Cost

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Lecture 3
Turbo Section
Elements of Economic Analysis II
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Technology and cost

This week we'll study how firms produce

1. Today we'll study production *itself per unit of time*
   - We won't focus on why firms produce or how for how
2. On Thursday we'll study supply and its duration

Today we'll learn two dual/equivalent ways to represent

1. Production technology
   - Makes easy to represent many inputs, choices about them
   - Intuitive based on information on creating products
   - More difficult when thinking about choices, many outputs
2. Costs of production
   - Makes easy to represent many *outputs*
   - Intuitive based on output prices
   - Difficult when thinking about choices of inputs

We'll learn links, properties and uses of these

Getting into college: a motivating example

An example close to home may be useful motivation

- You are trying to get into a good college
- Imagine all colleges evaluate you based on a score s
  - Higher s, better college you get into
- s can be created by being good at:
  1. Extracurricular activities x, which costs t_x hours
  2. Academic work a, which costs t_a hours
- We'll think a lot about two ways of representing this
  1. Production of college admission
     - $s = f(x, a)$ determined by thinking about what colleges value
     - Helps determine which of a and x to spend time on
  2. Cost of being accepted
     - $t = C(s)$ is total time it takes to achieve score of s
     - Useful in making trade-off between prepping and all else
     - Partying, sleeping, time with family, etc.

Some extreme types of technology

1. Perfect complements (“Leontief” or “fixed proportions”)?
    - $f(x, a) = \tilde{f}(\min\{\xi x, \alpha a\})$: schools want perfect balance only
    - Important in many industries with complex products
    - O-Ring story/weakest link
2. Independent (“perfect substitutes”); mathematical form?
    - $f(x, a) = \tilde{f}(\xi x + \alpha a)$: schools’ value adds two components
    - Relevant mostly when different dimension are fungible
    - Advertisements in different newspapers
3. Either-or (“ultra-substitutes”); mathematical form?
    - $f(x, a) = \tilde{f}(\max\{\xi x, \alpha a\})$: school values only strength
    - Relevant when you have to choose one mode of production
    - Oil v. coal power plant, developing for android v. iPhone
    - Typically assume this away, but relevant in many settings
        - Also less extreme form of “more-than-perfect substitutes”

Properties of more common technologies

Thus, commonly, we'll assume that technology is

1. Monotonic (has free disposal)
   - No school holds being better against you
   - You can always "dispose of these"
2. Convex: define?
   - Preference for diversity; always combine and get as much

   $\Longrightarrow$ Along isoquant $-\frac{f_x}{f_a}$ declines as x increases
   - Called *marginal rate of technical substitutions* (MRTS)
3. Declining marginal product: define?
   - Usually we go further and assume $f_{xx}, f_{aa} < 0$
   - Value of each product usually declining
   - Also justified by optimization (next lecture)
4. Smooth
   - Often avoid kinks of simple production technology
   - Allows us to use calculus for everything

Cobb-Douglas function and other forms

Most common form is Cobb-Douglas $f(x, a) = kx^{\xi}a^{\alpha}$

- Equivalently: elasticity of f wrt x, a constant at ξ, α
- Extremely tractable, easy to use
- Generalizations a bit mathematically messy, but...
  - *MRTS* for Cobb-Douglas $= -\frac{\xi a}{\alpha x} \propto \frac{a}{x}$, elasticity of 1 in $\frac{a}{x}$
  - What if it still had constant elasticity, but different $\left(\frac{a}{x}\right)^{\frac{1}{\sigma}}$?
  - Called *constant elasticity of substitution* $\sigma = \frac{1}{\epsilon}$, takes form

$$f(x, a) = k\left(\frac{\xi}{\xi+\alpha}x^{1-\epsilon} + \frac{\alpha}{\xi+\alpha}a^{1-\epsilon}\right)^{\frac{\xi+\alpha}{1-\epsilon}}$$

1. Perfect complements: $\epsilon \to \infty$
2. Cobb-Douglas: $\epsilon = 1$; come derive
3. Independent: $\epsilon \to 0$
4. Either-or: $\epsilon \to -\infty$

$\Longrightarrow$ Messy, but lets you find range of examples

The standard taxonomy of costs

Another way is to think about costs; typical types:

1. Variable costs
   - Increase more-or-less continuously with output
   - Time spent studying or practicing sports
2. Lumpy (quasi-fixed) costs
   - Vary discontinuously with output levels
   - *Necessary* or *useful* to increase output in some range
     - Enrolling in a new class or taking up new activity
     - Paying for a tutor or buying new equipment
3. Fixed costs
   - Necessary to produce at all, but still can be avoided
     - Enrolling in school or starting club you devote all time to
4. Sunk costs
   - Fixed costs undertaken in past you can't undue
   - Shouldn't affect on future decisions: tuition already paid

Other distinction among costs

Costs can come from many sources:

1. Money costs
   - Most intuitive and common source: have to buy things
   - Equipment, textbooks, school supplies, etc.
2. Opportunity cost of time or other resources
   - Classic topic in economics
   - Time can be used for other purposes
   - Other scarce resources: energy, will power
3. Capital costs
   - Depleting fixed resources with future value
   - Problem set will ask you to explore; often neglected
   - Relationships, reputation, physical appearance, health

Why costs rather than technology?

Costs are useful relative to technology because:

1. Help think about *how much to produce*
   - Trade-off between college and other goals
   - One, parsimonious number summarizing
   - Easy to graph as we'll see
2. Help determine prices/interface with product markets
   - We want to determine what score you need for each school
   - Means we need to aggregate across all students
   - Find an equilibrium level to get in
   - Difficult using technology; easy using cost (next lectures)
3. With fancier math, we can even do this for many outputs
   - But we usually won't

Simple examples of equivalence

Nonetheless, the two concepts are equivalent ("dual")

- This is what economists mean by "duality" in economics
  - If only one input, price p_i just invert; $o = f(i) = i^\gamma \implies C(o) = p_i o^{\frac{1}{\gamma}}$
- When many inputs, two steps
  1. Find cheapest way to achieve given output
  2. Call this the *cost* of the output
- Consider the simple "extreme" technologies above
  1. Perfect complements?
     - Here you have to do both so cost $C(s) = \tilde{f}^{-1}(s)\left(\frac{t_x}{\xi} + \frac{t_a}{\alpha}\right)$
  2. Independent?
     - Can choose which method cheaper: $\frac{t_x}{\xi}$ or $\frac{t_a}{\alpha}$
     - So $C(s) = \tilde{f}^{-1}(s)\left(\min\left\{\frac{t_x}{\xi}, \frac{t_a}{\alpha}\right\}\right)$
     - Same as either-or, except might use both

Cost minimization: the mathematical problem

Let's now consider the more general two-input case:

$$C(s) \equiv \text{argmin}_{(x,a)} t_x x + t_a a \text{ subject to } f(x,a) = s$$

- Classic constrained minimization problem
- Let's review economic approach to this problem

1. We think of there being an *opportunity cost* of not producing
2. Minimize total (including opportunity) costs unconstrained
    - This opportunity cost is also called *Lagrange multiplier*
    - However, to emphasize economic intuition I will avoid this
3. Finally, we solve for opportunity cost
    - This is marginal cost of additional output, so useful in itself

Cost minimization and Lagrange multipliers

Take-away

What lessons do we learn from this?

1. Classical productive efficiency conditions:

For any two inputs i_j, i_k $MRTS_{jk} = \frac{f_k}{f_j} = \frac{p_k}{p_j}$

2. This is exactly what tangency interpretation is
3. Lagrange multipliers are just (marginal) opportunity costs
   - Solve marginal cost; can be used for *any* (opportunity) cost
4. Specific cases: won't bore with calculations, but for CES

$$C(s) = \left[\left(\frac{\xi}{\xi+\alpha}\right)^{\frac{1}{\epsilon}} t_x^{\frac{\epsilon-1}{\epsilon}} + \left(\frac{\alpha}{\xi+\alpha}\right)^{\frac{1}{\epsilon}} t_a^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \left(\frac{s}{k}\right)^{\frac{1}{\alpha+\xi}}$$

C-D special case: $C(s) = (\alpha+\xi)\left(\frac{t_x}{\xi}\right)^{\frac{\xi}{\alpha+\xi}} \left(\frac{t_a}{\alpha}\right)^{\frac{\alpha}{\alpha+\xi}} \left(\frac{s}{k}\right)^{\frac{1}{\alpha+\xi}}$

Standard cost curves

Mostly we will consider costs in per-unit terms:

1. Marginal cost
   - The cost of producing one more unit $MC(o) \equiv C'(o)$
2. Average costs
   - Average (total) cost $AC(o) \equiv \frac{C(o)}{o}$
   - Sometimes decomposed into average *fixed* and *variable*
   - $AFC(o) = \frac{FC}{o}$ and $AVC(o) = \frac{VC(o)}{o}$
   - $AC(o) = AFC(o) + AVC(o)$
   - $AC'(o) = \frac{TC'(o)o - TC(o)}{o^2} = \frac{MC(o) - AC(o)}{o}$
     - How I met my wife
3. Total cost
   - We rarely draw this curve, so not a focus
   - But can be decomposed into fixed and variable components

Typical shapes and relations

Example of cost curve: online ad auctions

Athey and Nekipelov (2010) study online search ad auctions

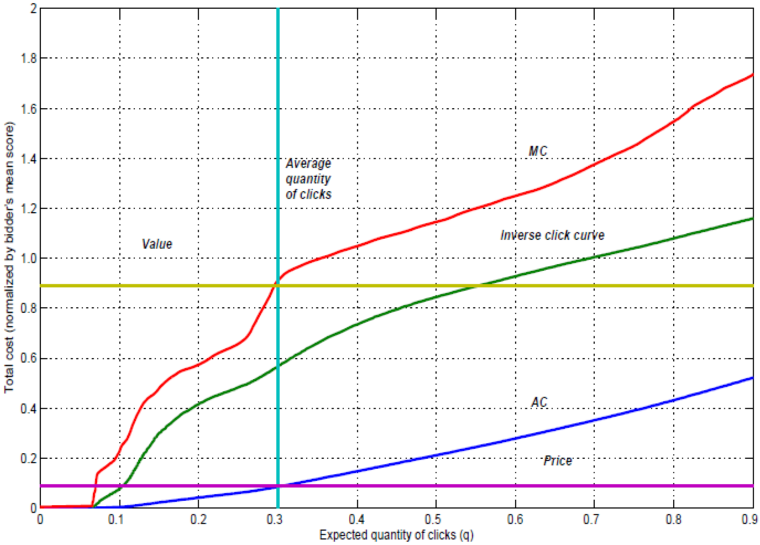
- Bidders offer price per-click for words like “car insurance”
- Leave constant for week or two
- Win slots based on score (from engine), competition, etc.
    - Are people likely to click based on experience?
    - Is the ad annoying or useful?
- Higher slot more likely to get higher score

    $\Longrightarrow$ Higher bid leads smoothly to higher average clicks
    - Inverting relationship gives average cost of clicks
    - From this, we can derive marginal cost of click

$\Longrightarrow$ Used to determine optimal bids

- Estimate using proprietary data from Microsoft

Athey-Nekipelov cost curve for common search term

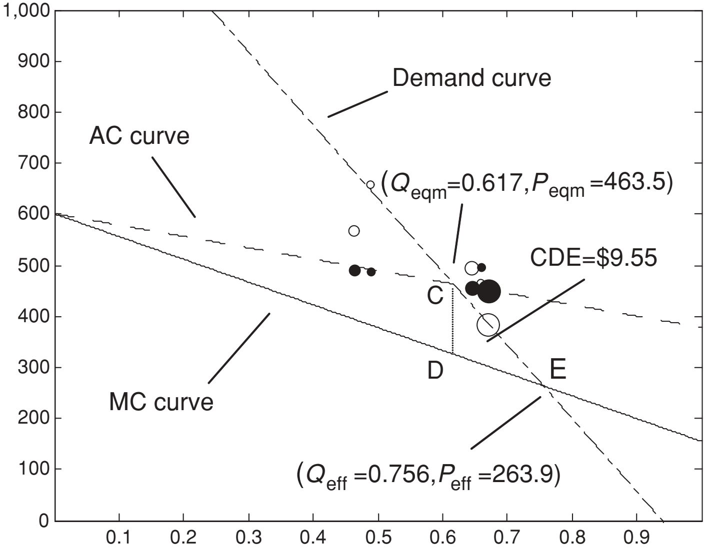


Example: insurance and adverse selection

Consider cost to the government of insurance coverage

- Quantity is fraction of population covered
- Sets a price to determine how many covered
- As it lowers price, more people will sign up
- Cost not linear because different people
    - Typical story: most sick people will seek insurance first
        - Called *adverse selection* or the *lemons problem*
        - Gresham's law: bad money drives out good (if treated same)
    - Might also be opposite: health conscious are first
        - Called *advantageous selection*
    - We'll return to policy implications in Lecture 12
    - Meantime: let's consider Einav et al. (2010)'s estimation
        - Consider divisions at large company
        - Price for insurance (more or less) randomly assigned
        - Generates variation in quantities, estimates of cost curves

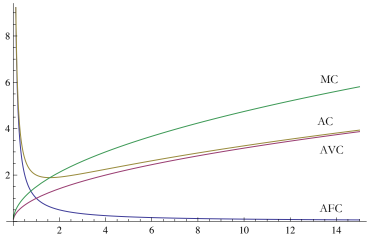
Einav et al.'s cost curves for insurance



Cobb-Douglas with fixed costs

Note that neither of these fit standard cost curve shape

- Standard story: fixed costs, but decreasing returns
  - Entrepreneurial talent spread thin
- Let's consider example: Cobb-Doulgas with fixed costs
- $C(s) = F + \kappa s^{\gamma}$
  - $AFC = \frac{F}{s}$; $AVC = \kappa s^{\gamma-1}$; $AC = \frac{F}{s} + \kappa s^{\gamma-1}$; $MC = \kappa\gamma s^{\gamma-1}$
  - With $F = 1, \kappa = 1, \gamma = 1.5$:



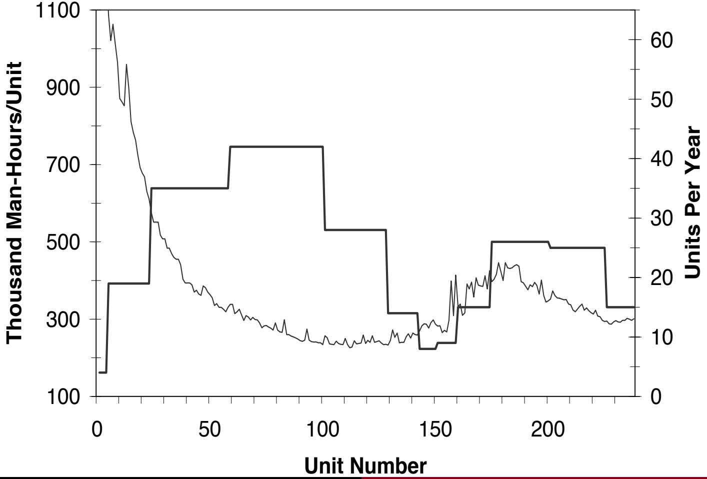
Comparative statics

Economies and diseconomies of scale

Standard assumption: AC first declines, then increases

- Declining AC called *economies of scale*: reasons?
  1. Fixed or lumpy costs for efficiency: Andres Carnes de Res
  2. Specialization: Smith's pin factory
  3. Learning-by-doing: Airplane manufacturing
  4. Network effects: Facebook (from consumers)
  5. Adverse selection: insurance as above (industry wide)
- Increasing MC called *diseconomies of scale*?
  1. Entrepreneurial talent spread thin: restaurants
  2. Agency and monitoring costs: developing world problems
  3. Market power: internet auctions, more later
- Nonetheless, most common is *constant returns to scale*
  - Often can replicate activities
  - Most large expansion requires replicating fixed costs

Benkard (2000)'s data on aircraft manufacturing



Other "economies" and diseconomies

Measurement of economies of scale:

1. *Elasticity of scale*: $\epsilon_S = \frac{AC}{MC}$
   - $> 1 \implies$ economies; $< 1 \implies$ diseconomies
2. *Homogeneity of degree k*: $C(2o) = 2^{\frac{1}{k}} C(o)$
   - Equivalent to constant elasticity of scale k
   - Technically: double all inputs, you get 2^k outputs
   - So don't change inputs as you grow in size

When multiple outputs (rarely) we also talk of scope

1. Economies of scope: cheaper to produce together
   - Common inputs: delivery systems for WalMart
   - Bi-products: crop rotation
   - Spillovers and agglomeration: cities (industry wide)
2. Diseconomies of scope: cheaper to produce apart
   - Clash of cultures: INDECOPI in Perú