Double Marginalization in Two-Sided Markets*

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*A few of the results contained here were circulated in a working paper "The Price Theory of Two-Sided Markets" in December of 2006 and appeared as part of my senior thesis of the same name. The bulk of that part of the research was conducted while serving as an intern at the United States Department of Justice Antitrust Division Economic Analysis Group. I am grateful to the Justice Department for their financial support and for inspiring my interest in this topic, but the views expressed here, and any errors, are my own. The first draft of this paper (which contained a mistaken claim, corrected here) was circulated in February 2008 under the title "Vulnerability and Double Marginalization in One- and Two-Sided Markets" and was included as the second chapter of my dissertation "Essays in Industrial Organization and Economic Methodology" under the same name. That version of the paper was researched and written while I was visiting the University of Chicago through the Becker Center on Chicago Price Theory. I am grateful to the Becker Center for their support and especially to Kevin Murphy who oversaw my research during my time there and conversation with whom helped provoke many ideas in this paper. Most of the results here were included in a working paper "Pass-Through, Double Marginalization and Two-Sided Markets" circulated in April 2008. I completed that paper during my yearly visit to the Toulouse School of Economics where my work was advised by Jean Tirole, who has been kind enough to sit on my dissertation committee from half way around the world and whose thinking has (obviously) tremendously influenced this work. I also appreciate the helpful comments and advice on this research supplied by Gary Becker, Dennis Carlton, Wouter Dessein, Avinash Dixit, Joe Farrell, Jeremy Fox, Matthew Gentzkow, Ali Hortaçsu, David Martimort, Canice Prendergast, Alex Raskovich, Markus Reisinger Jean-Charles Rochet, Bill Rogerson, Carl Shapiro, Howard Shelanski, Lars Stole, Chad Syverson, Mike Whinston, Bobby Willig and seminar participants at the Justice Department, Princeton University (especially Dilip Abreu) and Stanford's Graduate School of Business (especially Jeremy Bulow and Bob Wilson), as well as an anonymous editor at the *RAND Journal of Economics*. I am particularly grateful to Debby Minehart, under whose supervision my work at the Justice Department was conducted and whose advice lead me to this topic. Most of all, I am indebted to my advisors at Princeton (Roland Bénabou, Hyun Shin and especially my principal adviser José Scheinkman) for their advice and support in all of my research. All confusion and errors are my own.

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Abstract

Should banks (through Visa) be allowed to own debit clearing networks? This problem combines the classic Cournot (1838)-Spengler (1950) double marginalization problem with the more recent literature on two-sided markets (Rochet and Tirole, 2003). Because both the double marginalization (Weyl, 2008a) and two-sided markets (Weyl, 2008b) depend crucially on the pass-through rate, the analysis is natural and leads to strong over-identification given simple assumptions. Vertical integration does not generally erode (and often enhances) platform mark-ups. Therefore its (price level) benefits are more robust than those of competition in two-sided markets.

1 Introduction

The debit card industry[1] has two major players on the supply side[2]: banks which issue cards to consumers and debit clearing networks, like Star, Plus, NYCE and the other "bugs" that appear on the back of your debit card. While there are many competing debit clearing networks and a web of competing regional debit networks dominated the market in the 1990's, by 2002 a single national, for-profit network, called Star, had 57% market-share in the US according to the eft (2007). Since 2002, however, Star has been progressively supplanted by Visa's Interlink; in 2007, Interlink surpassed Star's market share and appears to be on a continuing upward trajectory. Because Visa was run as a not-for-profit joint venture of commercial banks and now, after its initial public offering, is still majority controlled by a collection of banks. Interlink dominance of the market can therefore roughly be seen as vertical integration of banks with the debit clearing networks.

Should antitrust officials be suspicious or attempt to prevent such vertical integration, even if the alternative is a near-monopolistic for-profit network? Should merchants worry that integration between the debit clearing network and card-holder banks will unfairly favor the card-holders or give banks more power to raise the "interchange" fees they receive from merchants? What socially benign motive would Visa have to accomplish such integration anyway?

The banks and the debit clearing networks in many ways resemble classic vertical monopolies and the logic of double marginalization therefore suggests that their integration is socially desirable . However, this argument is difficult to make because the logic of double marginalization, supported in standard markets by the models of Cournot (1838) and Spengler (1950), has not be worked out in two-sided markets. The policy literature on two-sided markets has argued (Wright, 2004; Evans, 2003) that antitrust intuitions from standard

[1] Hayashi, Sullivan, and Weiner (2003) provide a comprehensive overview of the debit card industry and policy issues surrounding it.

[2] "Accepting" banks which represent merchants are generally though to be competitive and play little role in the market and thus are left out of the analysis. However, it would be possible to incorporate them into this analysis by adding a third layer of intermediaries. See Evans and Schmalensee (2005).

markets may be inapplicable or only applicable in complex ways to two-sided markets. For example, Weyl (2008b) shows that competition may increase the total price charged to consumers in a two-sided market. Might the vertical integration of two monopolies, have a similarly perverse effect, such as systematically increasing prices to one side of the market as some have argued (Cayseele and Reynaerts, 2007)?

An answer is important not only because of this particular question in the debit card industry, but also because similar issues arise in other two-sided markets, such as in the relationship between video game platforms and platform add-ons, in the credit card industry because of Visa's relationship to card-issuing banks and the internet service provision industry[3], given that content consumers and content producers may connect to the internet through different ISPs.

While Rochet and Tirole (2003) (RT2003) and Armstrong (2006) pioneered the analysis of competition in two-sided markets, work on vertical relations has been limited. Rochet and Tirole (2007) analyze the effects of bank market power on the right standards for regulating network determined fees and Lee (2007) studies (in a structural empirical model of the video game industry) the effects of "vertical integration" in the sense of integration between a network and consumers on one side of the market. However this paper is the first to study explicitly the strategic interactions between firms selling complementary (vertical) products in a two-sided market[4] and therefore the first which can address the effects on prices of

[3]This is particularly relevant in the context of the recent debate over net neutrality (Thierer, 2006; Lenard and Scheffman, 2006; Farrell, 2006).

[4]Independently, Cayseele and Reynaerts (2007) developed a model of complementary platforms in two-sided markets, also based on the RT2003 model, where firms segment consumers on one side and are complementary for consumers on the other side. However, their assumption that firms must use all platforms but at the same time that a reduction in demand on one platform does not reduce demand on other platforms appears inconsistent, at least with an assumption of perfect complementarity. Their analysis is therefore one about one-sided falls in vulnerability, and therefore a special case of Weyl (2008b). It is therefore not directly relevant to the issues considered here, where in order to allow a particular transaction to occur, all components must be available to the relevant participant. In fact, the results below can be considered a proof that under true perfect complementarity, when a reduction in one segment demand eliminates a transaction with all segments, Cayseele and Reynaerts (2007)'s result that integration is harmful to one side fails. This is because Cayseele and Reynaerts (2007) only considers linear demand, under which as shown below integration always benefits both sides. The specialness of their analysis to the case of linear demand is another contrast between our approaches, as I use general demand.

changes in vertical industrial organization in two-sided markets.

There are two basic problems in extending the Cournot-Spengler model to two-sided markets. The first is that prices charged on the two sides of the market may depend on the individual mark-ups of particular firms, rather than the total mark-up charged by both firms. A major incentive that a debit clearing network has to restrain the prices it charges to merchants is the profits they make off fees from card-holders on the other side of the market. These profits act as an effective subsidy in serving the other side of the market. A completer characterization of the Cournot-Spengler model in one-sided markets is needed to extend that model to two-sided markets. Second, a framework is needed for analyzing the equilibrium two-sided market effects of (arbitrary) changes in industrial organization, a framework that can be combined with a more complete analysis of the standard Cournot-Spengler model to extend it to two-sided markets.

Luckily, both of these problems are overcome in recent papers. In Weyl (2008a) and Weyl (2008b), I show that the same simple property of demand, its *pass-through rate* determines the crucial properties of both problems. The pass-through rate measures how much of an increase in her linear cost a monopolist finds it optimal to pass on to consumers as an increase in price. Whether pass-through is less than one-for-one (cost-absorption) or greater than one-for-one (cost-amplification) determines (Weyl, 2008a) whether mark-ups are strategic complements or substitutes in the double marginalization problem. It also determines (Weyl, 2008b) the effect of competition on the *price level*, the sum of prices charged to the two sides of the market, in the RT2003 model. And both models can be non-parametrically over-identified using exogenous cost variations, on the basis of simple assumptions, such as that demand is either cost-absorbing or cost-amplifying, but not both, over the relevant range of prices.

The fact that both of these depend on the same property of demand allows a natural fusion of the two problem which is both analytically simple and even more over-identified, despite is greater complexity, than either of the two components. I find that vertical integration has robust benefits[5] in its tendency to reduce the price level, the sum of prices charged to the two sides and a standard measure of competitiveness, charged to the two sides of the market, benefits that are in fact more robust than those of competition or price controls. While it is possible that vertical integration raises prices to merchants, (with a single, very narrow exception) this only occurs in the cases when prices fall more dramatically to consumers. This largely allays the concern that two-sidedness undermines the logic of double marginalization and confirms this basic economic intuition.

I begin in Section II by developing some preliminaries. I review the notion of pass-through in the simplest of monopoly pricing problems. I then turn to the Weyl (2008a) pass-through-based taxonomy of the double marginalization problem. Finally I discuss the RT2003 model of two-sided markets, where are pricing, costs and consumer preferences are per-interaction, as well as the Weyl (2008b) pass-through-based approach analyzing this model.

Section III lays out an intuitive model of double marginalization in an RT2003 two-sided market. Many vertical organizations are possible, but I focus on two that are analogous to the well-known Cournot-Nash and Spengler-Stackelberg formulations in a one-sided market, under the interpretation that double marginalization takes place one side of the market only. Either a separate (one-sided) firm sells a good that is perfectly complement to a two-sided service to consumers on one side of the market (the Spengler-Nash formulation) or there is an intermediary between the platform and one side of the market (the Spengler-Stackelberg formulation).

In Section IV I develop my main results. Prices and mark-ups on this "intermediated" side of the market, unsurprisingly, track those in the standard double marginalization problem. However, prices on the un-intermediated are determined, in reverse, by the intermediated-side mark-ups of the two-sided platform, as their mark-up to the intermediated side determines the cross-subsidy to the un-intermediated side. Because intermediated-side mark-ups

[5]I focus exclusively on the positive, price effects of integration, not on its normative properties. Those remain an interesting question for future research, a good starting place for which would be the useful constant pass-through demand class (Weyl, 2008b).

can be higher, but are usually lower, under separation, it is possible but unusual that prices to merchants rise as a result of integration. However, this occurs only when prices fall to card-holders.

The comparison of the price level, the primary goal of this exercise, are a bit more complicated as they may depend on whether pass-through is greater on the intermediated or un-intermediated side of the market, as well as how these each compare to one and whether pass-through increases or decreases on the intermediated side of the market with cost. However all these properties can be identified using simple tests given exogenous cost variations. The price level is lower under integration than under separation unless the separated organization is Stackelberg, intermediated-side demand is cost-absorbing, un-intermediated-side demand is cost-amplifying and intermediated side demand has decreasing pass-through; and even in this case the comparison is ambiguous. Thus the tendency of vertical integration to reduce the price level is more robust than the tendency of competition to do so. The Stackelberg price level is generally less than the Nash price level, unless both demands are cost-absorbing and pass-through is greater to the intermediated side than to the un-intermediated side. Profit comparisons mirror those in one-sided markets, for the most part, though are somewhat muddied by two-sided complications.

In section IV, I briefly speculate about potential vertical organizations and the effect of introducing competition into the model. I end Section IV by arguing that the inherent upstream moral hazard created by the two-sided pricing makes non-integrative solutions to the double marginalization problem more challenging. I conclude the paper in Section V by suggesting some potential directions for future research. Proofs not included in the text appear in two appendices.

2 Preliminaries

Pass-Through

Consider the problem of a monopolist in a standard market facing consumer demand[6] $D(\cdot)$ (assumed decreasing and twice continuously differentiable) and constant marginal cost of production c. The familiar first-order condition is given by:

$$\frac{p}{\epsilon(p)} = \gamma(p) \equiv -\frac{D(p)}{D'(p)} = p - c \equiv m \tag{1}$$

where $\epsilon(p)$ is the elasticity of demand. Thus γ is the ratio of price to elasticity of demand or, in mathematical terms, the inverse hazard rate of demand. I refer to this as the *vulnerability of demand*, because of its connection monopoly pricing: an upward shift in the vulnerability function encourages the monopolist to exploit consumer demand more by charging a higher mark-up. The second-order condition associated with this problem, the *mark-up contraction* (MUC) of Weyl (2008a) is $\gamma' < 1$, which I assume holds globally to ensure a global first-order solution.

The crucial parameter of demand in what follows is the pass-through rate[7], the elasticity of optimal price with respect to cost

$$\rho \equiv \frac{dp_M}{dc} = \frac{1}{1-\gamma'} \tag{2}$$

If $\rho < 1$ demand is said to be (locally) cost-absorbing (Rochet and Tirole, 2007) as the monopolist absorbs some of cost increases or subsidies. If $\rho = 1$ then demand is said to be constant mark-up. If $\rho > 1$ demand is said to be cost-amplifying. Of course these are local properties of demand and any particular demand function may be cost-absorbing at some prices and cost-amplifying at others. Nonetheless, many common demand functions (Weyl,

[6] Throughout this paper I assume that demand is globally decreasing and thrice continuously differentiable. One exception to the second property comes in some examples used to exhibit ambiguities in the few cases where they exist.

[7] For more about pass-through and its applications see Weyl (2008a).

2008a) are not only globally on one or the other side of the divide, but they also exhibit monotone pass-through (in one direction or the other) as a function of cost. While this assumption can be weakened while still obtaining similar identification (for example, it only need hold over the relevant range of prices), to make the analysis below clearer, I stick to the assumption that demand is either cost-absorbing or cost-amplifying and either increasing or decreasing pass-through.

Assumption 1. *Over the range of prices considered in the following applications, I assume all demand functions are either cost-absorbing, constant mark-up or cost-amplifying and have either monotonically increasing, decreasing or constant pass-through as a function of cost and price.*

Double Marginalization

The usefulness of this assumption is immediately apparent if we consider the classical double marginalization problem of Cournot (1838) and Spengler (1950). In both Cournot and Spengler's formulations of the problem, two monopolists with linear production costs produce goods that are perfect complements in consumption. In Cournot's formulation, pictured in Figure 1, both firms sell their goods directly to the consumer. In Spengler's, pictured in Figure 2, one firm, Upstream, sells to another, Downstream, which assembles the goods, selling them as a package to the consumer.

While there is no substantive economic difference between these scenarios, one does arise when the Spengler model is augmented by the common additional assumption that Upstream chooses its prices before Downstream in the spirit of von Stackelberg (1934) (ergo the name Spengler-Stackelberg) while the two Cournot firms choose their mark-ups simultaneously in the spirit of Nash (1951) (ergo Cournot-Nash). In the diagram, as below, all arrows represent a pricing relationship and those which are bolded indicate a temporally precedent action. Because their first-order conditions are symmetric, the two Nash firms at equilibrium charge the same mark-up $m^\star$ and earn the same profits $\pi^\star$; the industry mark-up over total

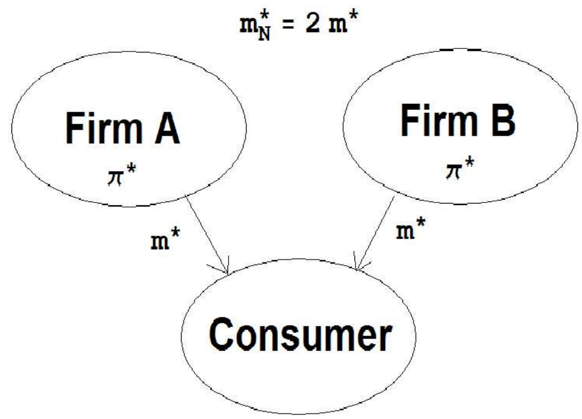

Figure 1: The Cournot-Nash industrial organization

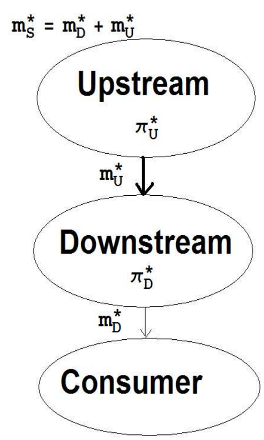

Figure 2: The Spengler-Stackelberg industrial organization

	$\rho < 1$		$\rho > 1$	
	Cost absorption		Cost amplification	
	Decreasing pass-through		Decreasing pass-through	
	$m_U^\star$		$m^\star$	
ρ'	$\vee$	$\pi_U^\star$	$\vee$	$\pi_D^\star$
$\wedge$	$m_I^\star < m_N^\star < m_S^\star$	$\vee$	$m_D^\star$	$\vee$
0	$\vee$	$\pi^\star$	$\vee$	$\pi_U^\star$
	$m^\star$	$\vee$	$m_U^\star$	$\vee$
	$\vee$	$\pi_D^\star$	$\vee$	$\pi^\star$
	$m_D^\star$		$m_I^\star < m_S^\star < m_N^\star$	
	Cost absorption		Cost amplification	
	Increasing pass-through		Increasing pass-through	
	$m_I^\star < m_N^\star < m_S^\star$		$m^\star$	
ρ'	$\vee$	$\pi_U^\star$	$\vee$	$\pi_D^\star$
$\vee$	$m_U^\star$	$\vee$	$m_D^\star$	$\vee$
0	$\vee$	$\pi^\star$	$\vee$	$\pi_U^\star$
	$m^\star$	$\vee$	$m_I^\star < m_S^\star < m_N^\star$	$\vee$
	$\vee$	$\pi_D^\star$	$\vee$	$\pi^\star$
	$m_D^\star$		$m_U^\star$	

Table 1: A taxonomy of the Cournot-Spenler double marginalization problem

industry cost is $m_N^\star = 2m^\star$. At equilibrium Upstream charges mark-up $m_U^\star$ and earns profits $\pi_U^\star$, Downstream charges mark-up $m_D^\star$ and earns profits $\pi_D^\star$ and the industry has total mark-up $m_S^\star = m_U^\star + m_D^\star$. Of course the natural benchmark against which these two separated organizations should be judged is that of vertical integration, in which case the industry mark-up is $m_I^\star$ (which solves equation 1).

Cost-absorption versus cost-amplification determines whether the mark-ups of the two firms in this problem are strategic substitutes or complements. To see this note that one firm increasing its mark-up raises the price to consumers by the amount of the increase, given any price charged by the other firm. It is therefore equivalent to a tax on the other firm, which in turn is equivalent to an increase in the other firm's cost. Whether the other firm absorbs this, by reducing her mark-up, or amplifies it, by raising her mark-up, is exactly the definition of strategic substitutability versus complementarity. From this, in Weyl (2008a) I deduce the results show in Figure 1: given a knowledge of whether demand is cost-absorbing or amplifying and whether it has increasing or decreasing pass-through, one can predict a full ranking of mark-ups and profits among firms within and between and a ranking of industry mark-ups between industrial organizations. These results form the foundation of my analysis in Section III, but because the justification behind them is not necessary for this analysis I do not discuss it further here and instead refer the reader to my other paper.

Two-Sided Markets

The two most popular models of two-sided markets are those proposed by RT2003 and Armstrong (2006). The models differ in their focus on fixed versus variable costs and consumer heterogeneity[8]. RT2003 assume that all consumer utility, all (linear) firm costs and all prices are per-interaction with consumers on the other side of the market and therefore consumers are heterogeneous in their valuation of interactions. On the other hand the Armstrong approach assumes that consumers are all identical in their (linear) valuation of interactions, but are heterogeneous in their fixed value of joining (or not) the service, and that all (linear) firm costs are per-consumer, not per-interaction. Because it is widely considered a more reasonable model of the payment card industry and my motivation here is in that industry, I adopt the RT2003 model.

The RT2003 model is most clearly understood in the context of the payment card industry. I make use of the monopoly version of their model, though this is essentially without loss of generality because, as they show, their model of symmetrically differentiated Bertrand competition is, like the corresponding model in a standard market, equivalent to monopoly[9] with higher elasticity (lower vulnerability). Every consumer is endowed with a debit card and every merchant with a machine that allows them to accept cards. A monopolistic card company charges a per-transaction price p^A to consumers and p^B to merchants. Consumer are randomly paired with a merchant, with whom they want to purchase a good, without

[8]The models also differ in the form of pricing they assume. However in Weyl (2008c) I show formally, as argued informally by RT2003 and RT2006, that this difference, unlike the others, is not substantive.

[9]See Weyl (2008b) for more details of this argument. A small caveat is that under competition, but not under monopoly, vulnerability on one side may depend on prices on the other side. In practice this poses essentially no complications, as the analytic tools developed in Weyl (2008b) do not rely on the assumptions that vulnerability depends only on own-price.

knowing whether or not they accept cards. Therefore merchants' only motivation to accept cards is that they want, for their own reasons, consumers to use cards[10]. A fraction of merchants $D^B(p^B)$, derived from these personal valuations, choose to accept cards. A fixed mass of consumers-purchase pairs is matched to these merchants, of which $D^A(p^A)$ choose to use cards, if they have the opportunity to do so; otherwise, they use cash. The monopolist has per-transaction costs c and thus earns profits $(p^A + p^B - c)D^A(p^A)D^B(p^B)$. By analogy to equation 1, the corresponding first-order condition is

$$p^A + p^B - c = \gamma^A\left(p^A\right) = \gamma^B\left(p^B\right) \tag{3}$$

An intuitive way of looking at this model is to note that it is exactly[11] that monopoly pricing problem with two goods where the only cross-effects between the goods is that the price of each good acts as a full cross-subsidy in the production of the other good. That is, every dollar collected from the merchants is equivalent to reducing by one dollar the cost of serving the card-holders. Because $p_I^A - (c - p^B) = p^B - (c - p^A)$, the mark-up over cost net of this cross-subsidy is by definition the same on the two sides of the market. Therefore given that vulnerability determines the optimal mark-up on one side of the market, the monopolist's equation of vulnerability on the two sides can be seen as a direct consequence of the fact that cross-subsidies are the only cross effects. This implies the most basic and robust result in the RT2003 model, namely the "seesaw" or "topsy-turvy" effect that competition (a lowering of vulnerability) or price controls on one side of the market lower prices on that side, while raising prices to consumer on the other side.

Perhaps the most natural (positive) question given these complexities is whether there is still a sense in which competition (lower vulnerability) and price controls reduce prices

[10]Rochet and Tirole (2007) relaxes this assumption, allowing merchants to accept cards in order to attract customers and essentially arrives at the same model, but with different normative interpretations.

[11]To see this note that for this to be the case it must be that optimizing on side A is equivalent to maximizing $(p^A + p^B - c)D^A(p^A)$ and maximizing on side B is equivalent to maximizing $(p^A + p^B - c)D^B(p^B)$. Clearly for both of these to be true any term involving opposite side prices, other than those entering the mark-up, must be multiplicative, yielding the multiplicative form.

“overall”. A natural notion of this overall’ price is the *price level* $\overline{p} \equiv p^A + p^B$. In fact a primary question I will ask below is what the effect of vertical integration is on the price level. Intuitively it should be clear that pass-through rates should be crucial in understanding the effects of price controls or competition on the price level, as they will determine how much of a cross-subsidy is passed through to the other side of the market. In fact in Weyl (2008b) I show that exactly two cases are possible, under Assumption 1.

1. Both demands are cost-absorbing: In this case, competition and price controls on either side always reduce the price level, as reductions in prices on one side are not fully passed through as increased prices on the other. Furthermore *balanced interventions* reducing the price level (those maintaining $\gamma^A = \gamma^B$); as would be caused by subsidies (or other reductions in cost), balanced competition or caps on the price level which allow the firms to continue to optimize on the balance; will always reduce both prices. Finally reductions in cost are less-than-fully passed through to the price level, so this can be thought of as a case of *total cost absorption*.

2. One demand is cost-absorbing, the other cost-amplifying: In this case, price controls on or a reduction in vulnerability on the cost-absorbing side will raise the price level as the resultant reduction in pries is more than fully passed-through to consumers on the other side of the market. On the other hand price controls or competition on the cost-amplifying side reduce the price level, as these are not fully passed-through to the cost-absorbing opposite side. Balanced interventions reducing the price level, being more than fully passed-through to the cost-amplifying side, lead to a rise in prices to the cost-absorbing side. Finally, a reduction in cost is more-than-fully passed through to the price level, thus this case can be thought of as one of *total cost amplification*.

A third natural possibility is that both demands are cost-amplifying. However, the analog of MUC in this problem is a condition ensuring that the passing of cross-subsidies back and forth across the two sides is stable. In Weyl (2008b) I call this cross-subsidy contraction (CSC) and it requires that $\rho^A \rho^B < 1$, ruling out both demands being cost-amplifying. Nonetheless, these two cases have very different policy implications: the benefits of competition, and the entire standard approach to industrial organization, are much more suspect in the second than the first case. While in the first case most of what we know holds up so long as the price level is thought of as the relevant "price", much of our intuition is destroyed by the second case. It is the new intuitions which emerge from the distinction between these cases and an understanding of the crucial role of pass-through which provides the other piece of the puzzle allowing the analysis of double marginalization in two-sided markets, to which I now turn.

3 Model

Motivated by the issues in the debit card industry I discussed in the introduction, I now combine the tools from the last section to extend the double marginalization problem to two-sided markets. While the vertical monopolies model does not quite fit the debit card industry given strong interbank competition, I suspect that interbank competition on debit card fees and incentives is fairly tame. Consumers can only get a debit card with their own bank and most consumers are locked-in to a bank (relative to the size of debit card fees or incentives) by the time they think of getting a debit card. Even in attracting new consumers, debit fees and incentives are such an insignificant and inaccessible part of the total decision of where to bank that they are likely to be a shrouded (Gabaix and Laibson, 2006), or otherwise only weakly competed-on (Holton, 1957; Lal and Matutes, 1989; Ellison, 2005) on, add-on. Therefore the vertical monopolies model is, perhaps, a reasonable approximation. Furthermore, as I argue in the following section, I suspect that many of the most important results developed here generalize to the introduction of the relevant forms of competition. The baseline against which I compare the vertically separated monopolies is, analogously to standard markets, the two-sided market monopolist of equation 3 above. Let c_I be the

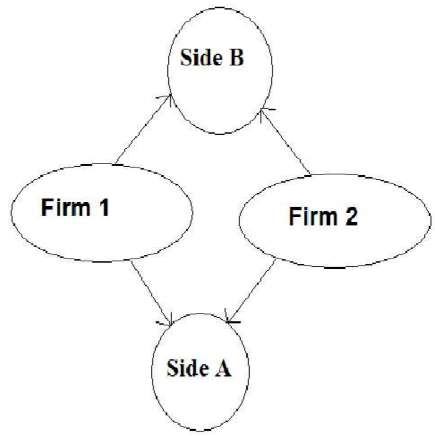

Figure 3: Cournot-Nash organization

integrated firm's cost, $p_I^{A\star}$ be their optimal price to side A and $p_I^{B\star}$ be their optimal price to side B. For notational economy I let $m_I^{A\star} \equiv p_I^{A\star} - c_I$ and $\overline{m}_I^{\star} \equiv \overline{p}_I^{\star} - c_I$.

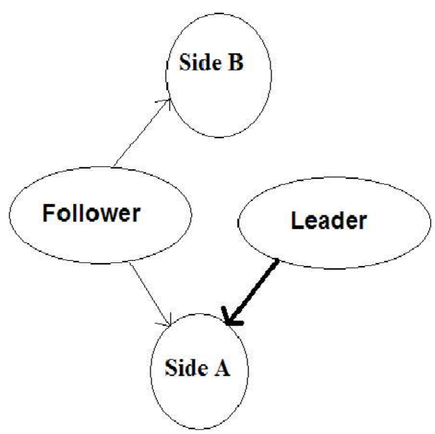

Figure 4: Spengler-Reverse Stackelberg organization

Because debit clearing networks traditionally have exclusive control over the "interchange" and "switch" fees charged to merchants, I consider two industrial organizations, shown in Figures 5 ("Spengler-Nash" or SN) and 6 ("Spengler-Stackelberg" or SS), that include this Spengler-like feature. Because of space limitations I do not consider every possible vertical industrial organization (Figure 3 shows one natural example, "Cournot-Nash", I do not consider), nor even every industrial organization with the Spengler-like feature that one side has exclusive control over prices charged to one side of the market (Figure 4 shows the case I neglect "Spengler-Reverse Stackelberg"). In all four figures each arrow represents a price charged and bolded arrows represent a temporally precedent action. Figures 5 and 6 are also labeled with the relevant price, profit and mark-up variables and definitions used in the results and proofs. I assume that the two firms have total linear costs c_I, but am agnostic about its division among the firms, because (as with the one-sided market case) this has no effect on anything of interest in equilibrium.

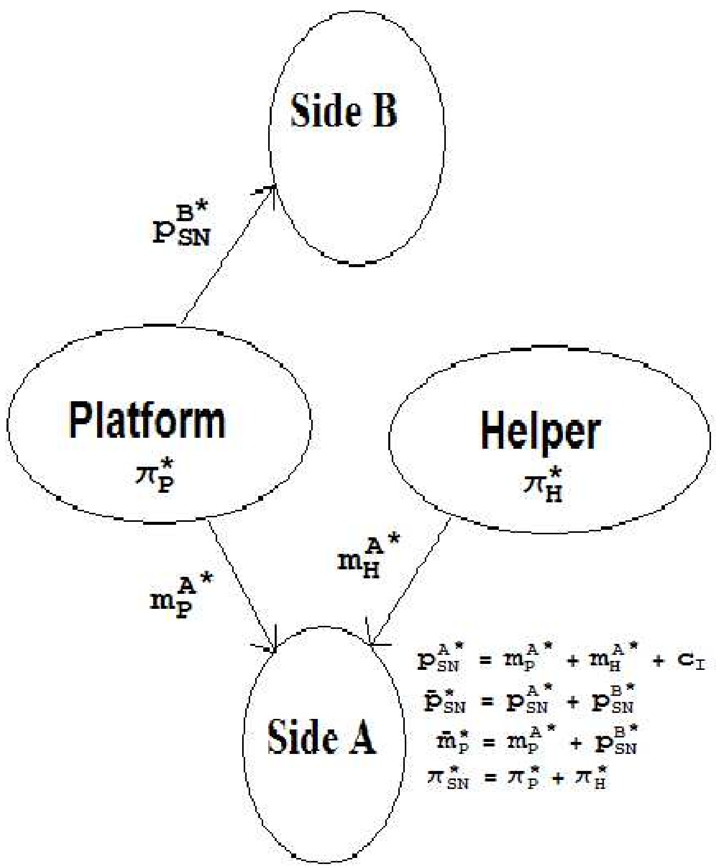

Figure 5: Spengler-Nash (SN) organization

The equilibrium conditions for Platform and Helper are given by

$$\overline{m}_P^{\star} = m_P^{A^\star} + p_{SN}^{B^\star} = \gamma^A\big(m_P^{A^\star} + m_H^{A^\star} + c_I\big) = \gamma^B(p_{SN}^{B^\star}) \tag{4}$$

$$m_H^{A\star} = \gamma^A(m_P^{A\star} + m_H^{A\star} + c_I) \tag{5}$$

These are perfectly analogous to the equilibrium conditions in the Cournot-Nash game in a one-sided market, except that now because Platform acts as a two-sided platform it must balance the vulnerability on side A with the vulnerability on side B. To ensure that the solution to equations 4 and 5 represent a stable equilibrium, one must (as in the one-sided case) make an additional stability assumption. This requires that if Platform raises its A-side mark-up by ϵ that tracing through the effects of this both on Platform's optimal B-side price and Helper's optimal mark-up to their effect of these on Platform's optimal A-side mark-up that the second-round increase is smaller than (contracts) the original increase:

$$(1 - \rho^A)^2 + \rho^A \rho^B < 1 \tag{6}$$

or equivalently

$$\rho^A + \rho^B < 1 \tag{7}$$

The equilibrium conditions for Upstream and Downstream are given by

$$\overline{m_U}^\star = m_U^{A\star} + p_{SS}^{B\star}\gamma^A\left(m_U^{A\star} + m_D^{A\star} + c_I\right)\left[1 - \gamma^{A'}\left(m_U^{A\star} + m_D^{A\star} + c_I\right)\right] = \gamma^B(p_{SS}^{B\star}) \tag{8}$$

$$m_D^A = \gamma^A(m_U^A + m_D^A + c_I) \tag{9}$$

which are the two-sided market equivalents of the Weyl (2008a) equations governing the Spengler-Stackelberg game in a one-sided market. The stability condition, analogous to inequality 6, in this context is

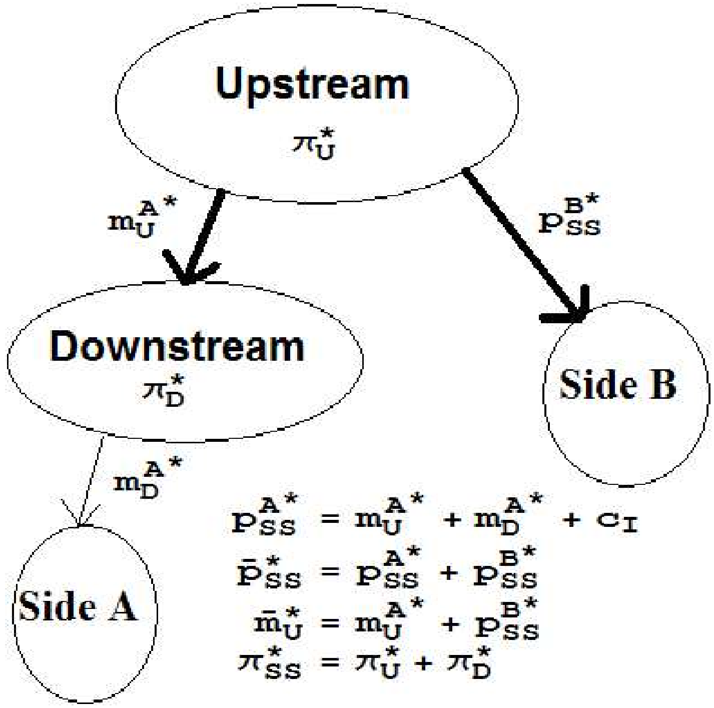

Figure 6: Spengler-Stackelberg (SS) organization

$$(1-\rho^A)(1-\rho_U^A)+\rho_U^A\rho^B<1 \tag{10}$$

where $\rho_U^A \equiv \frac{1}{1+\gamma^{A''}\gamma^A-\gamma^{A'}(1-\gamma^{A'})}$.

4 Results

The similarity between the conditions here and those in one-sided markets, along with the basic price theory results from above, help extend my results in one-sided markets to two-sided markets. I do this in several steps so as to make clear the ways in which I use each of the results.

Intermediated side prices

First, I use the seesaw effect to directly extend the results of Table 1 to the prices on the intermediated (A) side of the market, as shown in Table 2. This extension is natural, as $m_i^{A\star}$ is the analog of the mark-ups set by the various firms (a Nash firm, Upstream and Integrated) and $p_i^{A\star}$ that of the final prices to consumers in the various industrial organizations (Nash, Stackelberg and Integrated). It is therefore intuitive that these values track those in standard markets across industrial organizations. For example, under cost absorption on the intermediated side, $m_U^{A\star} > m_P^{A\star}$ as $m_U^\star > m^\star$ under cost absorption in a standard market. Similarly under either (strict) cost absorption or amplification on the intermediated side $m_D^{A\star} < m_H^{A\star}$ as $m_D^\star < m^\star$ in either case in a standard market. This is a direct result of the seesaw effect: because changes in industrial organization do not directly affect the un-intermediated side, it is the shifts in competitiveness on the intermediated side that determines the effects on prices. However, note that this reasoning does not allow comparison between mark-ups *within* industrial organizations across firms. Because Platform, Upstream and Integrated's mark-ups on the intermediated side are partly determined by the cross-subsidy from the un-intermediated side, their mark-ups on this side are not directly comparable with those of Downstream and Helper which are not (directly) affected by such cross-subsidies.

The only significant difference between the results in Table 2 and those in Table 1 therefore relates to the way in which it is possible to determine (using exogenous cost variations) the level and slope of pass-through on side A. To identify whether side-A demand is cost-absorbing or amplifying, from the results in the previous section it is sufficient to identify whether p^B is co-monotonic or anti-co-monotonic with cost[12]. A simple way to express this is to imagine that the price level falls (as a result of a fall in cost). Then we can calculate $\sigma^i \equiv \frac{dp^i}{d\overline{p}}$; co-monotonicity of p^B is then equivalent to $\sigma^B > 0$. We can also use the effects of cost shifts on prices[13] to determine whether pass-through is increasing or decreasing on the

[12]Just as in standard markets for this test to work in the case of the SS organization one must assume that $|\gamma''|$ is small.

[13]Here in the case of SS one must assume that $|\gamma^{(3)}|$ is small.

	$\rho^A < 1$	$\rho^A > 1$
$\rho^{A'} < 0$	$\sigma^B > 0$ $\frac{\overline{\rho}-1}{\overline{\rho}\sigma^A}$ decreasing $p_{SS}^{A\star} > p_{SN}^{A\star} > p_I^{A\star}$ $m_U^{A\star} > m_I^{A\star} > m_P^{A\star}$ $m_H^{A\star} > m_D^{A\star}$	$\sigma^B < 0$ $\frac{\overline{\rho}-1}{\overline{\rho}\sigma^A}$ decreasing $p_{SN}^{A\star} > p_{SS}^{A\star} > p_I^{A\star}$ $m_P^{A\star} > m_U^{A\star} > m_I^{A\star}$ $m_H^{A\star} > m_D^{A\star}$
$\rho^{A'} > 0$	$\sigma^B > 0$ $\frac{\overline{\rho}-1}{\overline{\rho}\sigma^A}$ increasing $p_{SS}^{A\star} > p_{SN}^{A\star} > p_I^{A\star}$ $m_I^{A\star} > m_U^{A\star} > m_P^{A\star}$ $m_H^{A\star} > m_D^{A\star}$	$\sigma^B < 0$ $\frac{\overline{\rho}-1}{\overline{\rho}\sigma^A}$ increasing $p_{SN}^{A\star} > p_{SS}^{A\star} > p_I^{A\star}$ $m_P^{A\star} > m_I^{A\star} > m_U^{A\star}$ $m_H^{A\star} > m_D^{A\star}$

Table 2: Intermediated side outcomes in two-sided markets double marginalization

intermediated side: the sign of the slope of ρ^A is the same[14] as the sign of the slope of $\frac{\overline{\rho}-1}{\sigma^A\overline{\rho}}$, where $\overline{\rho} \equiv \frac{d\overline{p}}{dc}$.

Proposition 1. *If $\rho^A < 1$ then $p_{SS}^{A\star} > p_{SN}^{A\star} > p_I^{A\star}$ and $m_U^{A\star}, m_I^{A\star} > m_P^{A\star}$. If $\rho^A > 1$ then $p_{SN}^{A\star} > p_{SS}^{A\star} > p_I^{A\star}$ and $m_p^{A\star} > m_I^{A\star}, m_U^{A\star}$. If either $\rho^A > 1$ or $\rho^A < 1$ then $m_H^{A\star} > m_D^{A\star}$.*

If $\rho^{A'} < 1$ then $m_U^{A\star} > m_I^{A\star}$. If $\rho^{A'} > 1$, then $m_I^{A\star} > m_U^{A\star}$.

Proof. Comparing equations 3, 4 and 8 they all have the same B-side vulnerability (as a function of price). By the seesaw effect the ranking of their A-side vulnerabilities therefore determines the ranking across industrial organizations of platform mark-ups on side A. Thus the analysis (which ranks vulnerability functions at any given price) of Weyl (2008a) justifying the results in Table 1 transfer exactly to this context (though the sum of mark-ups is, for notational simplicity, replaced here by the final A price to which its rankings are equivalent). □

[14] $\overline{\gamma}' = \frac{\overline{\rho}-1}{\overline{\rho}} = \frac{\gamma^{A'}\gamma^{B'}}{\gamma^{A'}+\gamma^{B'}}$. $\sigma^A = \frac{\gamma^{B'}}{\gamma^{A'}+\gamma^{B'}}$. Thus $\frac{\overline{\rho}-1}{\overline{\rho}\sigma^A} = \gamma^{A'}$.

Un-intermediated side prices

Next consider the comparisons of prices on the un-intermediated prices. This is equally simple because, in all three organizations I consider, this price is determined exclusively by one firm (Integrated, Platform or Upstream) and because competitiveness on the un-intermediated side is not directly effected by changes in industrial organization. Therefore the only effect is the indirect effect of intermediated side prices on un-intermediated side prices: the greater is the former, the greater the subsidy to the later, and the lower the price. This, in turn, is determined (in reverse, by the seesaw effect) by the ranking of $m_I^{A\star}, m_P^{A\star}$, and $m_U^{A\star}$.

Proposition 2. *If* $\rho^A < 1$ *then* $p_{SN}^{B\star} > p_I^{B\star}, p_{SS}^{B\star}$*,if* $\rho^A > 1$ *then* $p_{SN}^{B\star} < p_I^{B\star}, p_{SS}^{B\star}$*. If* $\rho^{A'} < 0$ *then* $p_{SS}^{B\star} < p_I^{B\star}$*; if* $\rho^{A'} > 0$ *then* $p_{SS}^{B\star} > p_I^{B\star}$*.*

Proof. Follows directly from the other side of the seesaw that established the ranking of $m_I^{A\star}, m_P^{A\star}$, and $m_U^{A\star}$ in the proof of Proposition 1. □

Thus integration may either raise or lower prices to the other side of the market, depending on whether demand is cost-absorbing or amplifying (starting from the Nash organization) or whether it is increasing or decreasing (starting from the Stackelberg organization). From a purely theoretical perspective, this suggests that integration has no systematic effect on prices on the other side of the market, though if we this the Nash organization and total cost absorption case somewhat more likely it would tend to decrease it. However, the precise effects can be (over-)identified as discussed above.

Total firm mark-ups

Comparing the price level across industrial organizations, my main goal, is subtler than the prices on the two sides of the market. This comparison relies on the more sensitive effects of shifts in vulnerability on the level of prices charged by two-sided firm and more generally on the details of the strategic interaction across the two sides of the market. Because of this

	$\rho^A < 1$		$\rho^A > 1$	
	$\sigma^A, \sigma^B > 0$		$\sigma^A > 0 > \sigma^B$	
	$\frac{\bar{\rho}-1}{\bar{\rho}\sigma^A}$ decreasing	$\frac{\bar{\rho}-1}{\bar{\rho}\sigma^A}$ increasing	$\frac{\bar{\rho}-1}{\bar{\rho}\sigma^A}$ decreasing	$\frac{\bar{\rho}-1}{\bar{\rho}\sigma^A}$ increasing
$\rho^B < 1$	$\overline{m}_U^\star > \overline{m}_I^\star > \overline{m}_P^\star = m_H^{A\star} > m_D^{A\star}$	$\overline{m}_I^\star > \overline{m}_U^\star > \overline{m}_P^\star = m_H^{A\star} > m_D^{A\star}$	$\overline{m}_P^\star = m_H^{A\star} > m_D^{A\star} > \overline{m}_U^\star > \overline{m}_I^\star$	$\overline{m}_P^\star = m_H^{A\star} > m_D^{A\star} > \overline{m}_I^\star > \overline{m}_U^\star$
	$\sigma^B > 0 > \sigma^A$		Not possible: violates CSC	
	$\frac{\bar{\rho}-1}{\bar{\rho}\sigma^A}$ decreasing	$\frac{\bar{\rho}-1}{\bar{\rho}\sigma^A}$ increasing		
$\rho^B > 1$	$\overline{m}_P^\star = m_H^{A\star} > \overline{m}_I^\star > \overline{m}_U^\star > m_D^{A\star}$	$\overline{m}_P^\star = m_H^{A\star} > \overline{m}_U^\star > \overline{m}_I^\star > m_D^{A\star}$		

Table 3: Firm total mark-ups in the double marginalization problem

additional complexity, it is (perhaps only pedagogically) useful to take this comparison in steps. I therefore first consider the comparison of total firm mark-ups (the intermediated side mark-up plus un-intermediated side price for the two-sided firms and intermediated side mark-up for Downstream and Helper), which invokes the price level reasoning in a straightforward way. This makes the final comparison of the price level across industrial organizations more intuitive.

Table 3 summarizes my results on total mark-ups. The intuition behind them is a direct combination of the results in Table 2 the taxonomy from Subsection II.C of the effects of shifts in prices and vulnerability on the price level. Consider first the results above the horizontal line, in the upper half of the table. Here the un-intermediated side is cost-absorbing so that changes in the intermediated side which tend to incentivize low pricing (fall in vulnerability) to the intermediated side lead to a fall in its total markup. Thus the firm's total mark-up in these cells tracks the intermediated side mark-up from Table 2. Because both Platform equates its total mark-up and Helper equates its mark-up to intermediated-side vulnerability, these two mark-ups must be the same. Combined with the fact, directly from Table 2, that $m_D^{A\star} < m_H^{A\star}$ this reasoning justifies all the rankings in the top half of Table 3.

Because it involves the un-intermediated side of the market being cost-amplifying, the bottom left box draws on the associated intuition that a fall in the effective vulnerability faced by a firm on a side of the market opposite one that is cost-amplifying tends to lead them to charge a higher total mark-up. Thus these results simply reverse the rankings of the upper left hand corner of total mark-ups for the two-sided firms, while preserving $m_D^{A\star} < m_H^{A\star}$. The ranking also draws on the fact that $m_D^{A\star} = \gamma^A < \gamma^A(1-\gamma^{A'}) = \overline{m}_U^\star$ when $\rho^A < 1 \iff \gamma^{A'} < 0$. These results are formalized in the following proposition, the proof of which follows precisely the logic given here.

Proposition 3. *1. If $\rho^B < 1$ then:*

(a) If $\rho^A < 1$, $\overline{m}_I^\star, \overline{m}_U^\star > \overline{m}_P^\star = m_H^{A\star} > m_D^{A\star}$; if $\rho^A > 1$, $\overline{m}_P^\star = m_H^{A\star} > m_D^{A\star} > \overline{m}_U^\star, \overline{m}_I^\star$.

(b) If $\rho^{A'} < 0$, $\overline{m}_U^\star > \overline{m}_I^\star$; if $\rho^{A'} > 0$, $\overline{m}_U^\star < \overline{m}_I^\star$.

2. If $\rho^B > 1$ then satisfying CSC requires $\rho^A < 1$ and $\overline{m}_P^\star = m_H^{A\star} > \overline{m}_I^\star, \overline{m}_U^\star > m_D^{A\star}$. If $\rho^{A'} < 0$ then $\overline{m}_I^\star > \overline{m}_U^\star$; if $\rho^{A'} > 0$ then $\overline{m}_U^\star > \overline{m}_I^\star$.

Proof. By the reasoning establishing Propositions 1 and 2, the effective A-side vulnerability faced by the two-sided firm (Upstream, Integrated, Platform) is ranked precisely as is the A-side mark-up of these firms (the opposite of their B-side price). By case 1 of the taxonomy in Subsection II.C (the case of $\rho^A, \rho^B < 1$) and the appropriate case of the case 2 (when $\rho^A > 1 > \rho^B$) the total mark-up $\overline{m}_i^\star$ charged by various firms also tracks this given that in both cases $\rho^B < 1$. This suffices, along with Proposition 1 and the fact that $\overline{m}_P^\star = \gamma^A\big(m_H^{A^\star} + m_P^{A^\star} + c_I\big) = m_H^{A^\star}$ to establish the result in the case of $\rho^A < 1$. In the case of $\rho^A > 1$ additionally note that

$$m_D^{A^\star} = \gamma^A\big(m_D^{A^\star} + m_U^{A^\star} + c_I\big) > \gamma^A\big(m_D^{A^\star} + m_U^{A^\star} + c_I\big)\Big[1 - \gamma^{A'}\big(m_D^{A^\star} + m_U^{A^\star} + c_I\big)\Big] = m_U^{A^\star}$$

and also

$$m_D^{A^\star} = \gamma^A\big(m_D^{A^\star} + m_U^{A^\star} + c_I\big) > \gamma^A\big(m_I^{A^\star} + c_I\big) = \overline{m}_I^\star$$

as $p_{SS}^{A^\star} > p_I^{A^\star}$. This establishes part 1 of the proposition.

For part 2 note that now $\rho^B > 1$ so that total mark-ups track effective vulnerabilities to the two-sided firm in reverse. In addition to this note that, because now $\gamma^{A'} < 0$ but still $p_{SS}^{A^\star} > p_I^{A^\star}$, but precisely my reasoning above $m_D^{A^\star} < \overline{m}_U^\star, \overline{m}_I^\star$, completing the proof. □

Price level

I now turn to the comparison of price level across industrial organizations, summarized in Table 4. First consider the upper left-hand box, where $\rho^A, \rho^B < 1$. Clearly the intermediated side price under either of the separated organizations is greater than the intermediated side price under integration by Proposition 1. Furthermore the pass-through to the un-intermediated side is less than one-for-one. Thus it must be that the price level is higher under separation than integration: $\overline{p}_{SS}^\star, \overline{p}_{SN}^\star > \overline{p}_I^\star$. To compare $\overline{p}_{SS}^\star$ and $\overline{p}_{SN}^\star$ note that Upstream charges a higher intermediated side mark-up (Proposition 1) than Platform does. The crucial question in determining the price level is whether this additional mark-up is more strongly passed on as a higher price to the intermediated side of the market or as a lower price to the un-intermediated side (thus the diagonal line in the upper left hand box of Table 4). This turns on whether ρ^A or ρ^B is larger; if ρ^A is larger then the higher A-side

	$\rho^A < 1$	$\rho^A > 1$
$\rho^B < 1$	$\sigma^A > \sigma^B > 0$ $\overline{p}^\star_{SS} > \overline{p}^\star_{SN} > \overline{p}^\star_{I}$ $\sigma^B > \sigma^A > 0$ $\overline{p}^\star_{SN} > \overline{p}^\star_{SS} > \overline{p}^\star_{I}$	$\sigma^A > 0 > \sigma^B$ $\overline{p}^\star_{SN} > \overline{p}^\star_{SS} > \overline{p}^\star_{I}$
$\rho^B > 1$	$\sigma^B > 0 > \sigma^A$ $\frac{\overline{\rho}-1}{\overline{\rho}\sigma^A}$increasing $\overline{p}^\star_{SN} > \overline{p}^\star_{SS} > \overline{p}^\star_{I}$	Not possible: violates CSC
	$\sigma^B > 0 > \sigma^A$ $\frac{\overline{\rho}-1}{\overline{\rho}\sigma^A}$ decreasing $\overline{p}^\star_{SN} > \overline{p}^\star_{SS}$? $\overline{p}^\star_{I}$	

Table 4: The price level in the two-sided markets double marginalization problem

mark-up is more strongly passed through to A-side consumers and thus $\overline{p}^\star_{SS} > \overline{p}^\star_{SN}$; if ρ^B is larger then the higher A-side market is more strongly passed-through to side B and thus $\overline{p}^\star_{SN} > \overline{p}^\star_{SS}$.

Now consider the upper-right hand box. Here clearly the intermediated-side pass-through is greater than the un-intermediated-side pass-through. Furthermore in this case, Platform charges a higher intermediated side mark-up than Upstream. Thus by the same logic (in reverse) as in the upper left-hand box, $\overline{p}^\star_{SN} > \overline{p}^\star_{SS} > \overline{p}^\star_{I}$ as $\rho^B < 1$ still holds.

Now consider the lower-left hand box. In this case the un-intermediated side is cost-amplifying so potentially weird things can happen. But remember that Platform's optimal mark-up is less than Integrated's on the intermediated side, as demand on this side is cost-absorbing. Thus $\overline{p}^\star_{SN} > \overline{p}^\star_{I}$. Furthermore clearly $\rho^B > \rho^A$ and Upstream charges a higher A-side mark-up than Platform so $\overline{p}^\star_{SN} > \overline{p}^\star_{SS}$. When $\rho^{A'} > 0$, Upstream charges a smaller intermediated-side mark-up than Integrated and thus by the same logic as in the Nash case, $\overline{p}^\star_{SS} > \overline{p}^\star_{I}$. However when $\rho^{A'} < 0$, Upstream charges a larger intermediated-side mark-up than Integrated. If ρ^B is sufficiently large (and larger than ρ^A) and Upstream charges a sufficiently larger intermediated-side mark-up than Integrated, it is possible (as shown in the proof of the proposition below) that $\overline{p}_I^\star > \overline{p}_{SS}^\star$. However it is clearly also possible that this effect is not large enough to overcome the mark-up added by Downstream, which is not present in the Integrated case, and therefore that $\overline{p}_{SS}^\star > \overline{p}_I^\star$. The following proposition states these results formally.

Proposition 4. *1.* $\overline{p}_{SN}^\star > \overline{p}_I^\star$.

2. *If* $\rho^B < \rho^A < 1$ *then* $\overline{p}_{SS}^\star > \overline{p}_{SN}^\star$. *If* $\rho^A = 1$ *or* $\rho^B = \rho^A < 1$ *then* $\overline{p}_{SS}^\star = \overline{p}_{SN}^\star$. *Otherwise* $\overline{p}_{SS}^\star < \overline{p}_{SN}^\star$.

3. *If* $\rho^B > 1 > \rho^A$ *and* $\rho^{A'} < 0$ *then it is possible, but not necessary, that* $\overline{p}_{SS}^\star < \overline{p}_I^\star$. *Otherwise* $\overline{p}_I^\star < \overline{p}_{SS}^\star$.

Proof. See Appendix A. □

Thus only in one very narrow case (Stackelberg organization, $\rho^B > 1 > \rho^A$, $\rho^{A'} < 0$) is it possible that vertical integration may raise the price level and even there this possibility relies on the dominance of one effect over another. Thus the price level effect of vertical integration are significantly more robust than those of competition. The reason is that it is not the case that every dollar of increase in intermediated side prices caused by vertical separation acts as a subsidy to the un-intermediated side; some of these are absorbed by the non-two-sided firm and thus are purely harmful. While this does not imply[15] that vertical integration is robustly welfare improving given the potential importance of price balance to welfare (Weyl, 2008b), it does show that the benefits of vertical integration may be *more* robust than those of competition in two-sided market.

Another, though perhaps not as important, curio is that the Stackelberg organization is more often the most "competitive" (leads to the lowest price level) in a two-sided market,

[15] The full welfare analysis of vertical integration in two-sided markets remains an open, and challenging, question. Analysis within the constant pass-through class might be an interesting starting point.

	$\rho^A < 1$	$\rho^A > 1$
$\rho^B < 1$	$\sigma^A > \sigma^B > 0$: $\pi_U^\star > \pi_P^\star = \pi_H^\star > \pi_D^\star$ $\pi_I^\star > \pi_{SN}^\star > \pi_{SS}^\star$ $\sigma^B > \sigma^A > 0$: $\pi_U^\star > \pi_P^\star = \pi_H^\star > \pi_D^\star$ $\pi_I^\star > \pi_{SN}^\star \; ? \; \pi_{SS}^\star$	$\sigma^A > 0 > \sigma^B$: $\pi_D^\star > \pi_U^\star > \pi_P^\star = \pi_H^\star$ $\pi_I^\star > \pi_{SS}^\star > \pi_{SN}^\star$
$\rho^B > 1$	$\sigma^B > 0 > \sigma^A$: $\pi_U^\star > \pi_D^\star \; ? \; \pi_P^\star = \pi_H^\star$ $\pi_I^\star > \pi_{SS}^\star \; ? \; \pi_{SN}^\star$	Not possible: violates CSC

Table 5: Firm and industry profits in the two-sided markets double marginalization problem

whereas in a standard market (if we assume cost-absorbing demand is more common than cost-amplifying demand) the reverse was the case. The reason is that the two-sided firm now has a special position of serving both sides of the market that makes its intermediated side mark-up more socially valuable than that of the non-two-sided firm. This strongly offsets the social disadvantages of its first-move advantage (when it exists).

Profits

I conclude this section by discussing the comparison of firm and industry profits within and across industrial organizations. The results are summarized in Table 5. Individual firm profits generally track those in a standard market with the appropriate properties of A-side demand. Because they both equate their (total) mark-ups to A-side vulnerability, Helper and Platform must earn the same profits. Upstream always earns more profits than Platform as it can imitate Platform and obtain the same response that Platform would from Helper itself from Downstream. Under cost-amplifying intermediated-side demand it is still the case that the total mark-up of Downstream is greater than that of Upstream, thus Downstream's profits must also be greater in this case. Most subtly it is still the case that under total cost absorption that Downstream does worse than Helper. The reason is that, starting at the Nash prices, if it were ever in the interests of Downstream to have Upstream raise her mark-up in order to reduce p^B then it would also be in the interest of Platform to lower p^B starting at these prices. The reason is that while it costs platform one dollar of mark-up to reduce p^B by one dollar, costs Downstream $\frac{1}{\rho^B} > 1$ (as $\rho^B < 1$ under total cost absorption) dollars to achieve the same effect. As Upstream raises her mark-up above this level and Downstream's mark-up becomes smaller, this trade-off is increasingly unattractive.

However, in the case where $\rho^B > 1$ the relationship is ambiguous. Now Downstream gets the first dollar of p^B reduction cheaper than Platform does and therefore it is initially in Downstream's interest for Upstream to raise its mark-up. However as this occurs and Downstream's mark-up contracts, the trade-off is less appealing. In cases when ρ^B is quite large and ρ^A is close to 1 (those which butt up against $\rho^A \rho^B < 1$) Upstream only raises its intermediated-side mark-up by a small amount (relative to Platform) and the first reasoning dominates. But in cases where ρ^B is close to 1 and ρ^A is small, Upstream raises their intermediated-side mark-up over what Platform would charge by a large amount and the second reasoning dominates.

Full industry profits require a bit more piecemeal analysis. Integrated profits are always higher than either vertically separated organization, as the Integrated firm could always imitate the separated firms. In the upper-right corner, Downstream and Upstream both earn higher profits than either SN firm, so clearly the SS industry has greater profits than the SN industry. In the upper-right half of the upper-left corner the $\overline{p}_I < \overline{p}^\star_{SN} < \overline{p}^\star_{SS}$ and the SN organization guarantees the correct price balance for industry profits (by equation 4 given this price level, while the SS organization leads to a sub-optimal (for industry profits) balance given that level (by equation 8). Thus SN is preferable both on the price level and balance, in terms of industry profitability, and thus must be better.

Unfortunately, in the remaining cases it is not possible (based on the simple criteria I have been using) to compare $\pi^\star_{SS}$ and $\pi^\star_{SN}$. Many things are now possible: if we are in the case where $\pi^\star_D > \pi^\star_H$ (requiring, but not implied by, $\rho^B > 1$) then clearly $\pi^\star_{SS} > \pi^\star_{SN}$. This is due to the fact that the price level is higher (further away from the industry profit optimum) under SN. But on the other hand the price balance is more distorted under SS, so in the cases when $\pi^\star_D < \pi^\star_H$ (possible with any of $\rho^B >, <$ or $= 1$), it can be[16] that $\pi^\star_{SN} > \pi^\star_{SS}$. As I show in the proof of the following proposition, which formalizes the results in Table 5, it is easy to construct examples of each of these situations (except, as I note in the proof, I have not found an example when $\pi^\star_{SN} > \pi^\star_{SS}$ and $\rho^B > 1$, nor a proof that this is impossible).

Proposition 5. *1.* $\pi^\star_I > \pi^\star_{SS}, \pi^\star_{SN}$.

2. If $\rho^A > 1 > \rho^B$ *then* $\pi^\star_D > \pi^\star_U > \pi^\star_P = \pi^\star_H$*; if* $\rho^A < 1$ *and* $\rho^B \leq 1$ *then* $\pi^\star_U > \pi^\star_P = \pi^\star_H > \pi^\star_D$*; if* $\rho^B > 1 > \rho^A$ *then* $\pi^\star_U > \pi^\star_D, \pi^\star_P = \pi^\star_H$ *but there may be any relationship between* $\pi^\star_P = \pi^\star_H$ *and* $\pi^\star_D$.

3. If $\rho^A > 1 > \rho^B$ *then* $\pi^\star_{SS} > \pi^\star_{SN}$*. If* $1 > \rho^B \geq \rho^A$ *then* $\pi^\star_{SN} > \pi^\star_{SS}$*. If* $\rho^A = 1 > \rho^B$ *then* $\pi^\star_{SN} = \pi^\star_{SS}$*. If* $1 > \rho^B > \rho^A$ *then any relationship between* $\pi^\star_{SN}$ *and* $\pi^\star_{SS}$ *is possible. If* $\rho^B > 1 > \rho^A$ *it is possible that* $\pi^\star_{SS} > \pi^\star_{SN}$ *and it may or may not be possible (I have*

[16]My computational simulations indicate that this is not possible in the constant pass-through class. However I have not attempted to prove this. I think that it is safe to say however that "typically" in this case $\pi^\star_{SS} > \pi^\star_{SN}$.

not shown) that $\pi^\star_{SN} > \pi^\star_{SS}$.

Proof. See Appendix A. □

Summary

The basic message of this paper is basic logic of double marginalization extends to two-sided markets. In fact, there are reasons to believe that its benefits in two-sided markets may be more robust than those of competition. While there are a few cases in which integration can lead to higher fees to merchants, this is generally not the case and when it is, it is precisely in these cases when integration brings the greatest reductions in consumer prices. While this does not clearly imply that vertical integration is always welfare enhancing (and certainly not for both sides of the market), it does provide a framework for determining what the potential effects of integration may be. Given exogenous cost variations and observations of costs and prices it is simple to determine where in the tables one lies. Given how many predictions of rankings of variables both within (in the case of the Spengler-Stackelberg organization) and across industrial organizations are possible on the basis of this knowledge, it is not only possible to identify what the effects of vertical integration should be. In fact, it should be, in principle and with the right data, also possible to non-parametrically test, in several different ways, the validity of the model. The over-identification here is even stronger than in a one-sided market double marginalization or the simple two-sided markets model because the fusion of the two models' over-identification multiplies these tests together.

5 Extensions and Robustness

I have only considered three possible vertical organizations (Integrated, SS and SN) of this industry. I focused on these because I believe they are the most policy relevant and because they have an elegant parallel to the industrial organizations in standard markets. However, there are several others, such Spengler-Reverse Stackelberg (SRS) and Full Nash

(FN) depicted in Figures 3 and 4. More complex organizations could also be possible: in the FN diagram, sides of the market might first (sequentially or non-sequentially) commit to prices on one side and then on the other. While I have not analyzed all of these cases, I suspect the basic results derived here would be sound: I suspect that under total cost absorption results vertical integration would lead to a reduction in the price level. The reason is that determining the full market pricing equation will always involve summing together the individual pricing equations of the two firms in some fashion, and therefore lead to a rise in vulnerability (see Appendix A). However, in the case of total cost-amplification, more careful attention would need to be paid to the consequences of integration for individual firm mark-ups in order to analyze the price level. In these cases, it is not clear whether the strong robustness of the (price level) benefits of vertical integration would carry over. Nonetheless I suspect that the details of these cases, and their comparisons to the cases I consider here, could be worked out in a straight-forward manner using the tools developed here, particularly the analysis of strategic complementarity and substitutability using pass-through rates.

Another potential robustness problem for the model above, discussed earlier, is the assumption of monopoly at both levels. A well-known property of the Cournot-Spengler model (Tirole, 1988) when there is competition at one or both of the levels is that vertical integration by the more competitive of the two levels is always weakly pro-competitive and strictly so if competition at the less competitive of the two levels is imperfect. As Bowman (1957) famously put it "a monopoly of bolts if nuts are competitive is as good as a monopoly of bolts and nuts." I have not explored these issues in two-sided markets here, but I suspect that using a model like RT2003 to do so would yield similar conclusions: unless the rise of Interlink is likely to reduce interbank competition, it seems unlikely that the fact of interbank competition would reverse the conclusion here (largely) in favor of vertical integration. However, if it were to reduce interbank competition, this could be a significant harm; in practice, though, given that banks already cooperate through Visa in the credit card and signature (offline) debit markets, I suspect that the emergence of Interlink in the online debit market is unlikely to provide significant marginal collusive opportunities. I therefore would suspect that my basic results are robust to the introduction of competition at one or even (as long as integration takes place as the more competitive level) both levels. Finding the right extension of RT2003 to allow appropriately for verticality with competition and particularly competition at multiple levels is an interesting question for future research. In the absence of this, my thoughts here are only conjectures. And of course these results could likely be weakened or reversed if more complicated effects of vertical integration (such as foreclosure, intra- and inter-brand competitive pressures, etc.) were built into the model.

Another objection that might be raised to my model is that it may be possible to solve the vertical problem without integration. However, it is crucial that whatever firm is made the residual claimant have control of prices on both sides of the market. Otherwise if non-linear pricing (franchising) leaves one firm (the debit clearing network) in control of prices on one (merchant's) side of the market and another (banks) in charge of pricing to the other (card-holders) this fails to solve the vertical problem: pricing on each side has an externality for the other firm. Inherently any firm that makes pricing decisions (after a contractual solution) on either side of the market has moral hazard and as has been emphasized in the literature on vertical relations (Tirole, 1988) and moral hazard in teams (Holmström, 1982), making both firms residual claimants is difficult. One solution is that the firms might maintain physical control over pricing, but jointly contract to constrain themselves to the optimal prices; however, one might worry that such an arrangement would be insufficiently responsive to high-frequency demand shifts. Alternatively all pricing decisions might be given to one residual claimant (the banks or the network). It seems difficult to imagine a scheme for giving such pricing authority to the banks which differs markedly from the cooperative arrangements instituted by Visa/Interlink. Giving full debit pricing authority to a for-profit clearing network like Star seems deeply anti-competitive, as it would essentially allow vertical integration at the less competitive level of the market (banks, concentrated as they are, are much less so than debit clearing networks). Thus it seems that, at least in the case of the debit industry, non-integrative means of solving the vertical problem may be more difficult to find than in a standard market (with no moral hazard). This analysis, if it could be formalized, would strengthen the case for vertical integration.

6 Conclusion

I conclude by briefly discussing some directions for future research. While the primary application I pursue here, and the one to which my model is most directly suited, is the payment cards industry, I hope it may shed some light on issues in other two-sided markets, such as the relationship between software platforms[17] (video game consoles, operating systems, etc.) and producers of complementary products (joysticks, hardware, etc.) or between different internet service provider, some connecting content consumers (like you and I), others connecting content supplies (like Google and Yahoo!). Along these lines it would be interesting to understand if and how the results here extend to the Armstrong (2006) model, which some consider a better model for those sorts of markets. Also, it would be very useful to understand the normative properties of double marginalization even in the RT2003 model, which were neglected here. A useful starting point might be the tractable constant pass-through class of demand functions (Bulow and Pfleider, 1983; Weyl, 2008a). Finally, empirical work based on the identification and over-idenficiation here is perhaps the most direct applications suggested by the results above.

[17]For an excellent survey of software platforms which raises some of these issues see Evans, Hagiu, and Schmalensee (2006).

Appendix

A Proof of Proposition 4

I begin with the comparison of $\overline{p}^{\star}_{SN}$ and $\overline{p}^{\star}_{I}$ as these are the easiest to compare. The reason is that what determines the balance of prices under SN is exactly $\gamma^A = \gamma^B$ by equation 4. But the price level is determined by adding equations 4 and 5 to yield

$$p^{A\star}_{SN} + p^{B\star}_{SN} - c_I = 2\gamma^A(p^{A\star}_{SN}) = \gamma^A(p^{A\star}_{SN}) + \gamma^B(p^{B\star}_{SN}) \tag{11}$$

Note that at equilibrium it is still the case that $\gamma^A = \gamma^B$, but vulnerability is higher for every pair of prices than it is under integration as $\gamma^I > 0$, so by Proposition 4 the price level must be lower under separation.

Both Helper and Downstream take the two-sided firm intermediated-side mark-up as given and therefore given any such mark-up, will charge the same mark-up to consumers. Furthermore every dollar increase in the two-sided market firm's A-side mark-up is equivalent to an increase in Helper/Downstream's cost. Thus for every dollar that, for example, $m^{A\star}_U$ exceeds $m^{A\star}_P$ the price to the A-side consumers will rise by $\hat{\rho}^A$ times this amount, where $\hat{\rho}^A$ is the average value of the pass-through over the appropriate price range. Similarly every dollar that $m^{A\star}_U$ rises above $m^{A\star}_P$ corresponds to a one dollar subsidy in the two-sided firm's cost of producing services to the un-intermediated side of the market. Thus we can write

$$\overline{p}^{\star}_{SS} = \overline{p}^{\star}_{SN} + \int_{m^{A\star}_P}^{m^{A\star}_U} \rho^A\big(m^A_D(m) + m + c_I\big) - \rho^B\big(p^B(m)\big)dm \tag{12}$$

where $m_D(\cdot)$ gives the Downstream's optimal reaction to a given Upstream mark-up and $p^B(\cdot)$ is the optimal p^B for Upstream given that they charge a mark-up of m on side A. Now consider the cases in Table 4. If $\rho^A < 1$ then $m^{A\star}_U > m^{A\star}_P$ by Proposition 1. Thus if $\rho^A > \rho^B$ then the second term on the RHS of equation 12 is strictly positive and thus $\overline{p}^{\star}_{SS} > \overline{p}^{\star}_{SN}$; if

$\rho^B > \rho^A$ then the opposite holds; if $\rho^A = \rho^B$ then $\overline{p}^\star_{SS} = \overline{p}^\star_{SN}$. If $\rho^A > 1$ then $m_U^{A^\star} < m_P^{A^\star}$ and the second term is strictly negative if $\rho^A > \rho^B$; but clearly $\rho^A > 1 > \rho^B$ is the only possible case of $\rho^A > 1$ so in this case $\overline{p}^\star_{SS} < \overline{p}^\star_{SN}$. If $\rho^A = 1$ the second term is always 0 so $\overline{p}^\star_{SS} = \overline{p}^\star_{SN}$.

My approach to comparing SS to integration is more piecemeal. First consider the two simple cases:

1. $\rho^{A'} < 0$ and $\rho^B \leq 1$: In this case $\overline{m}^\star_U \geq \overline{m}^\star_I$ and $m_D^{A^\star} > 0$. Given that $\overline{p}^\star_{SS} = \overline{m}^\star_U + m_D^{A^\star} + c_I$ and $\overline{p}^\star_I = \overline{m}^\star_I + c_I$, clearly $\overline{p}^\star_{SS} > \overline{p}^\star_I$.

2. $\rho^{A'} \geq 0$: In this case $p_{SS}^{B^\star} \geq p_I^{B^\star}$ and, as is always the case, $p_{SS}^{A^\star} > p_I^{A^\star}$. Thus clearly $\overline{p}^\star_{SS} > \overline{p}^\star_I$.

Now the more difficult case is when $\rho^{A'} < 0$ and $\rho^B > 1$. Clearly, by continuity, it is possible in this case that $\overline{p}^\star_{SS} > \overline{p}^\star_I$ as this ranking is strict in case 1 above which borders this case. However, the opposite ranking is also possible as the example below shows. Note that this example violates twice continuous differentiability (though it is once continuously differentiable). This is merely a way to provide a simple example, substituting piecewise linear for smoothly concave vulnerability. Because there are thrice continuously differentiable functions approximating the one I consider arbitrarily well and the ranking is strict, I strongly suspect that an example with thrice continuous differentiability is possible.

Example 1. *Suppose that*

$$D^B(p^B) \equiv (3 + p^B)^{-3}$$

$$D^A(p^A) \equiv \begin{cases} 2 - p^A & p^A \leq -2 \\ a\left(-\frac{49}{25} + p^B\right)^{-\frac{1}{100}} & p^A > -2 \end{cases}$$

where $a \approx 3.87$ is chosen to preserve continuity and that $c_I = 4$. Then $\overline{p}^\star_{SS} < \overline{p}^\star_I$.

To see this compute vulnerability functions:

$$\gamma^A(p^A) = \begin{cases} 2 - p^A & p^A \leq -2 \\ -196 - 100p^A & p^A > -2 \end{cases}$$

$$\gamma^B(p^B) = 1 + \frac{p^B}{3}$$

Plugging these in to equations 3, 9 and 8 under the assumption that $p_I^{A\star} \leq -2$ *and* $p_{SS}^{A\star} > -2$ *(my example was cooked so this would occur) yields (after some computation):*

$$\begin{aligned} p_I^{A\star} &= -3 \quad p_{SS}^{A\star} \approx -1.97 \\ p_I^{B\star} &= 12 \quad p_{SS}^{B\star} \approx 10.52 \\ \overline{p}_I^{\star} &= 9 \quad \overline{p}_{SS}^{\star} \approx 8.55 \end{aligned}$$

Thus my conjecture about $p_{SS}^{A\star} > -2 > p_I^{A\star}$ *is confirmed (note MUC holds in the problem so using the equations really does give the global optimum) and in this example* $\overline{p}_{SS}^{\star} < \overline{p}_I$*.*

This example, as mentioned above, relies on introducing a lot of concavity into the A-side vulnerability, inducing Upstream to raise its A-side mark-up significantly above that of Integrated (≈ -6.01 versus -7), most of which is absorbed by Downstream's reducing its mark-up (Downstream's mark-up is $\approx .045$ compared to a total mark-up of ≈ 4.55 for Upstream) and leading Upstream to more than pass-through most of that mark-up to side B consumers.

B Proof of Proposition 5

First note that clearly $\pi_U^\star > \pi_P^\star$ because Upstream can always imitate Platform, just as $\pi_U^\star > \pi^\star$ in Table 1. Because they face the same demand and charge the same total mark-up by Proposition 3, clearly $\pi_P^\star = \pi_H^\star$. If $\rho^A > 1$ Downstream faces the same demand as Upstream but charges a higher total mark-up so in this case $\pi_D^\star > \pi_U^\star$. If $\rho^A = 1$ both industries are the same and Upstream and Downstream charge the same total mark-up so $\pi_U^\star = \pi_D^\star = \pi_P^\star = \pi_H^\star$.

Now I want to compare $\pi_D^\star$ to $\pi_H^\star$ in the case when $\rho^A < 1$. Note that the only difference between these two firms that is not under their control is that Downstream faces a higher A-side mark-up from the two-sided firm (equivalent to a higher cost) and a lower B-side price. Thus one can write

$$\pi_D^\star = \pi_H^\star + \int_{m_P^{A\star}}^{m_U^{A\star}} \frac{d\Big[\big(p_D^A(c_I+m) - c_I - m\big) D^A\big(p_D^A(c_I+m)\big) D^B\big(p_U^B(m)\big)\Big]}{dm} dm$$

where $p_D^A(\cdot)$ and $p_U^B(\cdot)$ respectively represent the optimal Downstream price to side A and the optimal Upstream price to side B when Upstream's mark-up is fixed at m. Thus the comparison of $\pi_D^\star$ to $\pi_H^\star$ is determined by the comparison of

$$\int_{m_P^{A\star}}^{m_U^{A\star}} \frac{d\Big[\big(p_D^A(c_I+m) - c_I - m\big) D^A\big(p_D^A(c_I+m)\big) D^B\big(p_U^B(m)\big)\Big]}{dm} dm$$

to 0. Therefore if I can sign

$$\frac{d\Big[\big(p_D^A(c_I+m) - c_I - m\big) D^A\big(p_D^A(c_I+m)\big) D^B\big(p_U^B(m)\big)\Big]}{dm}$$

I can compare $pi_D^\star$ and $\pi_H^\star$. Dropping arguments for spacial economy

$$\frac{d\big[(p_D^A - c_I - m) D^A D^B\big]}{dm} = \rho^A\big(D^A D^B + D^{A'} D^B m_D\big) - D^A D^B - \rho^B D^{B'} D^A m_D$$

where $m_D \equiv p_D^A - c_I - m$. Because $D^{A'}, D^{B'} < 0$ dividing through by $D^{A'} D^{B'}$ yields an expression of the same sign.

$$\rho^A(\gamma^A\gamma^B - \gamma^B m_D) - \gamma^A\gamma^B + \rho^B\gamma^A m_D = \rho^A(\gamma^A\gamma^B - \gamma^A\gamma^B) - \gamma^A\gamma^B + \rho^B\gamma^{A^2} =$$

$$\gamma^A(\rho^B\gamma^A - \gamma^B) \tag{13}$$

Here the first equality is comes from the fact that $m_D = \gamma^A$ by Downstream's optimization. The terms multiplied by ρ^A drop out by the envelope theorem: effect of Upstream's mark-up on Downstream's price are irrelevant to Downstream's profits as this price is chosen optimally; only the effects on variables outside Downstream's control matter. Because $\gamma^A > 0$, the sign of expression 13 is determined by the sign of

$$\rho^B\gamma^A - \gamma^B \tag{14}$$

There are two cases. First suppose that $\rho^B \leq 1$. At $m = m_H^{A\star}, \gamma^A = \gamma^B$ by equation 4. Thus expression 14 is weakly negative evaluated at $m = m_H^{A\star}$. As m rises it becomes strictly negative if either $\rho^B = 1$ or $\rho^B < 1$. To see this note that if $\rho^B = 1$ then γ^B and ρ^B are constant and γ^A declines as m rises as $\rho^A < 1$ and p_D^A increases in m. By CSC for any $m > m_H^{A\star}$, $\gamma^B > \gamma^A$. Thus expression for any $m > m_H^{A\star}$

$$\rho^B\gamma^A - \gamma^B < \rho^B\gamma^A - \gamma^A < 0$$

as $\rho^B < 1$. Thus if $\rho^B \leq 1$, $\pi_P^\star = \pi_H^\star > \pi_D^\star$. On the other hand if $\rho^B > 1$, it is clearly possible that $\pi_P^\star = \pi_H^\star > pi_D^\star$ by continuity and the fact that the ranking is strict when $\rho^B \leq 1$. However, the other ranking is possible as well. I will show this below by example, but, so as to collect all relevant examples together, I defer this until after discussing what can be proved (more generally) about industry profit rankings, which I turn to now.

Clearly $\pi_I^\star > \pi_{SS}^\star, \pi_{SN}^\star$ as Integrated can always imitate the other two industrial organizations, but never chooses to. If $\rho^A > 1$ then both SS firms earn higher profits than either SN firm, so clearly $\pi^\star_{SS} > \pi^\star_{SN}$. If $\rho^A = 1$ all firms earn identical profits so $\pi^\star_{SS} = \pi^\star_{SN}$. If $1 > \rho^A \geq \rho^B$ then $\overline{p}^\star_{SS} \geq p^\star_{SN}$ by Proposition 4, but given any particular price level, SS leads to sub-optimal price balance by equation 8 while SN leads to optimal price balance by equation 4. Given that in this case both arrangements lead to a super-optimal price balance, SN is weakly better for industry profits in terms of price level and strictly better in terms of price balance. Therefore $\pi^\star_{SN} > \pi^\star_{SS}$. If $1 \geq \rho^B > \rho^A$ clearly by continuity it is possible that $\pi^\star_{SN} > \pi^\star_{SS}$. However the reverse is also possible, as I show in my second example below.

If $\rho^B > 1 > \rho^A$ clearly it is possible that $\pi^\star_{SS} > \pi^\star_{SN}$ so long as my claim that $\pi^\star_D > \pi^\star_H = \pi^\star_P$ is possible is true, as in this case the profits of both SS firms are greater than those of both SN firms; the first example below shows this case. However, it may also possible that that $\pi^\star_{SN} > \pi^\star_{SS}$ (the price balance effect dominates). I have not found an example of this (and have searched extensively in the constant pass-through class). However I have not either found a proof that $\pi^\star_{SS} > \pi^\star_{SN}$ always holds (even in the constant pass-through class). Thus this remains an unproven ambiguity that I have not tried more to resolve as I do not consider it of primary importance in any policy application I know of. It remains for future research.

Example 2. *Suppose that*

$$D^A(p^A) \equiv (1 - p^A)^\alpha$$

$$D^B(p^B) \equiv (1 + p^B)^{-\alpha-3}$$

and let $c_I = 2$. *In this case*

$$\pi^\star_D = \frac{\alpha}{\alpha+1}\left[(3+\alpha)^{-3-\alpha}\left(3+\alpha+\frac{1}{1+\alpha}\right)^\alpha\right]$$

$$\pi^\star_H = \frac{(2+\alpha)^\alpha(3+\alpha)^{-3-\alpha}}{8}$$

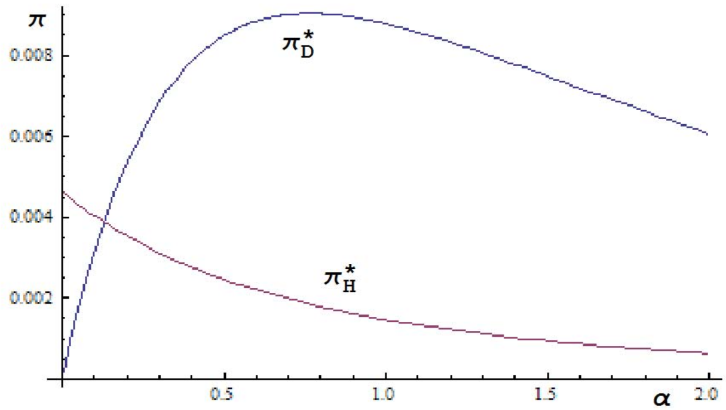

Figure 7: Example 2: $\pi_U^\star$ v. $\pi_H^\star$ as a function of α

Rather than trying to analytically compare them, in Figure 9 I plot the values against one another The non-monotonic curve is $\pi_D^\star$, which is clearly higher for many if not most of the values (approximately $\alpha > .2$). For all of these values it is clearly also the case that $\pi_{SS}^\star > \pi_{SN}^\star$. In fact I have not found in my computational simulations in the linear vulnerability class an example with $\rho^B > 1$ where $\pi_{SN}^\star > \pi_{SS}^\star$, but neither have I tried to prove one does not exist. However, I have found such an example (the next one) outside this class.

With my final example I show that $\pi_{SS}^\star > \pi_{SN}^\star$ is possible even with $\rho^B < 1$.

Example 3. *Suppose that*

$$D^A(p^A) \equiv (1 - p^A)^{\frac{1}{10}}$$

$$D^B(p^B) \equiv (1 - p^B)^{\alpha}$$

with $c_I = 0$. In this case

$$\pi_{SS}^\star = \frac{3\alpha^\alpha 2^{\frac{41}{10}+2\alpha}}{11(121+110\alpha)^{\frac{1}{10}}\left(\frac{11}{5}+2\alpha\right)^{1+\alpha}}$$

$$\pi_{SN}^\star = \frac{5\alpha^\alpha 2^{\frac{31}{10}+2\alpha}}{\left(\frac{21}{5}+2\alpha\right)^{\alpha}(21+10\alpha)^{\frac{1}{10}}}$$

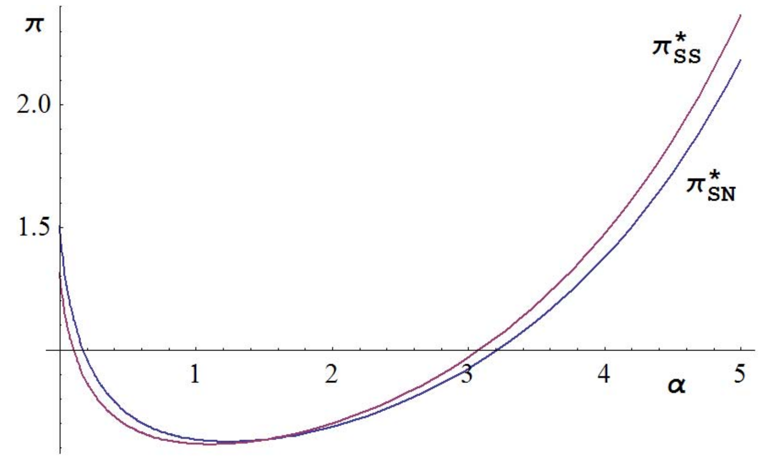

Figure 8: Example 4: $\pi^{\star}_{SS}$ v. $\pi^{\star}_{SN}$ as a function of α

Figure 10 shows a graph of these two against α*. Clearly for large* α *(large* ρ^B*)* $\pi^{\star}_{SS} > \pi^{\star}_{SN}$*, as I claimed.*

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