An approach to jointly estimating an endogenous marginal value of time function and a recreation budget share equation is developed. The specification requirements for the marginal value of time function, to ensure consistency with the maintained hypothesis of two binding constraints on choice, are articulated. The estimating model consistent with these requirements that nests several conventional treatments of the marginal value of time is highly significant. The general endogenous marginal value of time model outperforms other approaches and shows a more complex relationship between the marginal value of time and the individual's wage than has been used in previous work. (JEL J22, Q26)

I. INTRODUCTION

At least since the work of Becker (1965), it has been recognized widely that time may play an important role in consumer demand. One of the areas where this may be most significant empirically is in the valuation of natural resource amenities through their associated recreation demands. A principal reason is that recreation is a time-intensive commodity, so that the value of the time spent in recreation would be expected to play a large role in its overall value. The potential bias to consumer's surplus estimates of the net economic value of recreation from excluding time "prices" from the demand model has been recognized from the outset, for example by Knetsch (1963) and Clawson (1959).

To incorporate time into recreation demand models, researchers have generally adopted the practice suggested by McConnell (1975) of defining the "full price" of recreation as the money cost of a trip plus its monetized time cost, though the use of "full" budgets also called for by the Becker approach is less widespread. Bockstael et al. (1987) pointed out quite clearly that both full budgets and full prices are required in the demand function if the recreationist is jointly choosing labor supply with recreation at an exogenous marginal wage, but that the structure of demand was unclear if the individual was not making such marginal labor supply choices (i.e., if he or she was working fixed hours). Subsequently, Larson and Shaikh (2001) showed that the full prices–full budgets specification is sufficient to satisfy the constraints on demand parameters that arise because choice is made subject to two binding constraints, regardless of the individual's position in the labor market. They also pointed out that models that use full prices but only money income (in effect ignoring the role of time as a resource constraint) cannot be consistent with the requirements of choice subject to two constraints.

Although the outlines of how time should enter the structure of demand are becoming clear, a major unresolved issue is how to determine its value in practice. Several studies have followed the basic logic of Becker's early work, assuming that the opportunity cost of recreation time (i.e., the value of other time forgone in favor of recreation) is an exogenous parameter, such as the average wage rate or, more commonly, some fraction thereof. This fraction is either chosen arbitrarily or estimated as part of the recreation demand model, as in Cesario (1976), McConnell and Strand (1981), and Smith et al. (1983).

Such assumptions may be erroneous for a variety of reasons. Assuming the average wage is the appropriate opportunity cost of time
presumes that the individual faces no con-
straints on hours worked, derives no utility
or disutility from work, and has a linear
wage function, as has been noted by Chiswick
(1967), Bockstael et al. (1987), and Smith et al.
(1983). This is unlikely to be true for many
people. Assuming some constant fraction of
the wage is the opportunity cost of time, espe-
cially an arbitrarily chosen fraction, also seems
likely to be incorrect for most people. For all of
these reasons, an individual's average wage
does not necessarily reveal anything about
the shadow value of discretionary leisure
time, either as an upper or lower bound.

In a recent advance, Feather and Shaw
(2000) adapted the Heckman (1974) labor sup-
ply model to include information supplied by
recreationists about their labor supply status,
in particular whether they felt under- or over-
employed relative to their desired number of
hours. This information is used to identify sub-
groups in the data for which the shadow value of
leisure time is less than the wage or more than the
wage, which helps in the estimation of both a
wage function and a shadow value of leisure
time function. This approach produces indivi-
dual-specific estimates of the opportunity cost
of leisure time, based on their demographics
and labor market decisions, which are used in
a subsequent recreation demand analysis.

The Feather-Shaw work is a substantial
improvement in assessing the shadow value of
time by allowing it to diverge from the indi-
gual's wage rate in predictable ways. Like
most of the other literature, the Feather-
Shaw marginal value of time is determined
outside the recreation demand model; that is,
it is identified in an auxiliary relationship
(in their case, the individual's labor supply
choice) and is used as a predetermined value
in the recreation demand model. The marginal
value of time is not, then, responsive to changes
in the parameters of the recreation choice
problem, nor is it necessarily utility-consistent
with the recreation demand function.

This article takes a different approach to
incorporating the value of time into recreation
demand models by estimating a latent, endo-
genous marginal value of time function jointly
with recreation demand. In this approach, the
consumer's observed choices, made in response
to the constraints and prices he or she faces,
reveal the marginal value of time consistent
with those choices. The marginal value of
time function is internally consistent with the
demand model in that its functional form
is consistent with the homogeneity and other
curvature properties required for models of
consumer choice subject to two binding con-
straints (in this case, a money constraint and a
time constraint).

The article makes three contributions to the
recreation demand literature. First, it is the
first to estimate an endogenous marginal
value of time jointly with observable behavior
(in this case, recreation demands) in a utility-
consistent framework. To this end, the curva-
ture properties required for the marginal value
of time function to be consistent with consumer
choice subject to two constraints (on time and
on money) are identified, and an empirical
specification consistent with these properties
is developed. Second, the marginal value of
time function is specified so that it nests the
other empirical approaches used in the recrea-
tion demand literature, which allows for tests
of which approaches best explain the data. In
particular, it permits (but does not require) a
connection between the marginal value of time
and the individual's wage.

Finally, the accompanying demand model
that embeds the marginal value of time func-
tion is, to our knowledge, the first application
of the popular almost ideal demand system
(AIDS) model of Deaton and Muellbauer
(1980) to the case of choice subject to two bind-
ing constraints. We show how the functional
form of this demand model can be derived by
tracing through the dual relationships between
the two expenditure functions and the indirect
utility function that hold for the two-constraint
choice model.

The empirical setting for the demand-
marginal value of time model is California
whale watching, an increasingly popular activ-
ity that takes place along Western shores
during the months of December-April. Boats
in ports along the California coast offer day
trips of varying lengths that afford an oppor-
tunity for close-up viewing of the gray whale
migration between the Bering Sea off Alaska
and the Gulf of California in Mexico. In addi-
tion, one can often glimpse the migration from
many points on headlands along the coast.

II. BACKGROUND: THE TWO-CONSTRAINT
RECREATION CHOICE MODEL

The two-constraint model used to moti-
vate the analysis has its origins in the work
of Becker (1965) and DeSerpa (1971) and was used to analyze recreation choices by Bockstael et al. (1987). It can be interpreted, with minor modifications, for either the case where labor supply is predetermined relative to recreation choices or joint with them. The basic presentation is for predetermined labor supply, because many recreation choices are made conditional on (not jointly with) the individual's labor supply decision. The modifications needed to incorporate joint labor supply are then briefly noted.

When labor supply is predetermined, recreation demands are part of the second stage of a two-stage budgeting process, which allocates the money and time resulting from the first-stage labor supply decision to all consumption activities. The money-budget $M$ and time-budget $T$ applicable to the recreation demand choice are therefore fixed.

Let $u(x, s)$ be the individual's direct utility function, with $x$ an $n$-vector of consumption goods with corresponding money-prices $p$ and time-prices $t$. Second-stage recreation choices are made subject to strictly binding constraints on money-budget $M - px$ and time-budget $T - tx$.

Attention is confined to the case of two strictly binding constraints, which is not very restrictive for analyzing outdoor recreation, a time-intensive good whose time requirements have long been recognized to have an opportunity cost. For time to have a value or opportunity cost, the constraint must be strictly binding. Additionally, nonsatiation is a sufficient condition for the money constraint to bind. One could introduce cases in which one or both constraints go slack, though neither is very compelling for the analysis of recreation demand. It is not clear at least one numeraire good with respect to each constraint is sufficient for both constraints to bind continuously. The individual's utility is also influenced by an exogenous vector of individual and site characteristics $s$, which may include one or more unpriced quality variables.

The primal version of the problem leads to the indirect utility function $V(p, t, s, M, T)$, defined as

$$V(p, t, s, M, T) = \max_{x} u(x, s + \lambda \cdot \{M - px\} + \mu \cdot \{T - h - tx\}).$$

The solution to (1) yields the Marshallian demands $x_{i}(p, t, s, M, T), i = 1, \ldots, n$, along with the shadow values $\lambda(p, t, s, M, T)$ and $\mu(p, t, s, M, T)$ that are of particular interest. They are the marginal utilities of the money-budget and time-budget, respectively, whose ratio, $\mu/\lambda \equiv V_{T}/V_{M}$, is the marginal money value of time, as noted by McConnell (1975). Because $\mu$ and $\lambda$ are functions of the observable arguments $(p, t, s, M, T)$, their ratio is termed the Marshallian value of time, denoted by $\rho(p, t, s, M, T)$.

The presence of an additional binding (time) constraint implies additional structure on the consumer choice problem, as there are two versions of Roy's identity for this model. From the envelope theorem applied to (1),

$$V_{p} = -\lambda x_{i}, \quad V_{t} = -\mu x_{i}, \quad V_{M} = \lambda, \quad V_{T} = \mu,$$

so that for all goods in the estimated incomplete demand system one can write

$$x_{j}(p, t, s, M, T) \equiv -V_{p}/V_{M}$$

for $j = 1, \ldots, n$.

These two Roy's identities are a source of parameter restrictions in the empirical demand system and prove useful for specification and identification of the marginal value of leisure time from demand system coefficients. Larson and Shaikh (2001) show that even when $\rho = \rho(p, t, s, M, T)$ is an endogenous function of all parameters, these restrictions are satisfied by systems of demands that are functions of "full" prices $p_{j} + \rho \cdot t_{j}$, for $j = 1, \ldots, n$, and

1. If the time constraint goes slack, the money value of time is zero and the standard single-constraint consumer problem results. If the money constraint goes slack, the value of time goes infinite and a standard consumer choice problem of allocating time results. Moyer (1984) provides an example of the latter choice problem applied to outdoor recreation.

2. Examples are goods, such as walks on the beach (requiring time but no money), or making charitable contributions (requiring money but no—or little—time). LaFrance and Hanemann (1989) discuss the construction of numeraire goods for incomplete demand systems.

3. To consider labor supply choice joint with goods demands, define an element of $x$, say $x_{1}$, as hours worked, with a money price $p_{1} = w$, where $w$ is the wage, and $t_{1} = 1$ (Bockstael et al. 1987).

4. Subscripts that are parameters represent partial derivatives; for example, $V_{T} = \partial V/\partial T$. 


"full" budgets $M + \rho \cdot T,$

$\begin{align*}
(3) \quad x_i(p, t, s, M, T) &= x_i'(p + \rho \cdot t, s, M + \rho \cdot T), \\
& \quad i = 1, \ldots, n.
\end{align*}$

A result they derive, that is useful later, is that because the 'full-budget' Marshallian demands are homogeneous of degree zero in $(t, T),$ the relationship between the time and money-budget slopes of (3) is

$\begin{align*}
(4) \quad \partial x_i(p, t, s, M, T)/\partial T &= \rho(p, t, s, M, T) \\
& \quad \cdot \partial x_i(p, t, s, M, T)/\partial M.
\end{align*}$

Equation (3) can be viewed as a structural form for the empirical recreation demand function that satisfies the two-constraint choice requirements. By specifying a form for $\rho(p, t, s, M, T)$ that is also consistent with the two-constraint choice requirements, one can directly estimate the endogenous marginal value of time function jointly with the parameters of demand. However, the properties of $\rho(p, t, s, M, T)$ have not yet been articulated. Knowledge of these properties is needed to ensure that empirical specifications of $\rho(p, t, s, M, T)$ are internally consistent with Marshallian demands and that both are utility-theoretic.

To develop the properties of the marginal value of time, it is necessary to consider the dual structure of the two-constraint model. Because choice is subject to two constraints, the model has two dual expenditure functions, one minimizing money expenditure subject to utility and time constraints, the other minimizing time expenditure subject to utility and money constraints. The dual money expenditure function $e(p, t, s, T, u)$ is

$\begin{align*}
(5) \quad e(p, t, s, T, u) &= \min_p px + \phi \cdot [u - u(x, s)] \\
& \quad + \psi \cdot \{T - h - tx\}
\end{align*}$

where $\phi \equiv e_u$ is the marginal money cost of utility and, in this problem, $-\psi \equiv -e_T(p, t, s, T, u)$

5. For demand systems, the requirements are somewhat more stringent. The full-budget term must enter multiplicatively and have the same budget slope across all demands, and the (Marshallian) cross-full price effects must be symmetric (Larson and Shaikh 2001).

is the marginal value of additional time. The solution to (5) yields a set of money-compensated Hicksian demands, $x^h(p, t, s, T, u),$ that vary money income to keep utility constant given fixed time-budget $T.$

The dual time expenditure function is defined as

$\begin{align*}
(6) \quad \xi(p, t, s, M, u) &= \min_x tx + \phi \cdot [u - u(x, s)] \\
& \quad + \theta \cdot \{T - h - tx\}
\end{align*}$

and in this problem $\phi \equiv \xi_u$ is the marginal time cost of utility and $-1/\theta = -1/\xi_M(p, t, s, M, u)$ is the marginal value of additional time. The solution to (6) is the set of time-compensated Hicksian demands, $x^h_t(p, t, s, M, u),$ that vary time expenditure to maintain constant utility given fixed money-budget $M.$

Summarizing the different marginal value of time relationships in terms of the slopes of the objective functions instead of the shadow values, the marginal value of time is

$\begin{align*}
(7) \quad \rho(p, t, s, M, T) &= -e_T(p, t, s, T, u) \\
& \quad = -1/\xi_M(p, t, s, M, u).
\end{align*}$

This equivalence of the dual expressions for the marginal value of time at the optimum is useful for analyzing the comparative statics of the marginal value of time function estimated as part of the recreation demand model.

These dual expenditure functions have been defined and partially described for two-constraint models by Smith (1986) but have not (to our knowledge) been used to analyze the comparative statics of the marginal value of time, nor for specifying utility-theoretic empirical two-constraint demand functions. These topics are taken up in the next two sections.

6. This can be verified by noting the identity relating the indirect utility function in (1) to the money expenditure function in (5), $V(p, t, s, e[p, t, s, T, u], T) \equiv u.$ Partial differentiation with respect to $T$ yields $V_M \cdot e_T + V_T = 0,$ or $-e_T = V_M/V_T.$

7. Here the identity is $V(p, t, s, M, \xi[p, t, s, M, u]) \equiv u,$ and partial differentiation with respect to $M$ and rearranging yields the result.

8. In equation (7), each expression for the marginal value of time has a different set of arguments because each objective function holds a different set of parameters constant. At the optimum, of course, they all are equal.
III. COMPARATIVE STATICS OF THE MARGINAL VALUE OF TIME

What, in fact, should one expect to be the “correct” signs on the derivatives of \( p(p, t, s, M, T) \)? This section develops comparative statics of the marginal value of time function and shows that under the full prices, full budgets specification of demands, it must satisfy analogs to Roy’s identity: the ratios of price to budget slopes of the Marshallian marginal value of time function equal the negative of corresponding quantity consumed.

By letting utility vary in equation (7), the two identities

(8) \( \rho(p, t, s, M, T) = -e_T(p, t, s, T, V[p, t, s, M, T]) \)

and

(9) \( \rho(p, t, s, M, T) = -1/\xi_M(p, t, s, M, V[p, t, s, M, T]) \)

result. These are useful in evaluating the comparative statics of the marginal value of time.

Relating Money-Price and Money-Budget Slopes of the Marginal Value of Time

Consider first the money-budget and price slopes of the marginal value of time. Differentiating (8) with respect to \( M \) and \( p_{j}, j = 1, \ldots, n \), one obtains

(10) \( \partial p(p, t, s, M, T)/\partial M = -e_{Tu}(p, t, s, T, u) \cdot V_M(p, t, s, M, T) \)

and

(11) \( \partial p(p, t, s, M, T) / \partial p_j = -e_{Tr_j}(p, t, s, T, u) - e_{Tu}(p, t, s, T, u) \cdot V_{p_j}(p, t, s, M, T) \).

The cross-partial derivative \( e_{Tr_j} \) of the money expenditure function with respect to money-price \( p_j \) and time-budget \( T \) is also, by Young’s theorem, the time-budget slope of money-compensated Hicksian demand, that is,

(12) \( e_{Tr_j}(p, t, s, T, u) = \partial x_j^M(p, t, s, T, u)/\partial T \).

To evaluate (12), note the identity relating money-compensated demands to Marshallian demands,

\( x_j^M(p, t, s, T, u) = x_j(p, t, s, e[p, t, s, T, u], T) \).

Differentiating both sides with respect to \( T \), the time-budget slope of money compensated demand can be written as

(13) \( \partial x_j^M(p, t, s, T, u) / \partial T = \partial x_j(p, t, s, M, T) / \partial M \cdot e_T(p, t, s, T, u) + \partial x_j(p, t, s, T, u) / \partial T. \)

Substituting (4) and (7) simplifies (13) to

(14) \( \partial x_j^M(p, t, s, T, u) / \partial T = \partial x_j(p, t, s, M, T) / \partial M [-\rho(p, t, s, M, T)] + \rho(p, t, s, M, T) \cdot \partial x_j(p, t, s, M, T) / \partial M = 0. \)

This interesting result shows that with a full-prices/full-budgets Marshallian demand specification, the time-budget slope of the money-compensated Hicksian demand is always zero at given \((u, M, T)\) points. Because it is a local result rather than a global result about the demand function, though, the money-compensated Hicksian demands are generally functions of \( T \).

Because of (14), the derivative \( e_{Tr_j}(p, t, s, T, u) \) is always zero in (11). Taking the ratio of (11) to (10), a fundamental result emerges,

(15) \[ \frac{\partial p(p, t, s, M, T)/\partial p_j}{\partial p(p, t, s, M, T)/\partial M} ] = V_{p_j}/V_M = -x_j(p, t, s, M, T). \]

Equation (15) is striking for the structure it reveals about the endogenous marginal value of time function. The ratio of its money-price to money-budget slopes is the same as for the indirect utility function and equals the negative of corresponding quantity consumed, via Roy’s identity.

9. This can be seen by noting that the equality \( e_{Tr_j}(p, t, s, T, u) = -\rho(p, t, s, M, T) \) holds only at a point, unless utility is varied to keep money-budget constant (as in equation [8]). But the money-compensated Hicksian demand holds utility constant by definition.
Relating Time-Price and Time-Budget Slopes of the Marginal Value of Time

Parallel results hold for the time-budget arguments. By differentiating (9) with respect to time-budget $T$ and time-prices $t_j, j = 1, \ldots, n$,

$$
\frac{\partial \rho(p, t, s, M, T)}{\partial T} = \xi_{M} \rho(p, t, s, M, u) \cdot V_T(p, t, s, M, T) / [\xi_{M} (p, t, s, M, u)]^2
$$

and

$$
\frac{\partial \rho(p, t, s, M, T)}{\partial t_j} = \xi_{M} \rho(p, t, s, M, u) + V_t(p, t, s, M, T) / [\xi_{M}]^2
$$

where in (17) arguments of the functions common to (16) are suppressed. The cross-partial derivative $\xi_{M} \rho(p, t, s, M, u) = \partial x_T^f(p, t, s, M, u) / \partial M$ is the money-budget slope of the time-compensated demand for $x$, again by Young’s theorem. To evaluate this term, note the identity relating time-compensated demand to Marshallian demand,

$$
x_T^f(p, t, s, M, u) \equiv x_T(p, t, s, M, \xi(p, t, s, M, u)).
$$

Differentiating with respect to $M$,

$$
\partial x_T^f(p, t, s, M, u) / \partial M = \partial x_T(p, t, s, M, T) / \partial M + \partial x_T(p, t, s, M, T) / \partial T \cdot \xi_{M} (p, t, s, M, u).
$$

Again substituting from (4) and (7), this time into (18),

$$
\partial x_T^f(p, t, s, M, u) / \partial M = \partial x_T(p, t, s, M, T) / \partial M + [\rho(p, t, s, M, T) \cdot \partial x_T(p, t, s, M, T) / \partial M] \\
\cdot (-1/\rho(p, t, s, M, T)),
$$

which parallels the result in (14). As with the money-compensated Hicksian demands, the time-compensated Hicksian demands have zero money-budget slopes at given $(u, M, T)$ points.

As before, substituting (19) into (17) and taking the ratio of (17) to (16) yields the identity relating time-price and time-budget slopes,

$$
\frac{\partial \rho(p, t, s, M, T)}{\partial t_j} / [\partial \rho(p, t, s, M, T)] / \partial T = V_t / V_T = -x_T(p, t, s, M, T).
$$

Equations (15) and (20) are the two principal results useful in specifying the marginal value of time function. When the marginal value of leisure time is endogenous, a demand model with full prices and full budgets is sufficient (not necessary) to satisfy the two-binding constraint hypothesis. Given this demand specification, the marginal value of time satisfies analogs to the two Roy’s identities given in (2): the ratio of the price to budget slopes within each constraint is the negative of the corresponding quantity consumed.

Implications for the Structure of the Marginal Value of Time Function

To identify a specification for $\rho(p, t, s, M, T)$ consistent with the requirements of equations (15) and (20), it is useful to note that incomplete demand systems are estimated in practice. By introducing $N_M$ and $N_T$ as numeraire goods such that $p_M = 1, t_M = 0, p_T = 0$, and $t_T = 1$, the budget constraints can be expressed as $M = p_M + N_M$ and $T = t_M + N_T$ with $N_M$ and $N_T$ representing expenditures of money and time outside the $M$-good empirical demand system of interest, which explains $x_T$.

Given this, a general structural specification for the marginal value of time that satisfies (15) and (20) is

$$
\rho(p, t, s, M, T; x') = f(M - p_M x', T - t_M x', s).
$$

That is, given the parameters of the problem and the individual’s optimal choices $x$, the “revealed” value of time is a function of the discretionary money and time after expenditures are allocated to goods in the empirical incomplete demand system time and of the other shifters $s$.

10. This can be easily verified by noting that $\partial \rho(p, t, s, M, T; x') / \partial y = (\partial f / \partial N_M) (-x_M)$ and $\partial \rho(p, t, s, M, T; x') / \partial M = (\partial f / \partial N_M), so equation (15) holds. Similar results hold for $t$ and $T$, so that equation (20) also is satisfied.
Given the development of the marginal value of time function, equations (3) and (21) are collected together to form an \( M + 1 \) good system of structural equations in the endogenous variables \( \mathbf{x}' \) and \( \rho \),

\[
(3') \quad \mathbf{x}'(\mathbf{p}, t, s, M, T; \rho) = \mathbf{x}'(\mathbf{p} + \rho \cdot t, s, M + \rho \cdot T),
\]

\( i = 1, \ldots n \)

and

\[
(21') \quad \rho(\mathbf{p}, t, s, M, T; \mathbf{x}') = f(N_M, N_T, s).
\]

where the definitions of the numeraire goods \( N_M = M - p'x' \) and \( N_T = T - t'x' \) are used to simplify notation in (21). Because \( \rho \) is not observed, equation (21') cannot be estimated directly as part of the econometric system. However, it can be substituted directly into (3), yielding the \( M \)-good implicit system

\[
(22) \quad \mathbf{x}' = \mathbf{x}'(\mathbf{p} + f(N_M, N_T, s), M - f(N_M, N_T, s) \cdot T).
\]

Equation (22) can be estimated given the empirical marginal value of time function \( \rho = f(N_M, N_T, s) \) from (21) and suitable choices of functional forms for the goods \( \mathbf{x} \) in the incomplete demand system.

IV. AN EMPIRICAL TWO-CONSTRAINT DEMAND MODEL

This section develops the first empirical application of a two-constraint version of the AIDS model of Deaton and Muellbauer (1980). This demand model has an endogenous marginal value of time consistent with the two-constraint requirements. The implicit demand system with endogenous marginal value of time described in (22) is considerably more general than recreation demand models estimated in practice: in fact, single-equation models are more the norm. After describing the general form of a two-constraint demand model, then a specific form for the marginal value of time is adopted and a single-equation version of the model is estimated empirically.

Beginning with any one of the optimized choice functions \( v(\cdot), e(\cdot), \) or \( \xi(\cdot) \), one can derive the others from the dual structure of the optimization problem. It is most convenient to develop the dual structure in terms of the money cost function, in a manner similar to the development of the standard AIDS model. Let the money expenditure function be

\[
(23) \quad e(\mathbf{p}, t, s, T, u) = \exp\{\alpha_0 + \sum_k \alpha_k^p \log(p_k^M) + \sum_k \alpha_k^s \log(s_k) + (1/2) \sum_k \gamma_k^p \log(p_k^M) \log(p_k^M) + 2 \sum_k \gamma_k^s \log(s_k) + \sum_k \gamma_k^s \log(s_k) \} + \beta_0 \Pi(p^M)^{\beta_1} - p^M(\cdot) \cdot T,
\]

where \( p^M = p^M(\mathbf{p}, t, s, T, u) = -e_T(\mathbf{p}, t, s, T, u) \) is the (money-compensated) Hicksian value of time function, \( \Pi \) and \( p^M = p^M(\mathbf{p}, t, s, T, u) \cdot t_I \) are full prices formed using this value of time function.

The indirect utility function \( V(\mathbf{p}, t, s, M, T) \) dual to \( e(\mathbf{p}, t, s, T, u) \) is

\[
(24) \quad V(\mathbf{p}, t, s, M, T) = (\log(M^F) - \alpha_0 + \sum_k \alpha_k^p \log(p_k^F) + \sum_k \alpha_k^s \log(s_k) + (1/2) \sum_k \gamma_k^p \log(p_k^F) \log(p_k^F) + \sum_k \gamma_k^s \log(s_k) \log(s_k)) \cdot \beta_0^{-1} \Pi(p^F)^{-\beta_1}.
\]

where \( p^F = p^F(\mathbf{p}, t, s, M, T, u) \) and \( M^F = M + \rho(\mathbf{p}, t, s, M, T, u) \cdot T \) are full prices and full budget formed using the empirical (Marshallian) value of time function \( \rho(\mathbf{p}, t, s, M, T) \). It can be verified in (24) that \( V_T/V_M = \rho(\mathbf{p}, t, s, M, T) \), consistent with the definition of \( \rho(\mathbf{p}, t, s, M, T) \) in (7). The time expenditure function corresponding to (23)

11. It is straightforward to show that \( -e_T = \rho^M(\mathbf{p}, t, s, T, u) \) for this model, as required. Though \( T \) appears throughout (25) because it is an argument of \( p^F \), homogeneity of degree zero of money expenditure function in \( (t, T) \) ensures that all terms involving \( \partial p^M/\partial T \) sum to zero.

12. That \( e(\mathbf{p}, t, s, T, u) \) and \( V(\mathbf{p}, t, s, M, T) \) are \( M - u \) inverses can be verified by substituting (24) into (23) and noting that \( p^M(\mathbf{p}, t, s, T, V(\mathbf{p}, t, s, M, T)) = p^M(\mathbf{p}, t, s, M, T) \). When terms cancel, what remains is \( e(\mathbf{p}, t, s, T, V(\mathbf{p}, t, s, M, T)) = M \). Conversely, using (23) in (24) and noting that \( \rho(\mathbf{p}, t, s, e(\mathbf{p}, t, s, T, u), u) = \rho^M(\mathbf{p}, t, s, T, u) \), the result is \( V(\mathbf{p}, t, s, e(\mathbf{p}, t, s, T, u), T) = u \).
and (24) is

\[
(25) \quad \xi(p, t, s, M, u) = \left(1/p^T\right) \cdot \{\exp[\alpha_0 + \Sigma_i \alpha_i^p \log(p^T_i) + \Sigma_k \alpha_k^T \log(s_k) + (1/2)\Sigma_i \Sigma_j \gamma_{ij}^p \log(p^T_i) \log(p^T_j) + 2 \Sigma_k \Sigma_j \gamma_{jk}^T \log(p^T_i) \log(s_j) + u \beta_0 \Pi_i (p^T_i)^{\beta_j}] - M\},
\]

where \( p^T_i = p^T(p, t, s, M, u) \) is the time-compensated value of time function and \( p^T_i = p_i + p^T(p, t, s, M, u) \cdot \tau_i \) are full prices formed using this value of time function. The time expenditure function is a \( V - T \) inverse with (24) and an \( M - T \) inverse with respect to (23), and it can be verified that \( p^T(p, t, s, M, u) = -1/\Sigma(p, t, s, M, u) \), again consistent with (7).

The Implied Share Equations

Differentiating (25) with respect to the money-price \( p_i \) yields the Hicksian money-compensated demand \( x_i^M(p, t, s, T, u) = \partial e(p, t, s, T, u)/\partial p_i \). Because \( p_i \) is embedded in the full price variable \( p^M_i = p_i + p^M(p, t, s, T, u) \cdot u \), the chain rule applies and the derivative can be written as \( \partial e(p, t, s, T, u)/\partial p_i = \{\partial e(p, t, s, T, u)/\partial \log(p^M_i)\} \cdot \{\partial \log(p^M_i)/\partial p_i\} \). From (25), it can be seen that

\[
\partial e(p, t, s, T, u)/\partial \log(p^M_i) = \{\alpha_i^p + \Sigma_j \gamma_{ij}^p \log(p^M_j) + \Sigma_k \gamma_{ik} \log(s_k) + \beta_i u \beta_0 \Pi_i (p^M_i)^{\beta_j}\} \cdot \{e(p, t, s, T, u) + \rho^M(p, t, s, T, u) \cdot T\}
\]

and \( \partial \log(p^M_i)/\partial p_i = 1/p^M_i \). Combining these, the Hicksian money-compensated demand is

\[
x_i^M(p, t, s, T, u) = \{\alpha_i^p + \Sigma_j \gamma_{ij}^p \log(p^M_j) + \Sigma_k \gamma_{ik} \log(s_k) + \beta_i u \beta_0 \Pi_i (p^M_i)^{\beta_j}\} \cdot \{e(p, t, s, T, u) + \rho^M(p, t, s, T, u) \cdot T\} / p_i^M.
\]

By substituting the indirect utility function in (24) for the utility term everywhere, the Marshallian demand function is

\[
x_i(p, t, s, T, u) = \{\alpha_i^p + \Sigma_j \gamma_{ij}^p \log(p^f_j) + \Sigma_k \gamma_{ik} \log(s_k) + \beta_i \log(M^F/PI)\} \cdot M^F / p_i^f,
\]

because \( p^M(p, t, s, T, V[p, t, s, M, T]) = \rho(p, t, s, M, T) \) and, therefore, \( p_i^f \) becomes \( p_i^f \) and \( e(p, t, s, T, u) + \rho^M(p, t, s, T, u) \cdot T \) is \( M^F \). The term \( PI \) is the budget deflator.\(^{13}\) Multiplying both sides by \( p_i^f / M^F \), the Marshallian shares \( w_i = x_i \cdot p_i^f / M^F \) are

\[
(26) \quad w_i = \alpha_i^p + \Sigma_j \gamma_{ij}^p \log(p^f_j) + \Sigma_k \gamma_{ik} \log(s_k) + \beta_i \log(M^F/PI),
\]

which take a form identical to shares in the single-constraint AIDS model, aside from the fact that \( p_i^f = p_i + \rho(p, t, s, M, T) \cdot \tau_i \) are full prices and \( M^F = M + \rho(p, t, s, M, T) \cdot T \) is full budget.

One can also derive the same system from the time expenditure function (25), differentiating with respect to \( \tau_i \) and using the envelope result that \( x_i = \partial \xi(p, t, s, M, u)/\partial \tau_i \). Not surprisingly, no new information results because the two expenditure functions are jointly dependent. The shares in (26) are full shares because they are defined in terms of full prices \( p_i^f \) and full budget \( M^F \).

The form of the share model in (26) is essentially a generalization of the AIDS share model from single constraint choice. The arguments of the share equations are full prices and full budgets, and the conversion factor between time and money is the marginal value of time function that must satisfy the restrictions on its price and budget slopes in (15) and (20).

The Empirical Value of Time Function

Because so much of the recreation demand literature focuses on the connection between the marginal value of leisure time and the wage, it is natural to include the individual’s wage as an argument of the vector of shifters \( s \). The specification should be flexible enough,\(^{13}\)

\[
13. \text{ It is defined as } PI = [\alpha_0 + \Sigma_i \alpha_i^p \log(p^f_i) + \Sigma_k \alpha_k^T \log(s_k) + (1/2)\Sigma_i \Sigma_j \gamma_{ij}^p \log(p^f_i) \log(p^f_j) + 2 \Sigma_k \Sigma_j \gamma_{jk} \log(p^f_i) \log(s_j) + \Sigma_i \Sigma_j \gamma_{ij} \log(s_i) \log(s_j)],
\]
though, to allow for different effects of the wage on the marginal value of time for those who make a marginal labor supply decision (e.g., by working flexible hours) and those who don't (e.g., work fixed hours). Other individual characteristics may also be important. For example, it seems likely that the individual's age is a determinant of the marginal value of leisure time. The individual's avidity for the particular recreation activity could also play a role.

With these observations and equation (21) in mind, the empirical marginal value of time is

$$\rho(N_M, N_T, s) = \exp\{\theta_0 + \theta_1 N_M + \theta_2 N_T + \theta_3 \ln(Wage)\},$$

(27)

where $F$ is a dummy variable equaling 1 if the individual works fixed hours and 0 otherwise. This specification recognizes that the individual's average wage, as well as the amounts of the numeraire goods $N_M$ and $N_T$, are likely to affect the marginal value of time in the full prices and full budgets; but the effects can be different for those working fixed versus flexible hours. One would expect that wage plays less of a role for those working fixed hours (i.e., $\delta_1 = 0$ and $\delta_2 \neq 0$) and more of a role for those working flexible hours and offering discretionary labor supply (i.e., $\theta_3 \neq 0$ and $\theta_1, \theta_2 = 0$). If all individuals report their marginal wages correctly, one might further suppose that $\theta_3 = 1$.

In addition to Wage, two other shifters are included, Age and Avidity. Avidity is an index ranging from 1 (lowest) to 4 (highest) based on responses to several categorical variables probing the individual's strength of opinion about the importance of preserving marine mammal populations.

The specification in (27) encompasses several approaches used for treating time in the empirical recreation demand literature. One is the approach suggested by McConnell and Strand (1981), which estimates the constant fraction of the wage rate that the marginal value of time is assumed to represent. When $\theta_1 = \theta_2 = \theta_3 = 0$, the value of time for individuals working flexible hours ($F = 0$) is

$$\rho(N_M, N_T, s) = \exp(\theta_0) \cdot Wage,$$

(28)

and is the natural log of the constant wage fraction. More generally, of course, the fraction of the wage could vary with individual characteristics, which is accommodated by the less restrictive condition $\theta_3 = 1$, in which case the marginal value of time for those working flexible hours is

$$\rho(N_M, N_T, s) = \exp\{\theta_0 + \theta_1 N_M + \theta_2 N_T + \varphi_1 Age + \varphi_2 Avidity\} \cdot Wage.$$

One could also estimate the value of time for those with fixed hours as a fraction of the wage, as (27) indicates, though this is less compelling because presumably there is less of a connection between the marginal value of time and the wage for these people.

Another common approach is to assume that the marginal value of leisure time is a fraction of the wage, with 1/4 to 1/2 often used in practice by reference to the value of time saved in transportation studies (e.g., Cesario 1976). This type of assumption is also easily evaluated as the additional restriction on (28) that $\theta_0 = \ln(k)$, where $k$ is the value to be tested. To test whether time is valued at zero, one can replace $\exp(\theta_0)$ in (28) with a constant $k$ and test whether $k = 0$.\(^{14}\)

V. AN APPLICATION TO CALIFORNIA WHALE WATCHING

The data used to illustrate the model are from on-site intercepts of whale watchers at four sites in California during the winter 1991–92. The survey instrument was pretested using individuals who had gone whale watching in the previous year. It collected information on trips taken so far that season, expected future trips, travel time, travel costs, whether the trip was their primary destination, and so on. Also collected was information including actual contributions to marine mammal groups; time spent reading, watching, or thinking about wildlife and whales; as well as purchases of whale-related merchandise. Last, demographic information including work status, wage rates, and income was asked. The survey was presented in booklet form.

In total, 1,402 visitor surveys were handed out, and 1,003 were returned, for an overall response rate of 71.3%. The response rate

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14. We thank an anonymous reviewer for pointing out this simpler test.
was reasonably similar across the four locations, varying from a low of 65.2% for intercepts at Point Loma (San Diego) to a high of 80.3% for intercepts at Point Reyes. On-site refusals were not a problem. For example, at Point Reyes, only 10 people of roughly 600 contacted (about 1.6%) refused to take a survey packet.

The empirical demand system focuses on estimating the budget share for whale watching trips but recognizes there are related actions individuals can take with respect to whale and marine mammal populations. These are the expenditure of one's time or money to contribute to the maintenance or enhancement of gray whale populations. The relative prices of these other actions differ across the population and therefore may affect the frequency of whale watching trips taken, just as substitute prices affect consumption of other goods. Augmenting the single-equation incomplete demand/ share equation for trips are the two numeraire goods $N$ and $N$, the residual expenditures of time and money from their respective budgets after accounting for trips.

The own (full) price of whalewatching trips $(x)$ is the money cost of travel to the site and while on site, plus the time costs of travel (hours traveled and spent on site) valued at the (endogenous) marginal value of time. Thus the full price of visits is $P_1 = p_1 + p(N_M, N_T, s) \cdot t_1$, where $p_1$ is the money cost of the trip and $t_1$ is the time cost of the trip, or the trip length. Monetary donations have a money price because charitable contributions are tax-deductible at both the federal and state level, so the marginal price of a dollar donation is less than a dollar and varies across individuals according to household income. These donations do not have a significant time-price, so the full price of donations is just the money cost of donating; that is, $P_2 = \tau$, where $\tau$ is the marginal tax rate for the individual. Both California and federal taxes were included in figuring $\tau$ for each individual, based on their total household income category.

Time donations $(x_3)$ have a time-price $t_3 \geq 1$, measured by the total number of hours required to deliver an hour of time to the volunteer organization. Total time may be higher because of transactions time costs from driving back and forth, and so on. Because we don't have information on how this time-price will differ across individuals in our sample, we take the time-price $t_3 = 1$ for all. There may also be a money cost $p_3$ of time donations, which would represent the money costs of driving to and from the site where volunteering occurs and other transactions costs of donating time. We have no information on this variable from our sample but suspect it is small, thus it is taken to be zero. Thus price $p_3 = 0$, and the full price of time donations is $P_3 = \rho(N_M, N_T, s)$, which varies across the sample based on differences in individual characteristics.

In addition to the time- and money-prices, it is expected that the individual's whale watching success will influence both trips demand and potentially the willingness to make donations of time and money. The success variable $(z)$ is the individual's ex ante expectation of whale sightings for the trip when they were contacted. Money-budget $(M)$ is the household income before taxes, and the time-budget $(T)$ is the amount of nonworking time measured as the number of weekend and paid vacation days. Table 1 provides some descriptive information on these variables used in the estimation model.

Results

The empirical model consists of the demand share for whale watching trips (equation [26]) with the marginal value of time function (equation [27]) embedded in the full prices and budgets of the share equation. It was estimated using nonlinear least squares in SAS version 6.12. Stone's price index (Stone 1954) was used for the deflator.

Estimation results for the full model are given in Table 2. The model is highly significant overall, and most coefficients individually are significant at the $\alpha = 0.05$ level. The share coefficients indicate that demand is price inelastic (with mean own-full price elasticity of $-0.31$) and income elastic (with mean full income elasticity of $0.66$). Expected sightings have a significant positive effect on trips share, and time donations are a substitute for trips, whereas money donations are complementary with trips. There are significant site-specific effects at the two northernmost sites, Point Reyes and Half Moon Bay.

15. Time on site is taken to be exogenous because all trips covered in this analysis are day trips and roughly half of all whale watching trips represented are boat trips of fixed duration.
TABLE 1
Some Characteristics of the Whale Watcher Sample (n = 393)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household income</td>
<td>$/year</td>
<td>65,178</td>
<td>5,000</td>
<td>150,000</td>
</tr>
<tr>
<td>Paid vacation</td>
<td>days/year</td>
<td>13</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>Time-budget</td>
<td>hours/year</td>
<td>6,771</td>
<td>6,672</td>
<td>7,272</td>
</tr>
<tr>
<td>Wage</td>
<td>$/hour</td>
<td>25</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>Age</td>
<td>years</td>
<td>36.6</td>
<td>17</td>
<td>64</td>
</tr>
<tr>
<td>Avidity</td>
<td>—</td>
<td>3.26</td>
<td>1.5</td>
<td>3.75</td>
</tr>
<tr>
<td>Expected sightings</td>
<td>no./trip</td>
<td>6.54</td>
<td>0.1</td>
<td>50</td>
</tr>
<tr>
<td>Whale watching trips</td>
<td>number</td>
<td>2.20</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>Money-price of trips</td>
<td>$/trip</td>
<td>109.31</td>
<td>0.465</td>
<td>203</td>
</tr>
<tr>
<td>Time-price of trips</td>
<td>hrs/trip</td>
<td>3.05</td>
<td>1.1</td>
<td>17.7</td>
</tr>
<tr>
<td>Money-price of donations</td>
<td>—</td>
<td>0.69</td>
<td>0.64</td>
<td>0.84</td>
</tr>
<tr>
<td>Time-price of donations</td>
<td>hrs/$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Money-price of volunteering</td>
<td>$/hr</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Time-price of volunteering</td>
<td>—</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE 2
Parameter Estimates for the Full Trips Share-Value of Time Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Asymptotic Student's t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trips share equation</td>
<td>$\beta_0$</td>
<td>5.1214</td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>-0.1176</td>
<td>-1.84</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2$</td>
<td>-0.5997</td>
<td>-2.91</td>
</tr>
<tr>
<td></td>
<td>$\alpha_3$</td>
<td>0.1186</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{11}$</td>
<td>0.7255</td>
<td>10.50</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{12}$</td>
<td>-0.4663</td>
<td>-5.08</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{13}$</td>
<td>0.1312</td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{14}$</td>
<td>0.0476</td>
<td>2.55</td>
</tr>
<tr>
<td></td>
<td>$\beta_p$</td>
<td>-0.3626</td>
<td>-3.52</td>
</tr>
<tr>
<td>Marginal value of time function</td>
<td>$\delta_0$</td>
<td>-42.4001</td>
<td>-3.15</td>
</tr>
<tr>
<td></td>
<td>$\delta_1$</td>
<td>0.0068</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>$\delta_2$</td>
<td>0.0070</td>
<td>3.50</td>
</tr>
<tr>
<td></td>
<td>$\delta_3$</td>
<td>-0.2022</td>
<td>-0.56</td>
</tr>
<tr>
<td></td>
<td>$\theta_0$</td>
<td>-3.4330</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td>$\theta_1$</td>
<td>-0.0142</td>
<td>-2.46</td>
</tr>
<tr>
<td></td>
<td>$\theta_2$</td>
<td>1.0461</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$\theta_3$</td>
<td>0.8866</td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td>$\psi_1$</td>
<td>-0.7297</td>
<td>-2.76</td>
</tr>
<tr>
<td></td>
<td>$\psi_2$</td>
<td>-0.0194</td>
<td>-1.32</td>
</tr>
</tbody>
</table>

N = 393
Log L = -807.04
Log L (constants only) = -949.15

*Trips share is scaled up by a factor of 1,000.
Within the value of time function, it is clear that the parameter vectors are different for those who work fixed versus flexible hours. For those who work fixed hours, the time constraint was the most important determinant of the marginal value of time. As one would expect, wage is not a significant determinant of the marginal value of time for these individuals. In contrast, for those working flexible hours, the most important factors in the marginal value of time are the wage rate and the money-budget constraint. Age has a significant negative effect on the marginal value of time, controlling for budget and other effects, and avidity also has a negative sign but is not statistically significant.

Table 3 presents the results of hypothesis tests on the treatment of the marginal value of time. The first hypothesis test is whether personal characteristics (in our case, Age and Avidity) make a significant difference to an individual’s marginal value of time. The \( \chi^2 \) statistic for the restrictions that coefficients of these variables are zero is 9.90, exceeding the critical \( \chi^2_{0.05} \), which is 5.99. The hypothesis that personal characteristics are irrelevant is therefore rejected.

The rest of Table 3 explores the implications of using various conventional treatments of the value of time. These are nested within the endogenous value of time model and can be tested as parameter restrictions on the more general model. If the parameter restrictions corresponding to alternative conventional treatments are not rejected, one can say that there is no additional explanatory power from the more general model. If, on the other hand, the parameter restrictions are rejected, one can conclude that the more general endogenous marginal value of time model is preferred.

From Table 3, one can see that assuming the marginal value of time is a fraction of the wage is rejected, because the \( \chi^2 \) test statistics for four different variants of this assumption exceed the \( \alpha = 5\% \) critical values. The most general assumption, that those working fixed and flexible hours have a different fraction, with both estimated in the model, is rejected, as is the additional restriction that the fraction be the same for both groups. Similarly, using two common assumptions in the literature (that time is valued at 1/3 the wage and at zero) are also rejected. These results indicate that the endogenous marginal value of time model does a significantly better job of explaining trips shares than do the conventional treatments.

It is interesting to ask what relationship the model estimates of the marginal value of time have with the individual’s reported wage category. Table 2 indicates a strong link to the wage for individuals working flexible hours, as one would expect. The parameter
estimate on ln(Wage), \( \theta_3 \), is the elasticity of the marginal value of time with respect to wage, which is 0.886. This means that for every 1% increase in the wage, the marginal value of time for individuals working flexible hours increases by 0.89%. This is a partial effect, however, that doesn’t account for other changes in money income and discretionary time-budget that are induced by changes in hours worked.

Figure 1 graphs the model estimates of mean marginal value of time by wage category and labor class, along with error bars representing the 95% confidence intervals. The mean estimates are conditional sample means and take the estimated parameters as given, so the confidence intervals reported reflect the effects of variations in individual characteristics on the mean value of time estimates. The numbers used in Figure 1 are also presented in Table 4.

Figure 1 illustrates two interesting findings of the empirical model. First, marginal values of time for both labor classes (fixed and flexible hours worked) diverge significantly from the wage at moderate to high wages (above about $17.50/h). There is a modest increase in mean marginal value of time with wage for flexible hours workers but no discernible trend for fixed hours workers (Table 4). Overall, the relationship of the marginal value of time to the wage is more complex than most models in the literature allow, with the marginal value of time exceeding the wage for low wage rates and less than the wage for high wage rates. That is, the ratio of the marginal value of time to the wage decreases with increasing wage.

The second interesting finding is that there is not a statistically significant difference between the marginal values of time for fixed versus flexible hours workers. This is due in part, no doubt, to the fact that when the data are disaggregated into many subsets, the confidence intervals widen because the number of observations used for each statistic decreases. It may also reflect the fact that schedules are not strictly fixed for any workers in the long run, so that when a serious imbalance between the wage and the marginal value of time arises, an adjustment of hours worked occurs to bring them more in line.

The key point is the flexibility of the endogenous marginal value of time approach to better represent the complex relationship of the marginal value of time to individual’s constraints and opportunities. Figure 1 illustrates

![Figure 1: Values of Time by Wage Group](image)

### TABLE 4
Estimated Marginal Values of Time by Wage Category and Flexibility of Work Time

<table>
<thead>
<tr>
<th>Wage Category ($/hr)</th>
<th>Flexible Hours</th>
<th>Fixed Hours</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time Value ($/hr)</td>
<td>Number</td>
<td>Time Value ($/hr)</td>
</tr>
<tr>
<td>&lt;4</td>
<td>1.83 (1.17)(^a)</td>
<td>2</td>
<td>10.07 (6.40)</td>
</tr>
<tr>
<td>4–6</td>
<td>7.02 (0.95)</td>
<td>2</td>
<td>6.26 (1.50)</td>
</tr>
<tr>
<td>6–10</td>
<td>7.91 (1.14)</td>
<td>17</td>
<td>7.24 (0.80)</td>
</tr>
<tr>
<td>10–15</td>
<td>9.95 (1.14)</td>
<td>30</td>
<td>7.47 (0.58)</td>
</tr>
<tr>
<td>15–20</td>
<td>11.07 (1.10)</td>
<td>29</td>
<td>19.26 (7.54)</td>
</tr>
<tr>
<td>20–25</td>
<td>12.53 (2.21)</td>
<td>16</td>
<td>8.85 (0.82)</td>
</tr>
<tr>
<td>25–30</td>
<td>11.25 (1.94)</td>
<td>13</td>
<td>9.70 (0.78)</td>
</tr>
<tr>
<td>30–50</td>
<td>15.88 (1.53)</td>
<td>20</td>
<td>13.72 (2.99)</td>
</tr>
<tr>
<td>50–100</td>
<td>16.23 (1.78)</td>
<td>15</td>
<td>11.89 (1.94)</td>
</tr>
<tr>
<td>&gt;100</td>
<td>18.81 (2.43)</td>
<td>9</td>
<td>9.57 (3.94)</td>
</tr>
<tr>
<td>Grand total</td>
<td>12.08 (0.58)</td>
<td>153</td>
<td>10.75 (1.34)</td>
</tr>
</tbody>
</table>

\(^a\)Standard errors of the means in parentheses.
This flexibility of the model to represent elements of two approaches to the marginal value of time in the literature: the conventional assumption of a constant fraction of the wage, and the approach of inferring a constant travel time value for all individuals in the sample (e.g., Hausman et al. 1995). The patterns of time values are revealed by the data on choices, prices, and constraints, rather than being imposed a priori.

VI. CONCLUSIONS, LIMITATIONS, AND EXTENSIONS

This article has developed an approach to estimate an endogenous marginal value of time function jointly with recreation demands. It identifies the structural requirements that endogenous marginal values of time must satisfy to be consistent with the hypothesis of choice subject to two binding constraints. An analog to Roy's identities for the two-constraint model holds for the endogenous marginal value of time function, wherein the ratios of time-price slopes to time-budget slope of the marginal value of time function equal the negative of corresponding quantities consumed, which also equal the ratios of money-price slopes to money-budget slope. We propose a functional form for the endogenous marginal value of time that nests the other main empirical specifications in the literature, and estimate this function jointly with the recreation demand share in a two-constraint version of the AIDS. The approach is attractive econometrically because joint estimation of the parameters of the marginal value of time and demand means that full information about the model is used in its estimation. The parameters of the marginal value of time function are those which best fit the data, in providing the closest match with recreationists' actual choices given relative prices and constraints on time and money they face.

The model is applied to data on gray whale watching along the California coast. The share model is specified using full prices and budgets, with the endogenous marginal value of time function as the terms of trade between time and money. Based on the econometric results, joint estimation of the marginal value of time and the demand (share) model seems superior to conventional treatments of this important variable in the literature. Allowing for differences in the structure of demand between individuals working fixed versus flexible hours, the demand share and value of time functions are highly significant. The marginal value of time appears to have a more complex relationship with the wage than is allowed for in most conventional treatments in the literature.

It is encouraging that very little additional information is required to implement a recreation demand model fully consistent with the two-constraint requirements. Principally, information on time-budgets is needed beyond what is routinely collected. When labor supply is predetermined relative to recreation choice, the relevant time constraint is on work time, and this can be obtained easily through surveys that ask about the respondent's work schedule: how many hours worked per week and how many weeks worked per year. The nonwork time is then the difference between total time in the period of analysis and the time spent working. When, alternatively, labor supply is joint with recreation choice, the relevant time-budget is simply all time during the period of analysis. This does not vary across respondents, but the monetized value of the time-budget does, because the marginal value of time varies. The other important time variables in the two-constraint model, time spent in travel and on site, are commonly collected in recreation demand surveys because of the widespread recognition of the importance of time-prices.

An area for further work is to incorporate the individual's labor supply decision into the framework of this article, as another source of information about the individual's choices that help reveal the marginal value of time. Another possible direction for further work is to allow for the presence of multiple constraints on use of nonwork time. We have presumed a single time constraint, but in reality most people face multiple constraints on their ability to get free for recreational activities. Development of empirical models of preferences that generate both demand and the marginal value of time from the same expenditure or indirect utility functions is a natural generalization to pursue. Hopefully the identification of structural requirements for shadow values herein will assist with and motivate such work. In a sense, it allows researchers to widen the optimization space for finding workable empirical models that better fit observed choices while maintaining consistency with theory.
REFERENCES