

# Cost

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Lecture 3  
Regular Section  
Elements of Economics II  
Fall 2011

# Technology and cost

This week we'll study how firms produce

- 1 Today we'll study production *itself per unit of time*
  - We won't focus on why firms produce or how for now
- 2 On Thursday we'll study supply and its duration

Today we'll learn two dual/equivalent ways to represent

- 1 Production technology
  - Makes easy to represent many inputs, choices about them
  - Intuitive based on information on creating products
  - More difficult when thinking about choices, many outputs
- 2 Costs of production
  - Makes easy to represent many *outputs*
  - Intuitive based on output prices
  - Difficult when thinking about choices of inputs

We'll learn links, properties and uses of these

# Getting into college: a motivating example

An example close to home may be useful motivation

- You are trying to get into a good college
- Imagine all colleges evaluate you based on a score  $s$ 
  - Higher  $s$ , better college you get into
- $s$  can be created by being good at:
  - 1 Extracurricular activities  $x$ , which costs  $t_x$  hours
  - 2 Academic work  $a$ , which costs  $t_a$  hours
- We'll think a lot about two ways of representing this
  - 1 Production of college admission
    - $s = f(x, a)$  determined by thinking about what colleges value
    - Helps determine which of  $a$  and  $x$  to spend time on
  - 2 Cost of being accepted
    - $t = C(s)$  is total time it takes to achieve score of  $s$
    - Useful in making trade-off between prepping and all else
    - Partying, sleeping, time with family, etc.

## Some extreme types of technology

- 1 Perfect complements (“Leontief” or “fixed proportions”)?
  - $f(x, a) = \min\{x, a\}$ : schools want perfect balance only
  - Important in many industries with complex products
  - O-Ring story/weakest link
- 2 Independent (“perfect substitutes”); mathematical form?
  - $f(x, a) = x + a$ : schools’ value just adds two components
  - Relevant mostly when different dimension are fungible
  - Advertisements in different newspapers
- 3 Either-or (“ultra-substitutes”); mathematical form?
  - $f(x, a) = \max\{x, a\}$ : school values you only on strength
  - Relevant when you have to choose one mode of production
  - Oil v. coal power plant, developing for android v. iPhone
  - Typically assume this away, but relevant in many settings
    - Also less extreme form of “more-than-perfect substitutes”

# Properties of more common technologies

Thus, commonly, we'll assume that technology is

- 1 Monotonic (has free disposal)
  - No school holds being better against you
  - You can always “dispose of these”
- 2 Convex: define?
  - Preference for diversity; always combine and get as much
  - ⇒ Isoquants always “point towards” origin
  - Called *marginal rate of technical substitution* (MRTS)
- 3 Declining marginal product: define?
  - Usually we go further and assume concave in each input
  - Value of each product usually declining
  - Also justified by optimization (next lecture)
- 4 Smooth
  - Often avoid kinks of simple production technology
  - Allows us to use calculus for everything

# Cobb-Douglas function

Most common Cobb-Douglas  $f(x, a) = kx^\xi a^\alpha$ ; properties

- 1 Constant elasticity of production
  - An  $p\%$  increase in  $x$  increase output by  $\xi p\%$ ; prove?
- 2 Homogeneous of degree  $\xi + \alpha$ 
  - $f(2x, 2a) = 2^{\xi+\alpha} f(x, a)$ ; derive from last
- 3 Linear in logarithms
  - $\log(f) = \log(k) + \xi \log(x) + \alpha \log(a)$
- 4  $MRTS_{x,a} = \frac{f_a}{f_x} = \frac{\alpha x}{\xi a} \propto \frac{x}{a}$ : complements
  - Ratio of inputs determines relative desirability
  - More you have of one, more valuable is other

How are these properties related?

# The standard taxonomy of costs

Another way is to think about costs; typical types:

- 1 Variable costs
  - Increase more-or-less continuously with output
  - Time spent studying or practicing sports
- 2 Lumpy (quasi-fixed) costs
  - Vary discontinuously with output levels
  - *Necessary* or *useful* to increase output in some range
    - Enrolling in a new class or taking up new activity
    - Paying for a tutor or buying new equipment
- 3 Fixed costs
  - Necessary to produce at all, but still can be avoided
    - Enrolling in school or starting club you devote all time to
- 4 Sunk costs
  - Fixed costs undertaken in past you can't undue
  - Shouldn't affect on future decisions: tuition already paid

## Other distinction among costs

Costs can come from many sources:

- 1 Money costs
  - Most intuitive and common source: have to buy things
  - Equipment, textbooks, school supplies, etc.
- 2 Opportunity cost of time or other resources
  - Classic topic in economics
  - Time can be used for other purposes
  - Other scarce resources: energy, will power
- 3 Capital costs
  - Depleting fixed resources with future value
  - Problem set will ask you to explore; often neglected
  - Relationships, reputation, physical appearance, health

# Why costs rather than technology?

Costs are useful relative to technology because:

- 1 Help think about *how much to produce*
  - Trade-off between college and other goals
  - One, parsimonious number summarizing
  - Easy to graph as we'll see
- 2 Help determine prices/interface with product markets
  - We want to determine what score you need for each school
  - Means we need to aggregate across all students
  - Find an equilibrium level to get in
  - Difficult using technology; easy using cost (next lectures)
- 3 With fancier math, we can even do this for many outputs
  - But we usually won't

# Simple examples of equivalence

Nonetheless, the two concepts are equivalent (“dual”)

- This is what economists mean by “duality” in economics
  - If only one input, price  $p_i$  just invert;  
$$o = f(i) = i^\gamma \implies C(o) = p_i o^{\frac{1}{\gamma}}$$
- When many inputs, two steps
  - 1 Find cheapest way to achieve given output
  - 2 Call this the *cost* of the output
- Consider the simple “extreme” technologies above
  - 1 Perfect complements?
    - Here you have to do both so cost  $C(s) = s(t_x + t_a)$
  - 2 Independent?
    - Can choose which method cheaper:  $t_x$  or  $t_a$
    - So  $C(s) = s(\min\{t_x, t_a\})$
    - Same as either-or, except might use both

# Cost minimization: the mathematical problem

Let's now consider the more Cobb-Douglas two-input case:

$$C(s) \equiv \operatorname{argmin}_{(x,a)} t_x x + t_a a \text{ subject to } kx^\xi a^\alpha = s$$

- Classic constrained minimization problem
- Let's review economic approach to this problem
  - 1 We think of there being an *opportunity cost* of not producing
  - 2 Minimize total (including opportunity) costs unconstrained
    - This opportunity cost is also called *Lagrange multiplier*
    - However, to emphasize economic intuition I will avoid this
  - 3 Finally, we solve for opportunity cost
    - This is marginal cost of additional output, so useful in itself

# Cost minimization and Lagrange multipliers

# Take-away

What lessons do we learn from this?

- 1 Classical productive efficiency conditions:

$$\text{For any two inputs } i_j, i_k \text{ } MRTS_{jk} = \frac{f_k}{f_j} = \frac{p_k}{p_j}$$

- 2 This is exactly what tangency interpretation is
- 3 Lagrange multipliers are just (marginal) opportunity costs
  - Solve marginal cost; can be used for *any* (opportunity) cost
- 4 Cobb Douglas formula:

$$C(s) = (\alpha + \xi) \left( \frac{t_x}{\xi} \right)^{\frac{\xi}{\alpha + \xi}} \left( \frac{t_a}{\alpha} \right)^{\frac{\alpha}{\alpha + \xi}} \left( \frac{s}{k} \right)^{\frac{1}{\alpha + \xi}}$$

Optimal factor proportions are  $\frac{a}{x} = \frac{t_x \alpha}{t_a \xi}$

# Standard cost curves

Mostly we will consider costs in per-unit terms:

## 1 Marginal cost

- The cost of producing one more unit  $MC(o) \equiv C'(o)$

## 2 Average costs

- Average (total) cost  $AC(o) \equiv \frac{C(o)}{o}$
- Sometimes decomposed into average *fixed* and *variable*
- $AFC(o) = \frac{FC}{o}$  and  $AVC(o) = \frac{VC(o)}{o}$
- $AC(o) = AFC(o) + AVC(o)$
- $AC'(o) = \frac{TC'(o)o - TC(o)}{o^2} = \frac{MC(o) - AC(o)}{o}$ 
  - How I met my wife

## 3 Total cost

- We rarely draw this curve, so not a focus
- But can be decomposed into fixed and variable components

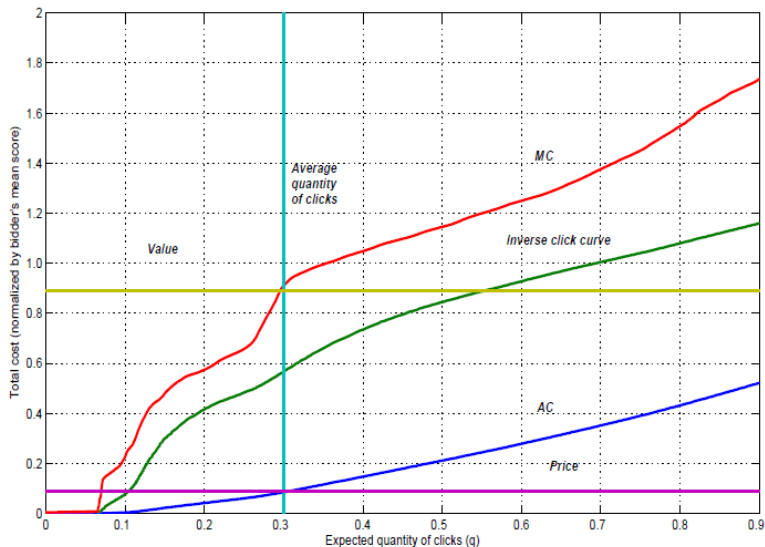
# Typical shapes and relations

## Example of cost curve: online ad auctions

Athey and Nekipelov (2010) study online search ad auctions

- Bidders offer price per-click for words like “car insurance”
- Leave constant for week or two
- Win slots based on score (from engine), competition, etc.
  - Are people likely to click based on experience?
  - Is the ad annoying or useful?
- Higher slot more likely to get higher score
  - ⇒ Higher bid leads smoothly to higher average clicks
    - Inverting relationship gives average cost of clicks
    - From this, we can derive marginal cost of click
- ⇒ Used to determine optimal bids
  - Estimate using proprietary data from Microsoft

# Athey-Nekipelov cost curve for common search term

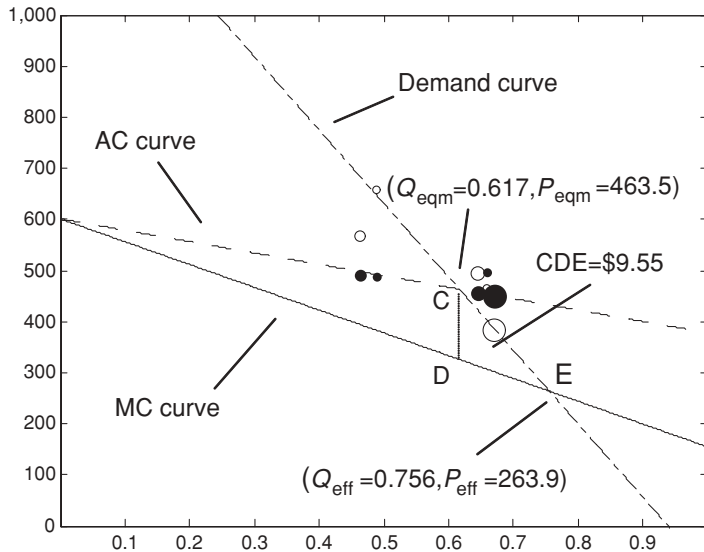


## Example: insurance and adverse selection

Consider cost to the government of insurance coverage

- Quantity is fraction of population covered
- Sets a price to determine how many covered
- As it lowers price, more people will sign up
- Cost not linear because different people
  - Typical story: most sick people will seek insurance first
    - Called *adverse selection* or the *lemons problem*
    - Gresham's law: bad money drives out good (if treated same)
  - Might also be opposite: health conscious are first
    - Called *advantageous selection*
  - We'll return to policy implications in Lecture 12
  - Meantime: let's consider Einav et al. (2010)'s estimation
    - Consider divisions at large company
    - Price for insurance (more or less) randomly assigned
    - Generates variation in quantities, estimates of cost curves

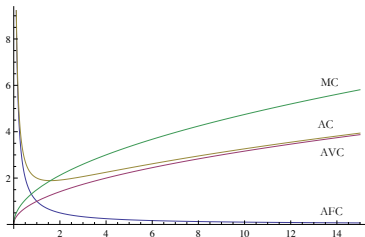
# Einav et al.'s cost curves for insurance



## Cobb-Douglas with fixed costs

Note that neither of these fit standard cost curve shape

- Standard story: fixed costs, but decreasing returns
  - Entrepreneurial talent spread thin
- Let's consider example: Cobb-Douglas with fixed costs
- $C(s) = F + \kappa s^\gamma$ 
  - $AFC = \frac{F}{s}$ ;  $AVC = \kappa s^{\gamma-1}$ ;  $AC = \frac{F}{s} + \kappa s^{\gamma-1}$ ;  $MC = \kappa \gamma s^{\gamma-1}$
  - With  $F = 1, \kappa = 1, \gamma = 1.5$ :



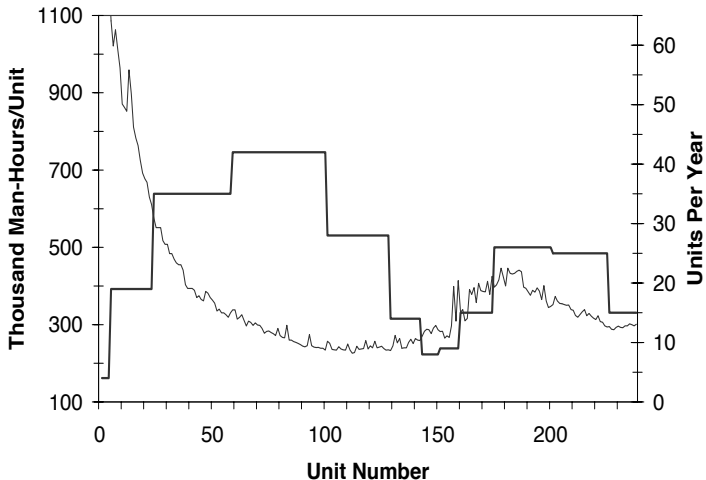
# Comparative statics

# Economies and diseconomies of scale

Standard assumption: AC first declines, then increases

- Declining AC called *economies of scale*: reasons?
  - 1 Fixed or lumpy costs for efficiency: Andres Carnes de Res
  - 2 Specialization: Smith's pin factory
  - 3 Learning-by-doing: Airplane manufacturing
  - 4 Network effects: Facebook (from consumers)
  - 5 Adverse selection: insurance as above (industry wide)
- Increasing MC called *diseconomies of scale*?
  - 1 Entrepreneurial talent spread thin: restaurants
  - 2 Agency and monitoring costs: developing world problems
  - 3 Market power: internet auctions, more later
- Nonetheless, most common is *constant returns to scale*
  - Often can replicate activities
  - Most large expansion requires replicating fixed costs

# Benkard (2000)'s data on aircraft manufacturing



## Other “economies” and diseconomies

Measurement of economies of scale:

- 1 *Elasticity of scale*:  $\epsilon_S = \frac{AC}{MC}$ 
  - $> 1 \implies$  economies;  $< 1 \implies$  diseconomies
  - Generalization of degree of homogeneity  $k$ 
    - Homogeneity  $\implies C(xo) = x^{\frac{1}{k}} C(o)$
    - Prove that when homogeneous,  $\epsilon_S = k$  everywhere

When multiple outputs (rarely) we also talk of scope

- 1 Economies of scope: cheaper to produce together
  - Common inputs: delivery systems for WalMart
  - Bi-products: crop rotation
  - Spillovers and agglomeration: cities (industry wide)
- 2 Diseconomies of scope: cheaper to produce apart
  - Clash of cultures: INDECOPI in Perú