Oligopoly

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Introduction

So far all market power models have one firm

- Today we want to bridge monopoly and competition
- Basic problem: lots of ways to relate oligopolistically
  - Strategic interactions tougher than markets, individuals

We'll look at a range of ways thinking about oligopoly

1. The most classical model: Cournot's quantity choice
2. Sequential quantity model of Stackelberg
3. Game theory as a broader language for strategyy
4. The Bertrand-Edgeworth critique: paradox of two firms
5. Differentiated products as a solution
6. Collusion and Stigler's theory
   - Incentives to collude and difficulty in achieving
   - Stigler's factors facilitating and hindering
7. Conjectural variations as a summary of collusion

Cournot's model of oligopoly

Most classic model of oligopoly is Cournot (1838)?

1. N firms produce a homogeneous product
   - Often we will assume same, constant MC c
2. Each firm takes as *the quantity of all others*
   - Sometimes stated as "choosing quantities" but misleading
     - Firm could think of itself as choosing quantity or price
     - Key is what it holds fixed from others, not chooses
3. Equilibrium is when each maximizes given others choices

Each firm earns profits $q_i \left[P\left(q_i + Q_{-i}\right) - c\right]$

- $Q_{-i} \equiv \sum_{j \neq i} q_j$, $Q \equiv \sum_j q_j$; MR? $q_i P'(Q) + P(Q)$

$\Longrightarrow$ Marginal revenue closer to demand than with monopoly!

- $q_i < Q$ so mark-up reduced; simple with symmetric firms:
  - $q_i = \frac{Q}{n}$ so $\frac{P-c}{P} = \frac{1}{n\epsilon}$

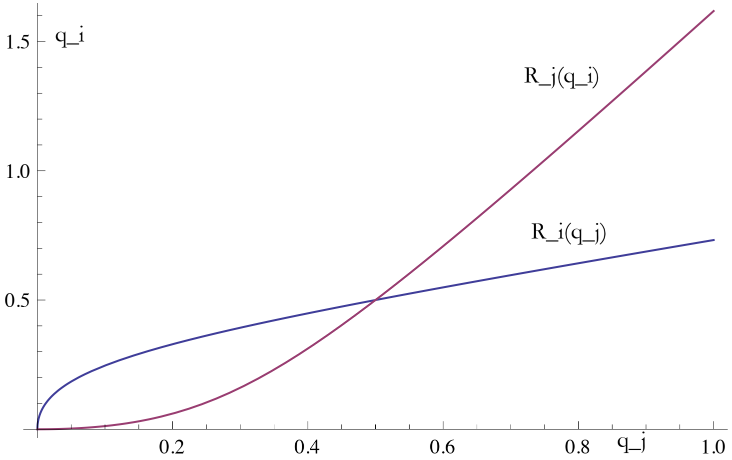
A simple example of Cournot oligopoly

Simple example: constant unit elasticity, $c = 1$; demand form?

- $P = \frac{1}{Q}$ so $P' = -\frac{1}{Q^2}$ and $MR = \frac{Q-q_i}{Q^2} = \frac{Q_{-i}}{Q^2}$
- Each firm sets to 1 so $\frac{q_i}{Q^2} = \frac{1}{Q} - 1$; aggregate across firms?
- $\frac{1}{Q} = \frac{n}{Q} - n \iff 1 = n - nQ \iff Qn = n - 1 \iff Q = \frac{n-1}{n}$

  $\implies$ Quantity goes to efficient $Q = 1$ (more on this Tuesday)
  - Per-firm output goes to 0: $q_i = \frac{Q}{n} = \frac{n-1}{n^2}$
- When just two firms, simple way to represent graphically?
  - $\frac{q_i}{(q_i+q_j)^2} = \frac{1}{q_i+q_j} - 1 \iff q_i = q_i + q_j - q_i^2 - q_j^2 - 2q_iq_j \iff q_i^2 + 2q_jq_i - q_j - q_j^2 = 0 \iff q_i = -q_j + \sqrt{2q_j^2 + q_j} = q_j\left(\sqrt{2 + \frac{1}{q_j}} - 1\right)$
  - *Reaction function*: serves similar role to supply/demand
  - Equilibrium intersection; cross in right direction (stable)

Cournot reaction functions and equilibrium



The Stackelberg model

Stackelberg argued that some firm may be leader

- Able to commit to, advertise level of production
- Other firm must then reply
- What incentives does this create?
  1. Follower is just a monopolist on *residual demand*
     - Follows reaction function from the leader
  2. Leader takes into account reactions of follower
     - Wants to encourage follower to *reduce quantity*
     - This leaves for demand to leader
- Does this require raising or lowering production?
  - In general depends on *strategic complements v. substitutes*
    - Driven by pass-through rate
    - Substitutes if $\rho < 1$, complements if $\rho > 1$
  - In our simple example CES has $\rho > 1$ so complements

A mathematical example of the Stackelberg model

Let's solve it out in our case

- $q_2 = q_1\left(\sqrt{2+\frac{1}{q_1}}-1\right)$ or $Q = q_1\sqrt{1+\frac{1}{q_1}}$; profits?
- $P = \frac{1}{q_1\sqrt{1+\frac{1}{q_1}}}$, $\left(\frac{1}{q_1\sqrt{1+\frac{1}{q_1}}}-1\right)q_1 = \frac{1}{\sqrt{1+\frac{1}{q_1}}} - q_1$; $q_1^\star$?
- $\frac{1}{2q_1^2\left(1+\frac{1}{q_1}\right)^{\frac{3}{2}}} - 1 = 0 \iff 2\left(q_1^{\frac{4}{3}} + q_1^{\frac{1}{3}}\right) = 1$
- Mathematica gives $q_1 \approx .1$, *way below* .5 from before
  - This is because reaction function slopes rapidly upward
  - With linear demand, you increase quantity

$\Longrightarrow$ Strategic substitutes v. complements (ρ) crucial

- Also key how much of others reactions taken into account

Basic conclusions from the quantity models

Notice a few things:

1. Even with many firms, prices still above cost
   - Though, as we'll explore on Tuesday, gradually fall
2. Prices between monopoly and perfect competition

One of the most interesting results comes from asymmetry

- Suppose different c_i, each firm sets $q_i P'(Q) + P(Q) = c_i$

$\implies$ Lower cost firms must produce more, higher mark-up

- But high cost firms still produce some; this is inefficient!
  - $\implies$ V. monopoly, competition, oligopoly misallocates production
- $QP'(Q) + NP(Q) = \sum_i c_i \implies \overline{q}P'(q) + P(Q) = \overline{c}$
  - $\implies$ Pass-through of any firm's cost like $\frac{1}{n}$
  - Only average cost matters, heterogeneity not key
    - Makes model more "robust" to heterogeneous costs
  - We'll compare this to other models shortly

The basic language of game theory

What we have been studying is called a "game"

- Strategic situation where small number make choice
  - Here prices/quantities, quality etc.
  - More broadly anything (who to date, where to eat, etc.)
- Participants' payoff depends on all actions
  - My profits depend on my quantity and yours
- Actions taken simultaneously (as in Cournot)....
  - This is called a *normal form game*
- Or in sequence as in as in Stackelberg
  - This is called an *extensive form game*
- Like competitive equilibrium we need *solution concept*
  - Competition: everyone takes prices, supply=demand
  - Here not obvious exactly what is analog
  - Most famous concepts build on work from Nash

Nash Equilibrium and other solution concepts

Two most prominent solution concept correspond exactly:

1. Nash equilibrium
   - Corresponds to how we solved Cournot
   - Each play takes as given *strategy* of other
     - Strategies are not what players “choose” but take as given
   - Each optimizes given this
     - Equilibrium Is intersection of reaction curves
2. Subgame-perfect equilibrium
   - Corresponds to Stackelberg model
   - One player acts first; other takes action as given
     - Could be whole sequence of actions as in chess
   - First player anticipates strategic effects
   - Game is solved by *backwards induction*
     - Just as we did: last player, next to last, etc.

$\Longrightarrow$ We won’t use much, but applies broadly

The Bertrand-Edgeworth model

If we have this broad concept, key question is strategies

- We assumed strategy was quantities but why?
- Do we think companies take others quantities as given?
- Why not take their prices as given?
  - This was the argument of Bertrand and Edgeworth
  - Consider several identical firms with CMC c
  - Each firm takes as given prices of rivals
  - What is equilibrium?
    - Suppose any firm makes strictly positive profits
    - Another firm can steal all these by charging slightly below
    - If any firm makes negative profits, can raise to avoid

$\Longrightarrow$ Bertrand and Edgeworth's Paradox

With $N > 1$, only equilibrium is $P = c$!

The differentiated products model: a solution

This seems a bit implausible:

- Requires that by undercutting by tiny amount, steal all
- This seems unlikely in the real world; why?
  - Consumers view products as *differentiated*
    - Non-price characteristics as last week
  - Identical, but consumers have to search
    - Prices are not transparent, costly to go to store

    $\Longrightarrow$ Once in store, monopolist competing against cost of 2nd visit
  - Either way we can write for firm i, $Q^i(p_1, \ldots, p_i, \ldots, p_N)$
  - Each monopolist on own product, but others substitute
    - $\frac{\partial Q^i}{\partial p_j} > 0$ for $i \neq j$
  - Very broad model, could quantities as strategies
    - However “Nash-in-prices” or “Bertrand” has become central
    - Called “Differentiated Products Nash-in-Prices” or (DPNiP)

Back to the Hotelling model

Simplest example is Hotelling from last class

- Rather than being in middle, though, imagine at two ends
- Consumer x chooses firm at 0 if $tx + p_0 < t(1-x) + p_1$
- Solve for cut-off x?
  - $2tx = p_1 - p_0 + t \iff x = \frac{p_1 - p_0 + t}{2t}$
- Thus demand, if uniform, is $Q_1(p_1, p_2) = \frac{p_1 - p_0 + t}{2t}$
  - Just linear demand! What is optimal price with $c = 0$?
    - Just half-way up, $p_0^\star = \frac{p_1 + t}{2}$
  - Equilibrium?
    - By symmetry $p_0^\star = p_1^\star$ so $p_0^\star = p_1^\star = t$
  - What about positive cost?
    - Pass-through of $\frac{1}{2}$ so $p_0^\star = \frac{p_1 + t + c}{2} = t + c$

    $\Longrightarrow$ Pass-through of 1 overall, because no exit

$\Longrightarrow$ Just an example; could also solve my reaction functions

Broader lessons about pricing with differentiation

Same basic principles apply broadly

- We can define *residual demand elasticity*:
  - Elasticity of demand *holding fixed other prices*?
    $\epsilon_i^r = \frac{\partial Q^i p_i}{\partial p_i Q^i}$
  - Then just follow Lerner rule: $\frac{p_i - MC_i}{p_i} = \frac{1}{\epsilon_i^r}$
- Nothing special about prices as strategies
  1. Just as easily defines firm's optimal quantity
     - Key is *holding fixed other firms' prices*
  2. Could also do à la Cournot/Nash-in-quantities
     - Then residual elasticity is *holding fixed other quantities*
     - Less common so we won't get into math, but very similar

$\Longrightarrow$ No reason differentiated products needs price strategies

- This is used extremely broadly in industrial economics

How oligopolists benefit from a cartel

Oligopolists create (pecuniary) externalities on one another?

1. Purely pecuniary under Cournot; why?
   - Believe all other firms' quantities are given
2. Purely real under DPNiP; why?
   - Believe other firms' prices are fixed, quantities change

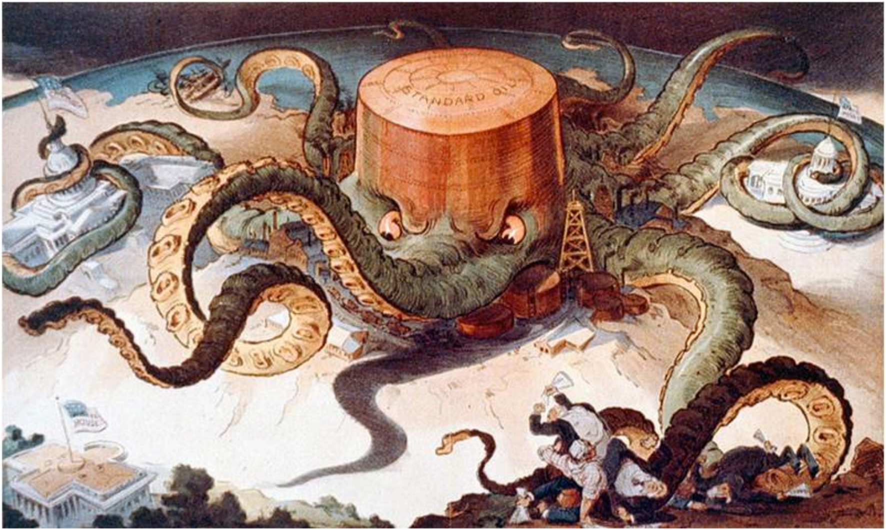
$\Longrightarrow$ Quantities always too high under Cournot but...?

- May be too high under DPNiP if other mark-ups larger

Regardless of social benefits or costs, firms benefit by avoiding

- Because substitutes, $q \downarrow / p \uparrow$ always benefits competitor
  - In fact, let's *define* two firms competing by being substitutes
- If they act to internalize this (pecuniary) externality we say?
  - They are "colluding" or "forming a cartel" or my favorite:
    - "Combination in the restraint of trade"
- Denounced since Adam Smith, illegal under Sherman Act

A cartoon that help catalyze the antitrust act



Legal restrictions on cartel formation

Once upon a time, cartel agreements were legally enforceable

1. In the US prior to the Sherman Act
2. In Continental Europe prior to the EU reforms
3. Hardly illegal in many developing countries (like Perú)
4. Britain all the way back to 18th century made illegal

Luckily the United States now leads the world on enforcement

1. Explicit agreement about prices/service illegal
   - Only competitors; complements are different as we'll see
2. Even without communication ("tacit") can be prosecuted
   - In practice much less because hard to prove
3. Criminal (jail time) and civil penalties
   - Usually "treble damages": probability of detection $\approx \frac{1}{3}$

$\Longrightarrow$ Many practical challenges in running a cartel today

$\Longrightarrow$ Collusion difficult, only possible in limited settings

Incentives to defect from a cartel

The basic problem is that cartel benefits all but...

- Each firm has incentive to betray; why?
  - This is what it means to be an (pecuniary) *externality*
  - By reducing price/increasing quantity each benefits
  - Can steal business, from others, benefit from higher prices

$\Longrightarrow$ The more ambitious cartel is, the less stable?

- Higher is the price, more incentive to steal the sales
- Near competitive level, little or no incentive to cheat

$\Longrightarrow$ Extent of collusion is ability to deter cheating

- Every cartel needs to create expectations of punishment

$\Longrightarrow$ Policy related to cartels all about this interplay

1. Goal of cartel is to ensure credible punishment of cheaters
2. Goal of agency, customers is to catch and prevent

- Stigler bases oligopoly theory on this back-and-forth

Detection, punishment and cartel enforcement

In order for the cartel to deter cheating it must?

1. Be able to determine what the optimal agreement is
    - Diffuse information, as in other externality problems
    - Hindered further here by government breathing down neck
2. Have clear what does and does not constitute cheating
    - Firms should be given some flexibility (private information)
    - But if too much flexibility then cartel does not function
3. Be able to detect cheating, distinguish from background
    - Many things might look like cheating but be innocent
4. Have cost-effective means of punishment
    - If you could tax and redistribute ideal
    - Shifting market share across firms works well
    - Price wars harm everyone so less effective
5. Be able to do this quickly (patience and frequency)

Stigler's factors facilitating/deterring collusion

Stigler's theory is based on these necessities; factors?

- He emphasizes factors facilitating/hindering these
  1. Large, heterogeneous buyers hurt, small homo help
     - Hard to track, easy to extract undermining concession
  2. Heterogeneity of product/firms hurts
     - Harder to define optimum, more incentive for one defect
  3. Price transparency helps track defections
     - Ability to offer secret price cuts key
     - Case against collecting industry, offering info to consumers
  4. Industry concentration helps reduce tracking, temptation
  5. Variability of demand hurts monitoring
  6. Frequency of interaction helps detect soon
  7. Growing demand helps, declining hurts
     - If declining, grab what you can while you can

Summarizing collusion through conjectures

All of these determine the extent of deterrence

- Most deterrence happens through price cuts/wars
- So simple way to summarize is firms' *conjectures*
  - If I lower price, how much will rival lower (or raise?) hers?
  - Also works with quantity...how do they respond?
- This varies depending on Stigler's factors

$\Longrightarrow$ Models incorporating called *Conjectural Variations* (CV)

- Usually capture by "parameter" of conjectured adjustment
  - Note this idea is useful to change price v. quantity models
  - Cournot is price model with conjectured "accommodation"
  - Price is quantity with conjectured "aggression"

$\Longrightarrow$ Thus CV is broad framework incorporating all theories

- Good because it can be used to talk about all them
- But to be useful requires more structure

Cournot pricing with conjectural variations

Let's consider simplest Cournot model

- Again profits $[P(Q_{-i} + q_i) - c]\, q_i$
- But now extra term, as other quantities not fixed: $\frac{dQ_{-i}}{dq_i}$
- Optimal pricing given by?

$$P + P' q_i \left(1 + \frac{dQ_{-i}}{dq_i}\right) = c$$

- If $\frac{dQ_{-i}}{dq_i} > 0$ then *MR* further below *P*
  - Therefore called a "conjectured accommodating reaction"
  - If you are nice and reduce quantity, others follow you
  - This is how collusion is facilitated
- On the other hand, if aggressive, $\frac{dQ_{-i}}{dq_i} < 0$, lower prices
  - This case was on your exam; if $\frac{dQ_{-i}}{dq_i} = -1$ then Bertrand
- Also starting from Bertrand, DPNiP

Back to our favorite example

Now let's return to our constant elasticity

- Now suppose that $\frac{dq_j}{dq_i} = .1$; MR?
  - $\frac{1}{q_i+q_j} - \frac{1.1q_i}{(q_i+q_j)^2}$; set to 1
- Now what are the reaction functions?
  - $1.1q_i = q_i + q_j - q_i^2 - q_j^2 - 2q_iq_j \iff q_i = -(q_j + .05) + \sqrt{2q_j^2 + 1.1q_j + .05^2}$
- Mess as before, but again we can plug into Mathematica
- Solution yields $q_1 = q_2 = .45 < .5$
  - Anticipated reciprocation reduces quantity
  - Just like in Stackelberg model!

    $\implies$ Strategic effects one reason conjectures seem reasonable
  - Anticipated substitute behavior has opposite effect

$\implies$ Shows how conjectures capture/enforce collusion