Dynamics in a Model of On-the-Job Search

Robert Shimer

June 23, 2003

1 Introduction

Recent papers have argued that standard theories of the labor market, e.g. the Mortensen and Pissarides (1994) matching model, cannot generate significant movements in unemployment and vacancies in response to shocks of a plausible magnitude (Hall 2003, Shimer 2003). This paper shows that introducing on-the-search in the manner of Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002) may help to reconcile labor market theories with empirical evidence by substantially amplifying and propagating shocks to the economy.

The driving force in this paper is cyclical changes in the composition of the searching population. By paying a fixed cost, a firm may contact one worker, randomly selected from the searching population, which consists of both unemployed and employed workers. Firms prefer to contact unemployed workers because they can be attracted at a lower wage; hiring an employed worker requires outbidding the incumbent employer. Now consider an aggregate shock that raises the present value of output from all matches, e.g. an increase in productivity or a decline in the exogenous separation rate. This raises the profit from contacting a worker, inducing firms to create job vacancies. In standard labor market models, this would have two effects, both of which reduce profits and mitigate the amount of job creation: wages increase and it becomes harder for firms to contact workers. For reasons discussed below, neither of these occur in this model. Instead, equilibrium is restored through a worsening of the composition of the searching population, i.e. an increase in the conditional likelihood of contacting an employed worker. Crucially, a small change in the composition of the searching population may be achieved only through a large change in the rate at which searching workers find jobs. Therefore, even a small increase in the present value of output causes a large decrease in the unemployment rate, amplifying the original shock.

There are several important elements in this model. The first, borrowed directly from

Postel-Vinay and Robin (2002), is that when a firm contacts a worker, it observes the worker's employment status and makes her a take-it-or-leave-it wage offer. If the worker is already employed, the incumbent firm simultaneously responds with a counteroffer. In equilibrium, firms engage in Bertrand competition for employed workers, and so the worker extracts all of the surplus from the match. This is an extreme form of the notion that contacting an unemployed worker is relatively more valuable; there is no value to contacting an employed worker. Conversely, when a firm contacts an unemployed worker, the firm extracts all of the surplus from the match, which includes not only current output but also the value of future job search opportunities to the worker. I show that these assumptions imply that the profit from contacting a worker depends only on the worker's employment status and the present value of output from a match, but is independent of the amount of vacancy creation in the economy.

Second, I assume that firms may contact a worker by paying a fixed cost k. Mathematically, this is equivalent to a matching model (Pissarides 1985) in which the aggregate flow of meetings is proportional to the stock of vacancies, $M(v) = \mu v$, and firms pay a flow cost $c = k\mu$ of maintaining an open vacancy. In other words, the cost of contacting a searching worker is independent of the amount of vacancy creation in the economy. Under a more standard assumption that the flow of matches is an increasing, constant returns to scale function of the stock of searching workers and vacancies, shocks are accommodated primarily through changes in the ratio of vacancies to searching workers. I choose to shut down that possibility in this model.[1]

Finally, I assume that workers do not search after receiving a second job offer without an intervening unemployment spell. This seems sensible since competition between employers implies that such workers earn the full surplus from the match and so have nothing to gain from continued job search. This leaves two groups of searchers, unemployed workers and employed workers in their first job following an unemployment spell. I show that there is a close link between the steady state unemployment rate u^* and the steady state fraction of searchers who are unemployed, x^*; assuming both groups of workers search equally effectively, $x^* = (2 - u^*)^{-1}$. This is key to the amplification mechanism. Recall that a small decline in productivity must be offset by a small improvement in the composition of the searching population i.e. a small increase in x^*, since other channels for restoring the zero profit condition have been shut down. The previous equation implies that this may require a large

[1] Existing estimates of matching functions, recently reviewed by Petrongolo and Pissarides (2001), do not address job search by employed workers. It is unclear how that biases the estimates.

change in the steady state unemployment rate. For example, if the fraction of searchers who are unemployed must increase from 0.51 to 0.52, the unemployment rate consequentially nearly doubles from 3.9 to 7.7 percent of the labor force.

The reader may justifiably complain that this model is rigged to amplify shocks. If this were the only result that the model delivered, the success of the exercise would be questionable. But interestingly, the model also helps to resolve another important shortcoming of the search model, that it has almost no internal propagation mechanism (Hall 1995, Pries forthcoming). In the Mortensen and Pissarides (1994) model, the unemployment rate adjusts back to its steady state value u^* following a one-time shock according to a first order autoregressive process, closing half the gap in $\frac{u^* \log 2}{s}$ periods, where s is the per-period rate at which workers lose jobs. Empirically, jobs last two or three years, so half the gap to a five percent steady state unemployment rate is closed in 0.07 to 0.10 years. This implies that search frictions do not deliver much slow adjustment in the unemployment rate.[2]

In this model, the rate at which workers find jobs λ endogenously adjusts slowly after a one-time shock so that the total number of job finders, λu, is constant during the transition to steady state. It follows that the unemployment rate is again a first order autoregressive process, but that the autoregressive parameter is only equal to the separation rate s. Equivalently, the unemployment rate adjust halfway to steady state in $\frac{\log 2}{s}$ periods, $\frac{1}{u^*}$ times as long as in the standard model. The model developed here therefore substantially amplifies and propagates a one-time shock to the economy.

A third success of the model is that it predicts a similar response to a variety of different shocks. Contrast this with the Mortensen and Pissarides (1994) model, which is qualitatively consistent with productivity shocks but not with changes in the exogenous separation rate. In that model, a reduction in separations directly reduces the unemployment rate but scarcely affects the ratio of vacancies to unemployment. This implies that if fluctuations in the separation rate are important at business cycle frequencies, as evidence from Davis, Haltiwanger, and Schuh (1996) and Blanchard and Diamond (1990) suggests, one should observe a positive correlation between unemployment and vacancies. Unfortunately, the correlation between these two variables is strongly negative (Abraham and Katz 1986, Blanchard and Diamond 1989, Shimer 2003). In the model developed in this paper, a reduction

[2]Cole and Rogerson (1999) propose that one should measure u^* as the nonemployment rate, a much larger number, thereby generating significant propagation in the Mortensen and Pissarides (1994) model. Hall (1995) and Pries (forthcoming) instead suggest that most new jobs end quickly, with the worker returning to the unemployment pool. They effectively measure s as the rate at which stable long-term relationships end, again yielding significant propagation.

in the exogenous separation rate raises the present value of a match, thereby inducing firms to create additional job openings. This raises the rate at which workers find jobs and reduces the unemployment rate, which implies a negative correlation between job vacancies and the unemployment rate, even in response to shocks to the job separation rate.

In the next section of this paper, I lay out the model of on-the-job search. Section 3 characterizes the steady state equilibrium of the model and performs comparative statics exercises, establishing the main amplification result. I then examine how the economy behaves following an unanticipated, permanent shock. In response to a positive shock, there is an instantaneous burst of job creation that discretely reduces the unemployment rate and the fraction of searchers who are unemployed. The economy thereafter adjusts slowly to its conditional steady state. In response to a negative shock, there is a brief period with no job creation, during which the unemployment rate and the fraction of searchers who are unemployed rises rapidly. The economy again quickly reverts to the slow adjustment pattern that generates significant propagation of shocks. Section 4 (incomplete) analyzes an economy subject to aggregate shocks, establishing that the amplification and propagation mechanism highlighted in the previous sections carries over to a stochastic environment. Section 5 comments further on the robustness of these results. I argue that one can endogenize search intensity with relatively modest effects on the results and I explain the extent to which the results carry over to the Burdett and Mortensen (1998) model, in which firms cannot condition wage offers on workers' employment status and cannot match outside wage offers.

2 Model

The economy consists of two types of agents, workers and firms. All agents are risk-neutral, infinitely-lived, and discount future payoffs at rate $r > 0$. Time is continuous and there is no aggregate uncertainty.

A worker may be either unemployed or employed. Workers produce output whether employed or unemployed, but market activity is more productive. In particular, each unemployed worker produces z units of output at home, while each employed worker produces $p > z$ units of output.

Each firm has access to a constant returns to scale production technology using only labor. A firm may hire an additional worker by paying a one-time fixed cost k. The new employee is drawn randomly from the searching population. Employed workers separate from their jobs for exogenous reasons at rate $s > 0$, which ensures the existence of an ergodic steady state equilibrium.

Because hiring a worker is costly, there are generally bilateral gains from existing matches. Standard competitive arguments equating a worker's wage w to her marginal product p therefore do not apply in this environment. In this paper, I borrow the wage setting procedure directly from Postel-Vinay and Robin (2002), who write:

> We make the following important four assumptions on wage strategies:
> (i) Firms can vary their wage offers according to the characteristics of the particular worker they meet.
> (ii) They can counter the offers received by their employees from competing firms.
> (iii) Firms make take-it-or-leave-it wage offers to workers.
> (iv) Wage contracts are long-term contracts that can be renegotiated by mutual agreement only. (p. 2302)

Postel-Vinay and Robin note that the first and second assumptions are different from the corresponding assumptions in Burdett and Mortensen (1998), but the third and fourth assumptions are the same.[3] In the present context, the first assumption implies that firms can observe whether a particular worker is employed or unemployed. If the worker is unemployed, the the third assumption implies that the firm can extract all of the surplus from the match. It does so by writing a long-term contract which is only renegotiated when the worker contacts another firm (the fourth assumption permits this), since otherwise it is not in the first employer's interest to renegotiate. When an employed worker contacts a firm, the second assumption allows the incumbent employer to match outside offers, which implies that the two potential employers bid up the worker's wage until the worker captures the entire surplus from the match. Again, this high wage is written into a long-term contract, which the worker never has an incentive to renegotiate.

To summarize, when an unemployed worker gets a job, the employer keeps all of the surplus from the match, so the worker is indifferent between employment and unemployment. After an employed worker gets a second job offer, the worker keeps all of the surplus from the match, leaving the firms indifferent about hiring the worker or letting her take the other job. The model does not tell us whether the worker switches jobs in this event.

I assume that two types of workers search, unemployed workers and employed workers who have not yet received a second job offer. The rate at which these workers find a job at

[3] See Mortensen (2003) for a thorough exposition of the Burdett and Mortensen model.

time t, $\lambda(t)$, is equal to the ratio of job vacancies to the stock of searching workers, and is determined endogenously in equilibrium.[4] After a worker receives a second job offer without an intervening spell of unemployment, she extracts all of the surplus from a match and so has no reason to search. The state of the economy at time t is defined by the measure of unemployed workers $u(t)$ and the measure of employed workers who are searching $e_0(t)$, with the measure of employed workers who have received two job offers equal to $1 - u(t) - e_0(t)$.

Finally, I impose a parameter restriction which ensures the existence of an interior steady state equilibrium:

Assumption 1 $\quad 1 > \dfrac{(r+s)k}{p-z} > \frac{1}{2}.$

3 Equilibrium

3.1 Bellman Equations

There is only one real economic decision in this model, whether to create a job vacancy. That determines the rate at which employed workers find jobs $\lambda(t)$. To characterize this, I write down a system of Bellman equations that describe the value of workers in different employment states and corresponding equations for jobs in different states. Let $U(t)$ denote the expected present value of income for an unemployed worker, $E_0(t)$ the corresponding value for a newly employed worker in her first job following an unemployment spell, and $E_1(t)$ the value for a worker who has received two job offers without an intervening spell of unemployment. These are related by three recursive equations. Start with the value of an unemployed worker:

$$U(t) = \int_t^\infty \big(z + \lambda(t')E_0(t')\big) e^{-\int_t^{t'} (r+\lambda(t''))dt''} dt' \tag{1}$$

We discount all future dates t' to account both for the rate of time preference and for the possibility that the worker has found a job during the intervening period. Her instantaneous payoff is the sum of her output from home production z and the instantaneous probability that she finds a job $\lambda(t')$ multiplied by the value of the first job $E_0(t')$.[5] Similarly, the value

[4] It is straightforward to extend the model to allow employed and unemployed workers to find jobs with different probabilities. This scarcely affects the results.

[5] Time-differentiating this equation yields the more familiar expression

$$rU(t) - \dot{U}(t) = z + \lambda(t)(E_0(t) - U(t)).$$

of a first job is

$$E_0(t) = \int_t^\infty \big(w(t') + sU(t') + \lambda(t')E_1(t')\big)e^{-\int_t^{t'}(r+s+\lambda(t''))dt''}dt' \tag{2}$$

Current payoffs come from wages $w(t')$ and the hazard of becoming either unemployed at rate s or getting another job offer at rate λ. Discounting accounts for impatience and the possibility of switching to either of the other states. Finally, the value of a second job is

$$E_1(t) = \int_t^\infty \big(p + sU(t')\big)e^{-\int_t^{t'}(r+s)dt''}dt, \tag{3}$$

with a similar interpretation.

Firms make unemployed workers a take-it-or-leave-it offer. This ensures that in equilibrium, workers are indifferent between unemployment and employment in a first job, $U(t) \equiv E_0(t)$ for all t. Substituting this into equation (1) and simplifying gives $U(t) = z/r$ for all t.[6] Equation (3) then implies a time-invariant value for $E_1(t)$:

$$E_1(t) = \frac{rp + sz}{r(r+s)}.$$

Finally, since $E_0(t) = U(t) = z/r$, equation (2) pins down the wage:[7]

$$w(t) = \frac{(r+s+\lambda(t))z - \lambda(t)p}{r+s}. \tag{4}$$

Turn next to the behavior of firms. When a firm contacts an employed worker, the worker extracts all of the surplus, and so the value of such a job is zero to the firm, $J_1(t) = 0$. Next

I write the value function in this nonrecursive form to be clear about the terminal condition, which is lost when taking the time derivative. Otherwise the two approaches are identical.

[6]Replace $E_0(t') = z/r$ in equation (1) to get

$$U(t) = \frac{z}{r}\int_t^\infty \big(r + \lambda(t')\big)e^{-\int_t^{t'}(r+\lambda(t''))dt''}dt'.$$

It is immediate that the integral is equal to 1. I use similar math throughout this paragraph.

[7]For plausible parameter values, the wage is negative. For example, if $r = 0$, the wage in steady state is negative whenever the unemployment rate u is smaller than $1 - z/p$, and so a five percent unemployment rate is consistent with a positive wage only if the value of home production is at least 95 percent of the value of market production.

let $J_0(t)$ denote the value of a job filled by a previously unemployed worker:

$$J_0(t) = \int_t^\infty \big(p - w(t')\big) e^{-\int_t^{t'} (r+s+\lambda(t''))dt''} dt.$$

The firm earns profit $p-w$ until either an exogenous separation ends the match or the worker contacts another firm. Substituting for w from equation (4) and simplifying gives

$$J_0(t) = \frac{p-z}{r+s}. \tag{5}$$

In other words, the value to a firm of contacting an unemployed worker is equal to the present value of the incremental output from market rather than non-market production, discounted to account both for impatience and exogenous separations, but not for the possibility that the worker contacts another firm. The first employer extracts all of the value of the worker's output during this employment spell, even if the worker later moves to another job.

Finally we can examine the free entry condition. When a firm contacts a worker, the worker is unemployed with probability $x(t) \equiv \frac{u(t)}{u(t)+e_0(t)}$, in which case the firm gets value $J_0(t)$ given in equation (5). Otherwise the worker is already employed, in which case the firm gets value 0 from the meeting. Jobs are created if the expected value of contacting a worker $x(t)J_0(t)$ is at least equal to the cost of contacting a worker k. Equivalently,

$$x(t) \begin{matrix} > \\ = \\ < \end{matrix} \frac{(r+s)k}{p-z} \Rightarrow \lambda(t) \begin{cases} = \infty \\ \text{free} \\ = 0 \end{cases} \tag{6}$$

Note that $x(t)$ is a state variable, and so condition (6) determines $\lambda(t)$ at each instant.

3.2 State Variables

The evolution of the state variables is easy to calculate given the $\lambda(t)$:

$$\begin{aligned} \dot{u}(t) &= s(1-u(t)) - \lambda(t)u(t) \\ \dot{e}_0(t) &= \lambda(t)u(t) - (s+\lambda(t))e_0(t). \end{aligned} \tag{7}$$

The stock of unemployed workers increases when the $1-u(t)$ employed workers suffer a separation and it decreases when the $u(t)$ unemployed workers find a job. The stock of employed workers getting the low wage increases when an unemployed workers finds a job and decreases due to separations and due to these workers getting a second wage offer. Combining these equations gives a differential equation for the fraction of searchers who are unemployed.

$$\dot{x}(t) = \frac{sx(t)(1 - x(t))}{u(t)} - \lambda(t)x(t)^2. \tag{8}$$

I am now in a position to define the equilibrium of this economy:

Definition 1 *A decentralized equilibrium is a triple $\{\lambda(t), u(t), x(t)\}$ for all t satisfying equations (6), (7), and (8). $\lambda(t)$ satisfies equation (6) given $x(t)$; $u(t)$ evolves according to equation (7) given $\lambda(t)$; and $x(t)$ follows equation (8) given $\lambda(t)$ and $u(t)$.*

3.3 Comparative Statics

In a steady state equilibrium, equations (6), (7), and (8) simplify slightly. If the job creation rate λ is at an interior value, condition (6) implies that the share of searchers who are unemployed satisfies

$$x^* = \frac{(r + s)k}{p - z}, \tag{9}$$

where the * indicates the steady state value of a variable. From Assumption 1, $x^* \in (\frac{1}{2}, 1)$.[8] Also, $\dot{u}(t) = \dot{x}(t) = 0$, and so equations (7) and (8) imply

$$u^* = 2 - \frac{1}{x^*}, \tag{10}$$

a number between zero and one. The preceding two equations explain the source of amplification in this model. Equation (9) implies that elasticity of x^* with respect to $p - z$ is -1, and so there is a modest response of the composition of the labor force to a change in the value of market productivity relative to non-market productivity. But equation (10) implies that the semi-elasticity of the unemployment rate with respect to labor market composition x^* is $2 - u^*$. In other words, a one percent decrease in $p - z$ causes a one percent increase in the steady state fraction of the searching population that is unemployed. This in turn results in nearly a two *percentage point* increase in the unemployment rate (since u^* is typically a small number), significantly amplifying the initial shock. We can also combine the

[8] If $(r + s)k \geq p - z$, $\lambda = 0$ and $u = x = 1$ in steady state. If $(r + s)k \leq \frac{1}{2}(p - z)$, $\lambda = \infty$, $u = 0$, and $x = \frac{1}{2}$ in steady state.

two equations to see this directly:

$$u^* = 2 - \frac{p - z}{(r + s)k}. \tag{11}$$

so

$$\frac{\partial u^*}{\partial \log(p - z)} = u^* - 2.$$

An increase in the separation rate s has a similarly large effect on the unemployment rate. For example, a one percent increase in the separation rate directly causes a $u^*(1-u^*)$ percentage point increase in the steady state unemployment rate, e.g. from 5.00 to 5.05 percent.[9] But this is amplified through the effect on job creation, which falls sharply when the separation rate increases. In total, the semi-elasticity of u^* with respect to s is $\frac{s(2-u^*)}{r+s} \approx 2$, so that a one percent increase in the separation rate raises the steady state unemployment rate from 5 to nearly 7 percent.

We can also study the behavior of other endogenous variables in steady state. Using equation (7) and (11), we get

$$\lambda^* = \frac{s\big((p - z) - (r + s)k\big)}{2(r + s)k - (p - z)},$$

which is a positive number under Assumption 1. An increase in market minus non-market productivity $p - z$ unambiguously raises the rate at which workers find jobs. A decrease in the cost of creating jobs k has a similar effect. The effect of the separation rate s is unclear, although as long as $r < s$, one can show that an increase in s reduces the rate at which workers find jobs.

One important variable that does not exhibit much amplification is the number of job openings (vacancies), equal to the flow of workers who contact firms, $\lambda^*(u^* + e_0^*)$. With a bit of algebra, we can rewrite this as $\lambda^*(u^*+e_0^*) = \lambda^* u^*/x^* = s(1-u^*)/x^*$ so, from equation (10),

$$v^* = s(1 - u^*)(2 - u^*). \tag{12}$$

An increase in market productivity or a decrease in the cost of contacting a worker reduces the unemployment rate u^* and therefore increases the stock of vacancies. This is consistent with the negative correlation between unemployment and vacancies observed in the data (Abraham and Katz 1986, Blanchard and Diamond 1989). The problem is quantitative. An

[9]This follows from the steady state equation $u^* = \frac{s}{s+\lambda^*}$, holding λ^* fixed.

increase in the unemployment rate from 5 to 6 percent reduces $(1-u^*)(2-u^*)$ from 1.85 to 1.82, which is scarcely discernible. In contrast, U.S. data indicate that the standard deviation of the deviation of log vacancies from trend is approximately the same as the corresponding standard deviation for unemployment (Shimer 2003); in other words, a shock that raises the unemployment rate from 5 to 6 percent should reduce vacancies by about 20 percent.

3.4 Deterministic Dynamics

This section considers the behavior of the economy in response to an unanticipated, permanent shock that changes the steady state fraction of the searching population that is unemployed from x_0 to x^*. According to equation (9), this includes a shock to productivity of market activity p, the productivity of non-market activity z, the exogenous separation rate s, and the cost of contacting a worker k.

I begin by examining the behavior of the economy when the unemployment rate is away from steady state, $u(t) \neq u^*$, but the share of unemployed in the searching population is constant at its steady state value: for all $t' \geq t$, $x(t') = x^*$. The main result is that the unemployment rate adjusts slowly and monotonically to its steady state value, following a first order autoregressive process with autoregressive parameter equal to the separation rate s. I then examine what happens if initially $x(t) \neq x^*$, and find that the variable rapidly — in some cases, instantaneously — adjusts to its steady state value, where it remains forever.

So to begin, suppose that for all $t' \geq t$, $x(t') = x^*$, defined in equation (9), although the unemployment rate is not necessarily at its steady state value. The free entry condition (6) does not restrict the behavior of job creation, since firms are always indifferent about contacting another worker when $x(t) = x^*$. Instead, the requirement that $\dot{x}(t) = 0$ pins down the amount of job creation. Equation (8) implies that when $x(t)$ is constant at x^*,

$$\lambda(t)u(t) = \frac{s(1-x^*)}{x^*}. \tag{13}$$

Substituting this into equation (7) gives a dynamic equation for the unemployment rate in terms of model parameters:

$$\dot{u}(t) = s\left(u^* - u(t)\right),$$

where u^* is defined in equation (11). Equivalently, if $u(0) = u_0$ and $x(t') = x^*$ for all $t \in [0, t]$, then

$$u(t) = e^{-st}u_0 + \left(1 - e^{-st}\right)u^*.$$

The unemployment rate converges monotonically towards its steady state value following a first order autoregressive process with autoregressive parameter equal to the separation rate s. This is a relatively slow speed of adjustment; for example, if about half of all jobs end each year, $s = 0.5$, the unemployment rate moves halfway from u_0 to u^* in $\frac{\log 2}{s} \approx 1.4$ years. In a standard model without on-the-job search, the speed of adjustment is more than an order of magnitude faster (Hall 1995, Pries forthcoming).

The next step is to show that if initially $x(0) \equiv x_0 \gtrless x^*$, market forces rapidly or even instantaneously drive this variable towards its steady state value. I handle the two cases separately, starting with $x_0 < x^*$. Equation (6) implies that as long as $x(t) < x^*$, $\lambda(t) = 0$, and so equations (7) and (8) reduce to

$$\dot{u}(t) = s(1 - u(t)) \text{ and } \dot{x}(t) = \frac{sx(t)(1 - x(t))}{u(t)}.$$

Solving these differential equations forward from date 0 gives

$$u(t) = u_0 e^{-st} + \left(1 - e^{-st}\right) \text{ and } x(t) = \frac{x_0\left(u_0 e^{-st} + \left(1 - e^{-st}\right)\right)}{u_0 e^{-st} + x_0\left(1 - e^{-st}\right)}.$$

Asymptotically, $x(t) \to 1$, while Assumption 1 implies $x^* < 1$. This means that at a finite time t^*, $x(t^*) = x^*$. By then, the unemployment rate will have risen to

$$u(t^*) = \frac{u_0(1 - x_0)x^*}{u_0(x^* - x_0) + x_0(1 - x^*)}.$$

In particular, assuming the economy starts at time 0 in steady state with one set of parameter values, $x_0 = \frac{1}{2-u_0}$. Then an unanticipated shock at time 0 that raises the steady state value of x to $x^* > x_0$ results in no job creation until period t^*, at which time the unemployment rate has risen to $u(t^*)$:

$$t^* = \frac{1}{s} \log\left(\frac{(1 - u_0)(1 + u_0 - u^*)}{1 - u^*}\right) \text{ and } u(t^*) = \frac{u_0}{1 + u_0 - u^*}.$$

Quantitatively, this initial adjustment period is short and, although the unemployment rate increases rapidly in the absence of job creation, the total effect on the unemployment rate is small. For example, a shock that raises the steady state unemployment rate from 5 to 6 percent leads to 0.001 years (less than 9 hours) without job creation (assuming $s = 0.5$), during which time the unemployment rate increases to 5.05 percent.[10]

Conversely, suppose $x_0 > x^*$. Now equation (6) implies that $\lambda(0) = \infty$, i.e. there is a sudden burst of job creation. To see what effect this has, suppose there is a (small) measure dj of jobs created. A fraction x will go to unemployed workers (who become employed) and the remaining $1-x$ to employed workers. This implies $du = -xdj$ and $de_0 = xdj-(1-x)dj = (2x-1)dj$. The change in the fraction of searchers who are unemployed is

$$dx = \frac{x}{u}\big((1-x)du - xde_0\big) = -\frac{x^3}{u}dj.$$

One can confirm that for any initial conditions u_0 and $x_0 > x^*$, the u and x are subsequently related by a simple expression:[11]

$$u = u_0 e^{x_0^{-1}-x^{-1}}.$$

In particular, since the burst of job creation stops when the share of unemployed in the searching population reaches x^*, the unemployment rate drops instantaneously from u_0 to

$$u_{0+} = u_0 e^{x_0^{-1}-x^{*-1}}.$$

If we again assume that the economy is initially in steady state with one set of parameter values, $x_0 = \frac{1}{2-u_0}$, before it is hit by an unanticipated shock at time 0, this reduces to

$$u_{0+} = u_0 e^{u^*-u_0}.$$

A shock that reduces the steady state unemployment rate from 6 to 5 percent results in a sudden burst of job creation that instantaneously reduces the unemployment rate to 5.94 percent.[12]

To summarize, market forces rapidly push the fraction of unemployed workers in the searching population towards its steady state value x^*. The same forces keep x^* at this value once it is obtained. Although the unemployment rate initially adjusts rapidly, for most of the adjustment process it follows a first order autoregressive process with parameter equal to the exogenous separation rate s.

[10]Define $\frac{u(t^*)-u_0}{u^*-u_0}$ to be the fraction of the adjustment in the unemployment rate that occurs during this initial period without job creation. By linearizing the equation for $u(t^*)$ around $u^* = u_0$, one can show that for a small shock, this is equal to u_0, the initial steady state unemployment rate.

[11]To verify this, note that $x = x_0$ implies $u = u_0$ and that $du = ux^{-2}dx = -xdj$.

[12]Linearizing the preceding equation around $u^* = u_0$ shows that, once again, approximately a fraction u_0 of the eventual adjustment in the unemployment rate from u_0 to u^* occurs instantaneously.

The adjustment process is slow because the total number of unemployed workers who find jobs $\lambda(t)u(t)$ is constant even when the economy is away from steady state (equation 13), except during the brief periods when $x(t)$ is away from its steady state value. Moreover, combining the definition of the steady state unemployment rate u^* with equation (13) implies

$$\lambda(t)u(t) = s(1 - u^*), \tag{14}$$

so a shock that raises the steady state unemployment rate slightly reduces the overall number of unemployed workers who find jobs. This is consistent with evidence from Blanchard and Diamond (1990) that the total flow of workers into employment is only very mildly procyclical (although the rate at which unemployed workers find jobs λ is strongly procyclical).

Once again, a significant gap between the model and U.S. data is the behavior of the vacancy rate. It straightforward to show that following the initial adjustment period that drives $x(t)$ to x^*, $v(t) = s(1-u^*)(2-u^*)$.[13] In other words, equation (12) holds both in and out of steady state, and so following a shock the vacancy rate jumps rapidly or immediately to its steady state value. This implies that vacancies rate do not exhibit the same persistence as the unemployment rate. By contrast, U.S. data show that vacancies and unemployment have approximately the same first order autocorrelation (Shimer 2003).

4 Stochastic Model

INCOMPLETE

5 Discussion

5.1 Endogenous Search Intensity

It is important for the amplification mechanism presented in this paper that unemployed workers search for a job while workers who are paid their full marginal product do not. If unemployed workers did not search for a job, everyone would by unemployed in steady state. If all employed and unemployed workers searched for jobs, then a fraction $u(t)$, rather than $x(t)$, of searchers would be unemployed, eliminating the key amplification and propagation

[13] To prove this, observe $v(t) = \lambda(t)(u(t) + e_0(t)) = \frac{\lambda(t)u(t)}{x^*} = s(1-u^*)(2-u^*)$, where the first equality uses the definition of the vacancy rate, the second uses the definition of $x(t) = x^*$, and the third uses equations (10) and (14).

mechanism from the model. I argued that it was reasonable that employed workers who are already paid their full marginal product should not search, since they have nothing to gain. But what about unemployed workers? In the equilibrium of this model, firms extract all of the profits from a meeting with an unemployed worker, so $U(t) = E_0(t)$. This implies that unemployed workers also have nothing to gain from searching. It may therefore be unclear whether this model could survive the introduction of endogenous search intensity.

There is a simple resolution of this quandary: endogenous search intensity may eliminate the equilibrium condition that $U(t) = E_0(t)$. To see why, suppose that workers can choose how hard to search, with greater search intensity imposing a more-than-proportional disutility cost. Also assume that firms cannot monitor worker's search intensity, although they continue to observe when a worker contacts another firm. Firms still make unilateral offers to unemployed workers, while employed workers who contact a second firm earn their full marginal product. Together these assumptions mean that firms might not want to impose the lowest acceptable wage on unemployed workers. By paying a higher wage, the firm might reduce the worker's search intensity, and therefore length the interval before the worker contacts another firm. In particular, a firm may choose to pay a high enough wage so that $E_0(t) > U(t)$, so unemployed workers have an incentive to search for a job. They do this not in order to encourage search by unemployed workers but rather to discourage search by their employees.

Develop this intuition more fully requires a careful analysis. Here I focus on steady states. Assume that a worker who searches with intensity $\sigma \geq 0$ contacts a firm at rate $\sigma\lambda$ but pays a flow utility cost $c(\sigma)$, increasing and convex with $c(0) = c'(0) = 0$. Let $E_0(w)$ denote the expected present value of income for an employed worker earning a wage w and let $\sigma(w)$ denote her choice of search intensity. These must satisfy

$$rE_0(w) = w - c(\sigma(w)) + s(U - E_0(w)) + \sigma(w)\lambda(E_1 - E_0(w))$$
$$\sigma(w) = \arg\max_{\sigma} \frac{w - c(\sigma) + sU + \sigma\lambda E_1}{r + s + \sigma\lambda}$$

In addition, the value of a worker earning her full marginal product satisfies the standard equation:

$$rE_1 = p + s(U - E_1).$$

Eliminating E_1 from the equation for preceding equations gives

$$E_0(w) = \frac{w - c(\sigma(w)) + \frac{\sigma(w)\lambda p}{r+s}}{r+s+\sigma(w)\lambda} + \frac{sU}{r+s} \tag{15}$$
$$\text{and } \sigma(w) = \arg\max_{\sigma} \frac{w - c(\sigma) + \frac{\sigma\lambda p}{r+s}}{r+s+\sigma\lambda} + \frac{sU}{r+s}.$$

This implies that the search intensity of employed workers, $\sigma(w)$, is independent of the value of unemployment, U. Since c is convex, the necessary and sufficient first order condition for a solution to this optimization problem is

$$p - w = \frac{r+s+\sigma(w)\lambda}{\lambda} c'(\sigma(w)) - c(\sigma(w)), \tag{16}$$

which implicitly defines σ as increasing function of w.

Next consider a firm's decision on the wage to give to an a previously-unemployed worker. The firm knows that the worker will choose her search intensity according to equation (16). This implies that the expected profit from offering a wage of w is

$$rJ_0 = \max_w \big(p - w - (s + \sigma(w)\lambda)J_0\big).$$

The firm earns profit $p-w$ until the match ends exogenously or the worker contacts another firm. Substituting for $p-w$ from equation (16), we can think of the firm choosing the worker's search intensity to solve

$$J_0 = \max_{\sigma} \left(\frac{c'(\sigma)}{\lambda} - \frac{c(\sigma)}{r+s+\sigma\lambda} \right), \tag{17}$$

Let σ^* denote this choice of search intensity. Since $c(0) = c'(0) = 0$, one can confirm directly that $\sigma^* > 0$. The actual wage choice is defined by (16).[14]

Turn next to the free entry condition $k = xJ_0$, where x is the fraction of searchers who are unemployed. Now the expression for x is slightly more complicated, reflecting the different

[14]This is predicated on an unemployed worker being willing to accept the job, $E_0(w) \geq U$. I show below that this is the case if

$$\frac{p-z}{r+s} > \frac{c'(\sigma^*)}{\lambda}.$$

If this condition fails, unemployed workers do not search and so the steady state unemployment rate is 1.

search intensity of employed and unemployed workers, σ^* and σ_u, respectively:

$$x = \frac{\sigma_u u}{\sigma_u u + \sigma^* e_0}.$$

Eliminate e_0 using the steady state equation for the stock of workers in initial matches,

$$\sigma_u \lambda u = (s + \sigma^* \lambda) e_0,$$

to get

$$x = \frac{s + \lambda \sigma^*}{s + 2\lambda \sigma^*}.$$

That is, the fraction of searchers who are unemployed depends on contact rate λ and the search intensity of employed workers, but not on the search intensity of unemployed workers. This implies the free entry condition determines λ for a given value of σ^*,

$$k = \frac{s + \lambda \sigma^*}{s + 2\lambda \sigma^*} \left(\frac{c'(\sigma^*)}{\lambda} - \frac{c(\sigma^*)}{r + s + \sigma^* \lambda} \right), \tag{18}$$

while σ^* is determined from the first order condition in equation (17). It is notable that productivity enters neither equation. This implies that both σ^* and λ^* are independent of productivity, where λ^* is the equilibrium contact rate.

Productivity does, however, affect the search intensity of unemployed workers. Substituting equation (16) into (15) gives

$$E_0 - U = \frac{p - rU}{r + s} - \frac{c'(\sigma^*)}{\lambda^*}.$$

Since unemployed workers choose their search intensity σ_u to solve

$$rU = \max_{\sigma_u} \big(z + \sigma_u \lambda^* (E_0 - U) - c(\sigma_u) \big),$$

search intensity must solve the necessary and sufficient first order condition

$$\lambda^* \left(\frac{p - rU}{r + s} - \frac{c'(\sigma^*)}{\lambda^*} \right) = c'(\sigma_u).$$

Substituting that back into unemployed workers' Bellman equation gives

$$rU = z + \sigma_u c'(\sigma_u) - c(\sigma_u),$$

which can in turn be used to simplify the first order condition:

$$\frac{\sigma_u c'(\sigma_u) - c(\sigma_u)}{r+s} + \frac{c'(\sigma_u)}{\lambda^*} = \frac{p-z}{r+s} - \frac{c'(\sigma^*)}{\lambda^*} \tag{19}$$

Since the left hand side is increasing in σ_u, this implicitly defines σ_u as an increasing function of p. And since $c(0) = c'(0) = 0$, unemployed workers search with positive intensity assuming the right hand side of the previous equation is positive. Finally, the steady state unemployment rate depends on productivity exclusively through the impact of productivity on unemployed workers' search intensity:

$$u^* = \frac{s}{s + \sigma_u \lambda^*}.$$

By manipulating the curvature of the cost function, one can easily change the responsiveness of the unemployment rate to productivity.

To summarize, an interior equilibrium is a triple $\{\lambda^*, \sigma^*, \sigma_u\}$ where firms choose σ^* to maximize the profit from employing a previously unemployed worker, given by equation (17), λ^* and σ^* satisfy the free entry condition (18), and unemployed workers optimally choose their search intensity σ_u to satisfy equation (19) given λ^* and σ^*.

Although the model behaves qualitatively similar when search intensity is endogenous, this robustness check is not a complete success. First, the channel highlighted in the main part of the paper is no longer present when search intensity is endogenous. A change in productivity no longer affects the equilibrium share of the searching population that is unemployed, but instead exclusively changes search intensity. This means the amplification mechanism is quite different. Second, the propagation mechanism disappears from the model. A productivity shock does not affect the contact efficiency λ^*, but instead leads to a jump in workers' search intensity. The unemployment rate therefore rapidly converges to its new steady state value, as in the standard Mortensen and Pissarides (1994) model.

5.2 The Burdett-Mortensen Model

The Burdett and Mortensen (1998) model is similar to the model in the main part of the paper, except that firms must make wage offers without knowing a worker's wage or employment status and cannot match outside offers. Assuming all workers, employed or unemployed, search with equal efficiency, there is a continuous wage distribution in equilibrium, with support between the value of leisure z and an upper bound $\bar{w} < p$. A firm that offers the lowest wage z earns profit

$$\frac{u(p-z)}{r+s+\lambda}$$

from each contact. It hires a worker only if that worker is unemployed, with probability u. The worker then remains with the firm, producing profit $p-z$, until a separation occurs either exogenously (rate s) or endogenously (rate λ of meeting another, higher wage, firm). Let k denote the cost of contacting a worker. Assuming $r=0$, the steady state equation for unemployment $u=\frac{s}{s+\lambda}$ implies free entry drives the unemployment rate to

$$u=\sqrt{\frac{sk}{p-z}}.$$

That is, the steady state unemployment rate is not very responsive to labor productivity.[15] What about out of steady state? Even if wages in existing jobs do not change when a productivity shock hits, I find that the Burdett and Mortensen (1998) economy transits very rapidly to its steady state unemployment rate.[16] Thus even paying attention to out-of-steady state dynamics, the unemployment rate scarcely responds to labor productivity in this model.

It seems that the different results from the Burdett and Mortensen (1998) model and the models discussed in this paper is largely due to the treatment of search intensity. In the Burdett and Mortensen (1998) model, search intensity is exogenous and the same for all workers. This eliminates the amplification and propagation mechanism from the model in Sections 2–4, in which most employed workers do not search. It similarly eliminates the amplification mechanism from the model in section 5.1, which is based on endogenous search intensity. Mortensen (2003) discusses a version of the Burdett and Mortensen (1998) with on-the-job search. It remains to be seen whether this model exhibits a significant amplification mechanism or interesting out-of-steady state dynamics.

References

Abraham, Katharine, and Lawrence Katz (1986): "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?," *Journal of Political Economy*, 94(3), 507–522.

[15] If $r>0$, the expression for the steady state unemployment rate is slightly more complicated but qualitatively unchanged.

[16] Details of the non-stationary behavior of this model are available upon request.

Blanchard, Olivier, and Peter Diamond (1989): "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1, 1–60.

——— (1990): "The Cyclical Behavior of the Gross Flows of U.S. Workers," *Brookings Papers on Economic Activity*, 2(85–143).

Burdett, Kenneth, and Dale Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39(2), 257–273.

Cole, Hal, and Richard Rogerson (1999): "Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?," *International Economic Review*, 40(4), 933–959.

Davis, Steven, John Haltiwanger, and Scott Schuh (1996): *Job Creation and Destruction*. MIT Press.

Hall, Robert (1995): "Lost Jobs," *Brookings Papers on Economic Activity*, 1, 221–256.

——— (2003): "Wage Determination and Employment Fluctuations," Mimeo.

Mortensen, Dale (2003): *Wage Dispersion: Why Are Similar Workers Paid Differently?* MIT Press.

Mortensen, Dale, and Christopher Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61(3), 397–415.

Petrongolo, Barbara, and Christopher Pissarides (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39(2), 390–431.

Pissarides, Christopher (1985): "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *The American Economic Review*, 75(4), 676–690.

Postel-Vinay, Fabien, and Jean-Marc Robin (2002): "Equilibrium Wage Dipspersion with Worker and Employer Heterogeneity," *Econometrica*, 70(6), 2295–2350.

Pries, Michael (forthcoming): "Persistence of Employment Fluctuations: A Model of Recurring Job Loss," *Review of Economic Studies*.

Shimer, Robert (2003): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies: Evidence and Theory," NBER Working Paper 9536.