'}\right) = M^X \sigma^X$$

Competition and its distortions

Competition just applies profit maximization to both margins

$$P = \widetilde{c}^X + \overbrace{\frac{N}{M^S + M^X}}^{\text{Cournot distortion}} + \overbrace{\frac{M^S \left(\widetilde{c}^S - \widetilde{c}^X\right)}{M^S + M^X}}^{\text{Akerlof-Einav-Finkelstein (adverse) selection distortion}}$$

$$N \left(\underbrace{\widetilde{u'}^{X+S}}_{\text{Spence distortion}} - \overline{c'} \right) = M^X \sigma^X + \underbrace{M^S \sigma^S}_{\text{Rothschild-Stiglitz cream-skimming}}$$

We can read many classic and new results off:

1. As perfect competition, ($M^S \to \infty$), no eq. as $RScs \to \infty$
2. Comp. improves Cournot and $\approx$ Spence (S like average)
3. Comp worsens selection and RScs (later infinitely)
   - Net on price likely good, bad on coverage with adverse
     - Price must be above AC with monopoly, blow up on coverage

Broadcast media and the lack of transfers

In broadcast media, no price to viewers

- This lack of “transferable utility” adds two wrinkles as above
- Let’s model soap operas to illustrate: $m \equiv$ melodrama
- Two *sides* to market: viewer and advertisers
- Viewers’ utility $u^{\mathcal{V}}\left(m; \boldsymbol{\theta}^{\mathcal{V}}\right)$, advertisers $\theta^{\mathcal{A}} S - P^{\mathcal{A}}$
- S is total spending power of readers $s\left(\boldsymbol{\theta}^{\mathcal{V}}\right)$
- Cost $C\left(m, N^{\mathcal{A}}, N^{\mathcal{V}}\right)$; two changes w/o transfers:
    1. Marginal utility holding fixed number by m: $\phi^{\mathcal{V}}_A = \frac{\widetilde{u^{\mathcal{V}}_A}}{\widetilde{u^{\mathcal{V}}_m}}$
    2. *Relative covar.*: $\sigma^{\mathcal{V}}(A, s) \equiv \mathrm{Cov}\left[\, u^{\mathcal{V}}_A - \phi^{\mathcal{V}}_A u^{\mathcal{V}}_m, s \,\middle|\, u^{\mathcal{V}} = 0\right]$

$$\lambda^A = \underbrace{\phi_A C_m}_{\text{direct externality}} + \underbrace{M^{\mathcal{V}} \frac{N^{\mathcal{A}} P^{\mathcal{A}}}{S} \sigma^{\mathcal{V}}(A, s)}_{\text{boomerang sorting externality to advertisers}}$$

Platform pricing without transfers

Profit-maximizing pricing/content provision then simple:

$$P^{\mathcal{A}} = \mu^{\mathcal{A}} + C_{N^{\mathcal{A}}} - \widetilde{a}\lambda^{A}$$

$$\underbrace{0}_{\text{Price}} = \underbrace{\frac{C_m}{M\widetilde{u_m^{\mathcal{V}}}}}_{\text{Cournot distoriton}} + \underbrace{C_{N^{\mathcal{V}}}}_{\text{marginal cost}} - \underbrace{\frac{P^{\mathcal{A}}N^{\mathcal{A}}}{S}}_{\text{per-income price for ads}} \left(\underbrace{\sigma^{\mathcal{V}}_{u_m,s}}_{\text{why soap operas}} + \underbrace{\widetilde{s}}_{\text{standard externality}} \right)$$

Can also derive socially optimal prices...

- But requires stand on interpersonal comparisons
- No transfers assumption to rely on
- How to measure? Important in many literatures

Fitting in non-linear pricing

Two-part tariff credit cards combine classic platform, multi-D

- Enrich, combine Rochet-Stole (02) and Rochet-Tirole (03)
  - Can extend to more general non-linear pricing
- Two sides: consumers $\mathcal{C}$ and merchants $\mathcal{M}$
- Fixed and linear prices $P^{\mathcal{C}}, p^{\mathcal{C}}$, linear to merchant $p^{\mathcal{M}}$
  - Consumers and merchants randomly matched
  - Merchants only choose to accept; consumers also intensive
- Merchant net value $\theta^{\mathcal{M}}$ from accepting
  - Accept iff $\theta^{\mathcal{M}} \geq p^{\mathcal{M}}$; fraction $N^{\mathcal{M}}$ join
- Consumers choose $q\left(p; \boldsymbol{\theta}^{\mathcal{C}}\right)$ conditional card purchases
  - Envelope: $S^{\mathcal{C}}\left(p^{\mathcal{C}}; \boldsymbol{\theta}^{\mathcal{C}}\right) = \int_{p^{\mathcal{C}}}^{\infty} q\left(\tilde{p}; \boldsymbol{\theta}^{C}\right) d\tilde{p} - p^{\mathcal{C}} q\left(p; \boldsymbol{\theta}^{\mathcal{C}}\right)$
  - Carry card if $S^{\mathcal{C}} N^{\mathcal{M}} \geq P^{\mathcal{C}}$; total fraction of purchases Q
- Cost $cQN^{\mathcal{M}}$, number of consumer $N^{\mathcal{C}}$

Magnitudes and intuitions with non-linear pricing

$P^{\mathcal{C}}, p^{\mathcal{M}}$ fairly boring; socially optimal linear $p^{\mathcal{C}} = c - \overline{\theta^{\mathcal{M}}}$

- Privately optimal follows straight from logic:

$$\underbrace{\left(p^{\mathcal{M}} + p^{\mathcal{C}} - c\right)}_{\text{Rochet-Tirole mark-up}} \left(\underbrace{M^{\mathcal{C}} \mathrm{Cov}\left[\widetilde{S^{\mathcal{C}}_{p^{\mathcal{C}}}, q}\right]}_{\sigma} + \underbrace{N^{\mathcal{C}} \overline{\frac{dq}{dp^{\mathcal{C}}}}}_{\text{intensive}} \right) = \underbrace{N^{\mathcal{C}}\left(\overline{q} - \widetilde{q}\right)}_{\text{Spence}}$$

- $p^{\mathcal{M}}$ replaces $\overline{\theta^{\mathcal{M}}}$ by Spence
- $S^{\mathcal{C}}_{p^{\mathcal{C}}} = q$ by envelope so $\sigma = \widetilde{\mathrm{Var}}[q]$
- To connect with literature, rearrange, define elasticities
  1. $\widetilde{\epsilon^{\mathcal{C}}_X} \equiv \frac{\widetilde{q} M^{\mathcal{C}} p}{Q}$, average quantity-weighted *extensive elasticity*
  2. $\overline{\epsilon^{\mathcal{C}}_I} \equiv -\frac{E[\epsilon q]}{E[q]}$, average quantity-weighted *intensive elasticity*

Linear component under non-linear pricing

$$\underbrace{\frac{p^{\mathcal{C}} - (c - p^{\mathcal{M}})}{p^{\mathcal{C}}}}_{\text{Lerner's mark-up}} = \frac{\overbrace{1 - \frac{\tilde{q}}{\overline{q}}}^{\text{Spence}}}{\underbrace{\overline{\epsilon_X^{\mathcal{C}}} \frac{\widetilde{Var[q]}}{\tilde{q}}}_{\text{Rochet-Stole sorting discipline}} + \underbrace{\overline{\epsilon_I^{\mathcal{C}}}}_{\text{Mussa-Rosen-Wilson}}}$$

- Nests several classic papers:
  1. If exiters average ($\tilde{q} = \overline{q}$) no distortion
     - Bedre-Defolie and Calvano (2010) use for credit cards
     - Assume no ex-ante knowledge, so marginals same
  2. If no platform, exit from bottom ($p^{\mathcal{M}} = \tilde{q} = \widetilde{\mathrm{Var}}[q] = 0$)...
     - Mussa-Rosen; Wilson formula $\frac{p-c}{p} = \frac{1}{\epsilon}$
- Suggests RS, selection effect with competition

Multiplicity, implementation and insulation

Problem with above analysis: ec's determined by users

- Given instruments, may be coordination problem
- Simple example: two sides $\mathcal{A}, \mathcal{B}$ with $u^{\mathcal{S}}\left(N^{-\mathcal{S}}; \boldsymbol{\theta}^{\mathcal{S}}\right)$
- Platform choose prices to each side, users coordinate
- Multiple **N** given **P**, but unique **P** given **N**
- $N^{\mathcal{B}} \implies$ distribution of values $\implies \frac{\partial P^{\mathcal{A}}}{\partial N^{\mathcal{A}}} < 0$
- If platform *could* choose quantities, easy
- But how can it *implement* more robustly to coordination?
- Condition prices on people on other side $P^{\mathcal{S}}\left(N^{-\mathcal{S}}\right)$!
  1. Choose target quantities $\left(\widetilde{N^{\mathcal{A}}}, \widetilde{N^{\mathcal{B}}}\right)$
  2. Charge *insulating tariff* $P^{\mathcal{S}}\left(N^{-\mathcal{S}}\right) \equiv P^{\mathcal{S}}\left(\widetilde{N^{\mathcal{S}}}, N^{-\mathcal{S}}\right)$
- Armstrong, Rochet-Tirole solutions special cases
  - Armstrong homogeneous utility, guarantee utility level
  - RT2003 proportional, price proportional

General conditions for insulation

What does this represent? White and Weyl (2011):

- Most internet companies had low initial prices

$\Longrightarrow$ Reduced-form for dynamic strategy (Cabral 2011)

Things are a bit more complicated in this paper

1. Many ec's, not just quantities
2. Need not have price instrument

Nonetheless natural analogy *insulating platform design*:

- Allow *all instruments* to condition on ec's
  - Reduced for dynamic adjustment of platform characteristics
- Technical question: when is this possible?
  - Intuitively: "enough" instruments to fix characteristics
  - Formally, $\left[\frac{\partial \mathbf{K}}{\partial \boldsymbol{\rho}}\right]^{-1} \frac{\partial \mathbf{K}}{\partial \hat{\mathbf{K}}}$ has bounded eigenvalues
  - Uniqueness: can't have too many ρ: also bound from below

Directions for future research

Paper aims to make three contributions:

1. General-purpose IO/contract model
2. Use rich heterogeneity to smooth, price theory approach
3. "Intuitive" labeling/categorizing of "effects"

Take away: don't be intimidated by multi-D screening, platforms

- Quite naturally amenable to simple empirical work
- Applied theory and empirical applications in progress:
  1. Gale-Shapley matching and college admissions
  2. Cream-skimming and women's' work-life balance
- Crucial to combine with competition
  - Heterogeneity endogenous through multihoming
  - Work with Alex White extends *AER* paper to competition
  - Uses insulation; combine with insulating platform design
- Working with Fabinger on general richness of Weyl-Tirole