A Short Course on Political Economics, taught by Roger Myerson at Central University of Finance and Economics, Beijing, 23-27 July 2007. OVERVIEW OF TOPICS:

Day 1. Multiple equilibria and the foundations of political institutions:
Leaders and captains: a model of competition to establish the state.
Leaders and governors: a model of moral hazard in high office.

Day 2. Inhibiting potential challengers:
A model of capitalist liberalization.
Federalism and incentives for success of democracy.

Day 3. Basic problems of social choice:
Public goods in the selectorate model.
Social choice impossibility theorems (Muller-Satterthwaite thm, Condorcet cycle).
The probabilistic voting model and utilitarianism, the bipartisan set.
Turnout with costly voting.
The Condorcet jury theorem and the swing voter's curse.

Day 4. Multicandidate elections:
Citizen-candidate model.
Propoportional representation, the M+1 law of single nontransferable vote
Comparing equilibria of 3-candidate voting rules (above fray, bad apple, Cox threshold).
Bipolar multicandidate elections with corruption.

Day 5. Voting in legislatures:
Sophisticated solutions of binary agendas.
Groseclose-Snyder lobbying model and Diermeier-Myerson legislative organization model.
Austen-Smith and Banks model of elections and post-election coalitional bargaining.

These notes are available online at
`http://home.uchicago.edu/~rmyerson/research/pec2007.pdf`
Computational models for use with these notes can be found at
`http://home.uchicago.edu/~rmyerson/research/pekingu.xls`
For a longer reading list see
`http://home.uchicago.edu/~rmyerson/econ361.htm`
Surveys papers:
1. "Fundamentals of social choice theory" *Northwestern U. working paper* (1996),
`http://home.uchicago.edu/~rmyerson/research/schch1.pdf`
2. "Analysis of democratic institutions" *J of Economic Perspectives* 9(1):77-89 (1995),
`http://home.uchicago.edu/~rmyerson/research/perspec.pdf`
3. "Economic analysis of political institutions", *Advances in Economic Theory and Econometrics* 1:46-65 (1997), `http://home.uchicago.edu/~rmyerson/research/japan95.pdf`
4. "Theoretical comparison of electoral systems,"*European Economic Review* 43:671-697 (1999)
`http://home.uchicago.edu/~rmyerson/research/schump.pdf`

NOTES FOR DAY 1

I reviewed the models of two papers
"The Autocrat's Credibility Problem and Foundations of the Constitutional State"
http://home.uchicago.edu/~rmyerson/research/foundatn.pdf
and "Leadership, Trust, and Power" http://home.uchicago.edu/~rmyerson/research/power.pdf
The main them was moral-hazard agency problems at the center of government and the foundations of the state.

Here are some other good readings in this area:

Gary Becker, George Stigler, "Law enforcement, malfeasance, and compensation of enforcers," J Legal Studies 3:1-18 (1974).
Joseph E. Stiglitz and Carl Shapiro, 'Equilibrium unemployment as a worker disciplinary device," American Economic Review 74:433-444 (1984).
George A. Akerlof and Lawrence F. Katz, "Workers' trust funds and the logic of wage profiles," Quarterly J of Economics 104:525-536 (1989).
Egorov, Georgy, and Konstantin Sonin. 2005. "The killing game: reputation and knowledge in the politics of succession." CEPR discussion paper.
Egorov, Georgy, and Konstantin Sonin. 2006. "Dictators and their viziers: endogenizing the loyalty-competence trade-off." CEPR discussion paper.
Myerson, Roger. 2004. "Justice, institutions, and multiple equilibria." Chicago Journal of International Law 5(1):91-107.
Schelling, Thomas C. 1960. The Strategy of Conflict. Harvard U. Press.
Acemoglu, Daron, and James Robinson. 2006. Economic Origins of Dictatorship and Democracy Cambridge: Cambridge University Press.
Basu, Kaushik. 2000. Prelude to Political Economy. Oxford: Oxford U Press.
Timothy Besley, Principled Agents. Oxford (2006).
Samuel E. Finer, The History of Government from the Earliest Times, Oxford (1997).

A model of leaders and supporters in contests for power (R,λ,s,c,δ)

The Autocrat's Credibility Problem and Foundations of the constitutional State
http://home.uchicago.edu/~rmyerson/research/foundatn.pdf

An island principality yields income R that can be consumed or allocated by the ruler.
The ruler is the leader who won the most recent battle on the island.
Battles occur whenever a new challenger arrives, at a Poisson rate λ.
(In any time interval ε, P(challenger arrives) = $1-e^{-\lambda\varepsilon} \approx \lambda\varepsilon$ if $\varepsilon \approx 0$.)
A leader needs support from captains to have any chance of winning a battle.
Pr(leader with n captains wins against a rival with m captains) = $p(n|m) = n^s/(n^s+m^s)$.
Let c denote a captain's cost of supporting a leader in battle.
The prince and the captains are assumed to be risk neutral and have discount rate δ.

Consider a leader who has n supporters, but expects all rivals to have m supporters.
(For simplicity, we will always assume stationary expectations about rivals.)
If the leader has promised to give each supporter an income y (as long as the leader rules)
then, when there is no challenger, a supporter's expected discounted payoff is
$U(n,y|m) = (y-\lambda c)/[\delta+\lambda-\lambda p(n|m)]$.
For these captains to rationally give support in battle, we need $p(n|m)U(n,y|m) - c \geq 0$.
The lowest income y satisfying this participation constraint is $Y(n|m) = (\delta+\lambda)c/p(n|m)$
The leader's expected disounted payoff is:
$V(n,y|m) = (R-ny)/[\delta+\lambda-\lambda p(n|m)]$ when he rules with no immediate challenge,
$W(n,y|m) = p(n|m)\, V(n,y|m) = p(n|m)(R-ny)/[\delta+\lambda-\lambda p(n|m)]$ on the eve of battle.

An absolute monarch is one who is released from all constraints of law.
An absolute leader who cheated a supporter would not be punished by anyone else,
although of course the cheated individual might be less likely to support him in the future.
(An absolutist would have no incentive to pay supporters if even those cheated don't react.)
So a leader is absolute when his relationships with all supporters are purely bilateral,
as if supporters have no communication with each other.
Against m, a force of n captains is <u>feasible for an absolute leader</u> iff
there exists some wage rate y such that $y \geq Y(n|m)$ and $V(n,y|m) \geq V(k,y|m)$ $\forall k \in [0,n]$.
First is participation constraint for captains, second is absolutist's moral-hazard constraint.

Let $v(n|m) = V(n,Y(n|m)|m) = [R-nc(\delta+\lambda)/p(n|m)]/[\delta+\lambda-\lambda p(n|m)]$,
and let $w(n|m) = W(n,Y(n|m)|m) = [p(n|m)R-nc(\delta+\lambda)]/[\delta+\lambda-\lambda p(n|m)]$.

<u>Proposition 1.</u> If n>0 and y satisfy the feasibility condition for an absolute leader against m, then
there exist $k > n$ such that $v(k|m) > V(n,y|m)$ and $w(k|m) > W(n,y|m)$.
<u>Proof.</u> [Easy if $y>Y(n|m)$.] $Y'(n|m) < 0$. AbsFeas => $V'(n,y|m) \geq 0$. [$'$ = deriv wrt 1st.]
So with $y=Y(n|m)$, $v'(n|m) = V'(n,y|m) - Y'(n|m)n/[\delta+\lambda-\lambda p(n|m)] > 0$.
So an absolute leader could always benefit by commitment to maintain a larger force.

Now suppose captains communicate at court, and a complaint by any captain could switch them to a distrustful equilibrium, where nobody trusts the ruler to reward supporters.
Complaining-only-if-cheated is incentive compatible, as captains expect U>0 on eqm path.
With challenges at rate λ and no support, the ruler's expected payoff would be $R/(\delta+\lambda)$.
So we say n is <u>feasible for a leader with a weak court</u> against m iff $v(n|m) \geq R/(\delta+\lambda)$.
$V(0,y|m) = R/(\delta+\lambda)$, so feasible for absolutist => feasible for leader with a weak court.
This court is called "weak" because it cannot change the arrival rate of new challengers.
But when a ruler is known to have no support, immediate challenges may be more likely.
Then loss of confidence at court could lead to a rapid downfall of the leader.
So we say n is <u>feasible for a leader with a strong court</u> against m iff $v(n|m) \geq 0$.

<u>Proposition 2.</u> Suppose that n is feasible for a leader with a weak court against m.
Then $nY(n|m)/R \leq p(n|m)\lambda/(\delta+\lambda)$ and $n \leq R\lambda p(n|m)^2/[c(\delta+\lambda)^2]$.
If n > 0 and s>0.5 then $m \leq M_0 = [R\lambda(2s-1)^{2-1/s}]/[4s^2c(\delta+\lambda)^2]$.

We may say that a force size m is <u>globally feasible</u> for leaders of some kind (absolute, or with weak courts, or with strong courts) iff m is feasible against m for such leaders.

<u>Proposition 3.</u> Suppose that $s \geq 2/3$.
If n is feasible against m for a weak-court leader and $0 < n \leq m$, then $w'(n|m) > 0$.
So if m is globally feasible for weak-court leaders then $\mathrm{argmax}_{k\geq 0}\, w(k|m) > m$.

We may say that m is a <u>negotiation-proof equilibrium</u> iff $w(m|m) = \max_{n\geq 0} w(n|m)$, so that any new leader before first battle would want to negotiate the same force size.
By Prop 3, such a negotiation-proof eqm cannot be globally feasible with weak courts.

<u>Proposition 4.</u> When $s \leq 2$, the negotiation-proof equilibrium is $m_1 = Rs/[c(4\delta+2\lambda+s\lambda)]$.
In this eqm, supporters get the fraction $m_1Y(m_1|m_1)/R = 2s(\delta+\lambda)/(4\delta+2\lambda+s\lambda)$ [$\to 1$ as $s \to 2$].
When $s \geq 0.763$, this equilibrium m_1 is greater than the bound M_0 from Proposition 2, and so an absolutist or a leader with a weak court could not get any support against this eqm.

What prevents courtiers from extracting more than the promised income $y = Y(n|m)$?
The courtiers are in a game with multiple equilibria. Each wants to support the leader as long as he trusts the leader and all others are expected to support the leader.
Before the first battle for power, the leader's speech could make the w-max'ing eqm focal.

But other cultural expectations might favor an eqm that is better for the captains.
The best alternative for the n supporters is to get $y = R/n$, leaving 0 for leader.
Then the n captains would be oligarchs, and their expected payoff against m would be
$\Omega(n|m) = -c + p(n|m)U(n,R/n|m) = -c + p(n|m)(R/n-\lambda c)/[\delta+\lambda-\lambda p(n|m)] = w(n|m)/n$.
We say that m>0 is an <u>oligarchic equilibrium</u> iff $\Omega(m|m) = \max_{n\geq 0} \Omega(n|m)$.
There is an <u>oligarchic equilibrium at 0</u> if $\Omega'(n|m) < 0$ $\forall(n,m)$ such that $n \geq m > 0$.

<u>Proposition 5.</u> When $1 < s \leq 2$ and $\lambda \geq \delta(2-s)/(s-1)$, the oligarchic equilibrium is $m_2 = R[(s-1)\lambda-(2-s)\delta]/[c(\delta+\lambda)s\lambda]$. Recall from Prop 4, m_1 is the negotiation-proof equilibrium for monarchs. If $s < 2$ then $m_2 < m_1$, but m_2/m_1 is increasing in λ/δ and s. If s=2 then m_2=m_1.
If $s \leq 1$ or $\lambda < \delta(2-s)/(s-1)$ then there is an oligarchic equilibrium at 0.

LEADERSHIP, TRUST, AND POWER http://home.uchicago.edu/~rmyerson/research/power.pdf

Why is the Exchequer so called? ...Because the table resembles a checker board... Moreover, just as a battle between two sides takes place on a checker board, so here too a struggle takes place, and battle is joined chiefly between two persons, namely the Treasurer and the Sheriff who sits to render account, while the other officials sit by to watch and judge the proceedings. FitzNigel 1180

What fundamental forces sustain the constitution of a political system?
Constitutional rules are enforced by individuals, who must have incentive to enforce them.
There must be specific agents who expect to be rewarded as long as they act to enforce constitutional rules, but who would lose these rewards and privileges if they did not fulfill their constitutional responsibilities. These are the high officials of government.
A political system can survive only if it solves some basic agency problems in motivating such officials, who are subject to moral hazard and imperfect observability.
So agency problems are essential to the constitution of any political system.

High officials will eschew temptations to abuse power only if they expect future rewards for loyal service, which creates a systematic reason for back-loading their reward.
So the leader should take on debts to his officials, which he may be tempted to repudiate.
A political leader must be like a banker whose debts are valued as rewards for current service.

Officials must lose credit when there is evidence of their malfeasance,
but the leader must be credibly restrained from false judgment to escape from his debts.
Thus, judgments of high officials will require close scrutiny in the leader's court,
so that high government officials should not fear being cheated and replaced.

We show that such agency problems can cause the leader to govern with a closed aristocracy, not based on any innate-inequality assumption of aristocrats being better than commoners,
but based on an innate-equality assumption that commoners are not better than aristocrats.

A model of incentives for governors [D, α, β, γ, K, H, δ]
Suppose a governor always has three options: to be a good governor, or to be corrupt,
or to openly rebel against the leader (flee abroad with local treasures).
Let D denote the expected payoff to a governor when she rebels.
The leader cannot directly observe whether a governor is good or corrupt,
but he can observe any costly crises that may occur under a governor's rule.
When the governor is good, crises will occur in her province at a Poisson rate α.
When the governor is corrupt, crises occur at a Poisson rate β, where $\beta > \alpha$,
and the corrupt governor also gains an additional secret income worth γ per unit time.

The position of governor is quite valuable, but candidates have only some limited wealth K,
and so they cannot pay more than K for the job. Suppose $K < D$.
The leader may derive some advantage from deferring payments to a governor,
but the leader's temptation to sack a governor increases with the debt owed to her.
So let H denote the largest credit owed to a governor that the leader can be trusted to pay.
We assume that each individual is risk neutral and has discount rate δ.

The governor observes any crisis in her province shortly before the leader does.
The governor can make short visits to the leader's court, where the governor cannot rebel.
The leader wants good governing always, because crises and rebellions are very expensive.

We now characterize *an optimal incentive plan* that minimizes the leader's expected cost subject to the constraints that a governor should never want to be corrupt or rebel.
The optimal incentive plan can be characterized by a stochastic process:
$\tilde{U}(t)$ = (the expected present discounted value of pay owed to the governor at time t).
This process will be discontinuous when a crisis occurs.
Let $\tilde{U}(t) = \lim_{\varepsilon\downarrow 0} \tilde{U}(t-\varepsilon)$, $\tilde{U}(t_+) = \lim_{\varepsilon\downarrow 0} \tilde{U}(t+\varepsilon)$.

In any short time interval ε, corruption yields benefits $\gamma\varepsilon$ but increases Pr(crisis) by $(\beta-\alpha)\varepsilon$.
To deter corruption, at each crisis the governor's expected value must drop by $\tau = \gamma/(\beta-\alpha)$.
That is, when there is a crisis at time t, we must have $E(\tilde{U}(t_+)) = \tilde{U}(t)-\tau$.

To deter rebellion, $E\tilde{U}(t_+)$ cannot be less than D after any crisis (governor sees crises first), and so the governor's credit before a crisis can never be less than $G = D + \tau = D + \gamma/(\beta-\alpha)$.
After a crisis, if $\tilde{U}(t)-\tau < G$, the governor should be called to court for a trial where, with probability $(\tilde{U}(t)-\tau)/G$ she is reinstated at $\tilde{U}(t_+)=G$, but otherwise is dismissed (to U=0).
After a dismissal at time t, the new governor must be given the initial credit $\tilde{U}(t_+) = G$, but the leader can recoup part of this value by making the new governor pay K.

When $\tilde{U}(t) < H$, the governor's credit grows between crises at rate $\tilde{U}'(t)=\delta\tilde{U}(t)+\alpha\tau$.
When $\tilde{U}(t) = H$, the governor is paid at rate $y=\delta H+\alpha\tau$ and has $\tilde{U}'(t)=0$ until the next crisis, when $\tilde{U}$ drops to $H-\tau$. So wages are deferred until the central moral-hazard constraint binds.

The leader's value function. Let V(u) be the leader's total expected discounted cost of paying governors in a province, when its current governor has credit u.
When $u < G$, a trial at court either restores the governor to credit G, with probability u/G, or disimisses her and gets a new governor who pays K for the same G status.
So for any $u < G$, $V(u) = V(G)-(1-u/G)K$.
If $G \le u < H$ then over the next short time interval ε we get
$V(u) \approx (1-\delta\varepsilon)[(1-\alpha\varepsilon)V(u+\varepsilon u')+\alpha\varepsilon V(u-\tau)] \approx V(u)+\varepsilon[V'(u)u'-(\delta+\alpha)V(u)+\alpha V(u-\tau)]$,
and so (with $u' = \delta u+\alpha\tau$) $V'(u) = [(\delta+\alpha)V(u)-\alpha V(u-\tau)]/(\delta u+\alpha\tau)$.
V' is discontinuously increases at G, from the left derivative K/G, to the right derivative
$[(\delta+\alpha)V(G)-V(G-\tau)]/(\delta G+\alpha\tau) = \delta(V(G)-K)/(\delta G+\alpha\tau) + K/G$.
$V(H) \approx (\delta H+\alpha\tau)\varepsilon + (1-\delta\varepsilon)[(1-\alpha\varepsilon)V(H)+\alpha\varepsilon V(H-\tau)] \approx V(u)+\varepsilon[\delta H+\alpha\tau-(\delta+\alpha)V(H)+\alpha V(H-\tau)]$,
and so $\delta H+\alpha\tau = (\delta+\alpha)V(H)-\alpha V(H-\tau)$.

Trick to compute V: let $\Psi(u) = [V(u) - uK/G]/[V(G)-K]$.
If $u \le G$ then $\Psi(u) = 1$. If $u \ge G$ then $\Psi'(u) = [(\delta+\alpha)\Psi(u) - \alpha\Psi(u-\tau)]/(\delta u+\alpha\tau)$.
This Ψ can be recursively computed and shown strictly convex for $u \ge G$.
Then to compute $V(u) = uK/G + \Psi(u)[V(G)-K]$, we need to know $V(G)-K$.
But $K/G + \Psi'(H)[V(G)-K] = V'(H) = [(\delta+\alpha)V(H) - \alpha V(H-\tau)]/(\delta H+\alpha\tau) = 1$.
So $V(G)-K = (1-K/G)/\Psi'(H)$ and $V(u) = uK/G + \Psi(u)(1-K/G)/\Psi'(H)$.
<u>Fact.</u> V(u) is increasing and convex in u, with $0 < V'(u) < 1$ when $G \le u < H$.

Optimality. The leader cannot gain by making extraneous bets on his debt u to the governor because V is convex on [G,H]. Randomization happens only in [0,G], where V is linear.
Paying ε to reduce the debt would change the leader's expected cost from V(u) to
$\varepsilon+V(u-\varepsilon) \approx V(u)+\varepsilon(1-V'(u)) > V(u)$, because $V'(u) < 1$ when $u<H$.
So the leader cannot gain by paying the governor when $u < H$.

Properties of the solution

The leader's ex ante expected cost $V(G)-K = (1-K/G)/\Psi'(H)$ is a strictly decreasing function of the bound H. So greater trust is good for the leader.

If H were less than G then it would be impossible for the leader to design an incentive plan such that governors are always deterred from corruption and rebellion in this problem.

The leader randomizes between dismissal and reinstatement after crises when the governor's credit is below $G+\tau$. The threat of dismissal must be moderated by randomization, or else it would incite governors to rebel after crises. Such randomization can be achieved by a "fair trial" at court.
But with K>0, the leader would actually prefer to dismiss than to reinstate $(V(G)-K < V(G))$.
If the leader could not be trusted to randomize appropriately, then he could not always deter corruption and rebellion.
For credibility, the leader may ask other high officials to witness the trial of any one of them (English exchequer, Hittite panku).

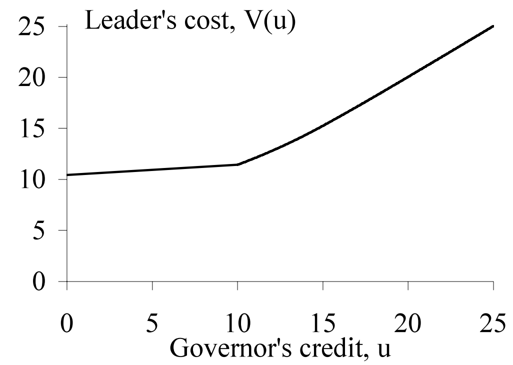
The leader cannot gain by punishing ex-governors. Governors' expected payoffs cannot be less than G, to avoid rebellions, and so any plan to unproductively punish an ex-governor would have to be offset by increasing her probability of reinstatement after a crisis, instead of reselling her office for K. So punishment would reduce the leader's expected income from K.

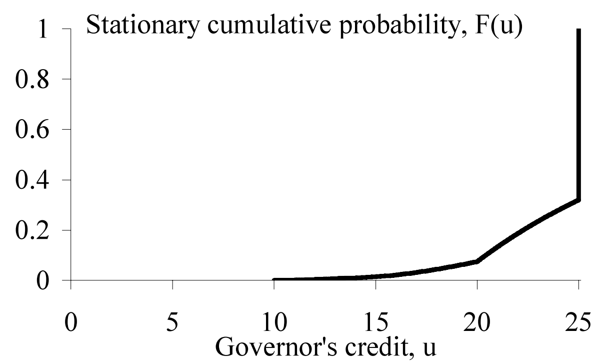
The long-run stationary distribution of credit

In the long-run stationary distribution of the governor's credit $\tilde{u}$, let $F(u) = P(\tilde{u}<u)$.
For any u between G and H, to equalize the rate of transitions upwards across u and downwards across u, we must have $(\delta u+\alpha\tau)F'(u) = \alpha[F(u+\tau)-F(u)]$.
Fact. $-F'(u) = \alpha(F(u)-F(u+\tau))/(\delta u+\alpha\tau)$, $F(G) = 0$, $1-F(H) = P(\tilde{u}=H) > 0$.
F can be computed from the formulas $F(u) = 1-\Omega(u)/\Omega(G)$ where
$\Omega(u)=0\ \forall u>H$, $\Omega(H)=1$, $-\Omega'(u) = \alpha[\Omega(u)-\Omega(u+\tau)]/(\delta u+\alpha\tau)\ \forall u<H$.
Fact. If $u \le H-m\tau$, then $F(u) \le [\alpha\tau/(\delta G+\alpha\tau)]^{m+1}$. With the stationary distribution F,
$E(\tilde{u}) \ge H-\alpha\tau^2/(\delta G)$ and $P(\tilde{u}=H) \ge (\delta H-\alpha\tau^2/G)/(\delta H+\alpha\tau)$.

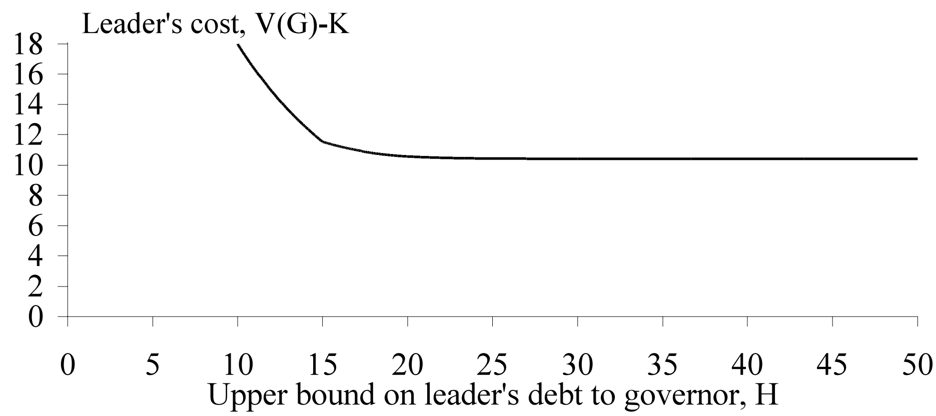
So $P(\tilde{u}=H) = 1-F(H)$ goes to 1 as $H\to\infty$.
If H is high then, in the long run, the leader usually pays high wages $\delta H+\alpha\tau$ to governors.
The long-run expected pay rate is greater than $\delta H-\alpha\tau^2/G$, which goes to ∞ as $H\to\infty$.
The governors' turnover rate $\int_{u\in[G,G+\tau]} \alpha(1-(u-\tau)/G))\, dF(u) \le \alpha F(G+\tau)$ goes to 0 as $H\to\infty$.
So with high H, successful governors tend to become entrenched in office.
Thus, even when the leader is secure in power and is as patient as his agents (same δ), agency costs give the leader an incentive to accumulate large expensive debts to governors.
This effect may also explain dynastic decline.

Example: Let $\delta = 0.05$, $\alpha = 0.1$, $\beta = 0.3$, $\gamma = 1$, $D = 5$, $K = 1$, $H=25$.
Then $\tau = \gamma/(\beta-\alpha) = 5$, $G = D+\tau = 10$, $V(G)-K = 10.44$, $1-F(H) = 0.68$,
E(pay rate) $= (\delta H+\alpha\tau)(1-F(H)) = 1.19$,
E(dismiss rate) $= \int_{u\in[G,G+\tau]} \alpha(1-(u-\tau)/G))\, dF(u) = 0.00030$.





Changing H (from 10 to ∞) could change $V(G)-K = (1-K/G)/\Psi'(H)$ from 18 to 10.42.



With H=10=G, we'd get pay $y=\delta G+\alpha\tau=1$, dismissal rate $\alpha(1-(G-\tau)/G)=0.05$, $V(G)-K=18$.

Discussion This model of high government officials is just a modest elaboration of Becker-Stigler (1974), Shapiro-Stiglitz (1984). As Becker-Stigler emphasized, powerful officials who are hard to monitor should get great rewards for a good record, and the leader who raises them to such great expectations should charge them ex ante for the privilege. They may pay with money, or with underpaid prior service, or with the discharge of a debt owed to an ancestor.
What I want to emphasize here is that this backloading of rewards makes the leader a debtor to his high officials. Thus, an effective leader accumulates debts to high officials,
and he must create institutions that give his officials some power to enforce these debts.

Who has such power over a monarch? The other high officials on whom his regime depends have such power, because they would rationally misbehave or rebel if they lost trust in the prince's promises of future rewards. So the prince needs a court or council where high officials witness his appropriate treatment of other high officials, and where they understand a shared identity among their relationships with the prince in a reputational equilibrium.

Such high councils of government seem universal in political systems (Finer, 1997). In them, the leader's reputation for rewarding his supporters is collectively maintained by his chief supporters

The prospect of high payoffs makes the governor's office a valuable asset that a leader should not waste on a talented person who cannot pay for the office (unless talent inequalities are big).
The leader's ideal would be to sell the office for $K=G$, so that appointments cover their cost.
But when $G=D+\gamma/(\beta-\alpha)$ is large, candidates who can pay so much may not exist.

There is a tension between selling office and randomizing dismissal:
But the prince's ability to resell the office (for $K>0$) means that the prince himself is not indifferent, that the prince would always prefer to dismiss the current governor after a crisis, rather than reinstate her at $U=D+\tau$. But if the governor knew that she'd be dismissed after a crisis, then she would rebel. The outcome of the trial must be unpredictable even to the governor who knows the facts of the case. So the prince's randomization in the trial must be closely monitored by others who have a power to punish the prince. "Fairness" of trials of governors must be actively monitored by others in the leader's court, as the correct outcome cannot be simply predicted from facts of the case.

The Roman emperor Septimus Severus began recruiting lower-class generals, but the Senate could guarantee fair trials only for members of the elite senatorial aristocracy. So our model could explain the increasing frequency of military rebellions after Septimus Severus.
The ultimate fall of the Western Roman Empire followed after the Valentinian III treacherously murdered the Roman general Aëtius.
Similarly, the Ming collapse followed after the unjust execution of Yuan Chonghuan.
The prince needs trust $H \geq D+\gamma/(\beta-\alpha)$, or else corruption and rebellion cannot be deterred.

Political institutions are established by political leaders, and political leaders need active supporters. Like a banker, a leader's promises of future credit must be trusted and valued as rewards for current service. Such a relationship of trust with a group of supporters is a leader's most important asset.

NOTES FOR DAY 2

The two main models today are "A Theory of Capitalist Liberalization" (new here) and "Federalism and Incentives for Success of Democracy" Quarterly Journal of Political Science, 1:3-23 (2006) http://home.uchicago.edu/~rmyerson/research/federal.pdf

These are models in which, even without a democratic election campaign, the possibility of being replaced in power creates some incentive for political leaders to serve broader groups in society.

Other related papers and books worth reading include:

Barro, Robert. 1973. "The control of politicians: an economic model." Public Choice 14:19-42.

Ferejohn, John. 1986. "Incumbent performance and electoral control." Public Choice 50:5-26.

Banks, Jeffrey S., and Rangarajan K. Sundaram. 1998. "Optimal retention in agency problems." Journal of Economic Theory 82:293-323.

Fearon, James D. 1999. "Electoral accountability and the control of politicians: selecting good types versus sanctioning poor performance." In Democracy, Accountability and Representation, edited by Adam Preszeworski, Susan C. Stokes, and Bernard Manin, pages 55-97. Cambridge: Cambridge U. Press.

Timothy Besley, Principled Agents. Oxford (2006).

Acemoglu, Daron, and James A. Robinson. 2000. "Why did the West extend the franchise? democracy, inequality and growth in historical perspective." Quarterly Journal of Economics 115:1167-1199.

Boix, Carles. 2003. Democracy and Redistribution. Cambridge: Cambridge U Press.

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Tiebout (JPE 1956) suggested that local governments could be motivated to provide efficient public goods by the desire to increase their tax base by attracting residents who are free to move from one locality to another.
Epple and Zelenitz (JPE 1981) asked the question: "does Tiebout need politics." Conversely: Can local political leaders be deterred from corrupt profit-taking by citizens who can vote with their feet as effectively as by citizens who can vote democratically to replace their leaders?
They find that the answer to this question is No, because local leaders have the ability to tax away rents of fixed local assets like land, and demand for local land is not infinitely elastic to tax cuts or improvements in local public goods because of congestion effects.
Next we consider a related model, showing how asset mobility can affect the quality of government.

A Theory of Capitalist Liberalization Model on CapitalistLiberalization page at http://home.uchicago.edu/~rmyerson/research/pekingu.xls

A fundamental problem for encouraging investment is that the government officials who enforce property rights may be tempted to abuse these powers and expropriate assets that are the results of others' investment. The rulers of a tightly controlled authoritarian state would have little or no fear of losing power if they expropriated investors' assets. But investors would be able to trust the government more in a political system where the current rulers would risk losing power if they tried to expropriate invested assets, but such risk presumes that investors have some (implicit) political influence. In more liberal state, where people have more freedom to speak and organize without government control, an expropriation of assets by government leaders would have a greater probability of creating a scandal that could cause a change of leadership.

For our purposes here, the probability of political change if the established rulers wrongfully expropriated investments may be considered as a measure of liberalization. A ruler may find some advantages in liberalizing the regime, even though such liberalization creates political risks for him, because such liberalization can encourage greater investment by providing more credible guarantees to investors, and greater investments increase the size of his tax base.

<u>The model</u>: [$Y()$,r,ρ,ψ,θ]
With any capital investment $k \geq 0$, let $Y(k)$ denote the net output production flow in the economy. Here $Y(k)$ is a flow per unit time. To produce the output $Y(k)$, the capital k must be used and controlled by many individuals in the general population, whom we may call capitalists, and their control over the capital would enable them to take it abroad at any time.
The capitalists' rate of time discounting is r. So to deter capital flight, the capitalists must enjoy an income flow worth rk from their capital holdings.
We may assume that $Y(k)$ is net of labor and resource costs, so the authoritarian rulers of the government can take (in taxes) the remaining flow $Y(k)-rk$.
Let ρ denote the rate of time discounting for a ruler who has not liberalized, which may be different from r because, for example, the ruler might face some exogenous risk of losing power without liberalization, which would increase ρ above r.
When the regime has liberalization λ, the probability of the ruler losing power if he tried to expropriate capital (or if he tried to reduce liberalization) would be λ.
But there are also false-alarm scandals that occur at some Poisson rate ψ, and people react to such scandals exactly as they would to a genuine attempt to expropriate capital.
So for a regime with liberalization λ, when the government is actually not trying to expropriate

capital, still even in any short time interval of length ε there is approximatly $\psi\varepsilon$ probability of a scandal and approximately $\psi\varepsilon\lambda$ of the rulers being replaced because of such a scandal.
So in a regime with liberalization λ, the current ruler discounts future revenue at rate $\rho+\psi\lambda$, and so with invested capital k, the ruler's present discounted value is $V(k,\lambda) = (Y(k)-rk)/(\rho+\psi\lambda)$.

But consider what would happen if the ruler tried to expropriate the capital. With probability λ the ruler would lose power and get 0 thereafter. Otherwise, with probability $1-\lambda$, the ruler would successfully seize some fraction θ of the capital, where $1-\theta$ denotes the fraction of capital that would be lost or destroyed in the struggle or taken abroad by fleeing capitalists. Thereafter the ruler would have lost any reputation for protecting capital and so may as well deliberalize, and so the value of his continuation in power, without any free investment or liberalization, would be $Y(0)/\rho$ after a successful expropriation. So the ruler's expected discounted value from trying to expropriate capital would be $W(k,\lambda) = (1-\lambda)(\theta k + Y(0)/\rho)$.

Capital k can be safely invested in a regime with liberalization λ iff k and λ satisfy the ruler's incentive constraint $V(k,\lambda) \geq W(k,\lambda)$. Of course capital must be nonnegative $k\geq 0$.
As liberalization λ here is a probability, it must satisfy the probability constraints $0 \leq \lambda \leq 1$.
The ruler's optimal regime (λ,k) should maximize $V(k,\lambda)$ subject to these constraints.

We assume that Y(k) is a continuously differentiable and strictly concave function of k, with $Y(0)\geq 0$ and $\text{limit}_{k\to\infty} Y'(k) = 0$. The other given parameters satisfy $r>0$, $\rho>0$, $\psi>0, 0<\theta\leq 1$.

Basic analysis.
Let K^* denote the ideal unconstrained investment level that satisfies $Y'(K^*) = r$.
If K^* were incentive-feasible with $\lambda=0$ then $(K^*,0)$ would be trivially optimal. To rule out trivial solutions, suppose that $(Y(K^*)-rK^*)/\rho < \theta K^* + Y(0)/\rho$, so that K^* is infeasible,
and also suppose that $Y(k)-rk > 0$ for at least some small k (as if $Y(0)>0$ or $Y'(0)>r$)

The basic incentive constraint $V(k,\lambda) = (Y(k)-rk)/(\rho+\psi\lambda) \geq W(k,\lambda) = (1-\lambda)(\theta k + Y(0)/\rho)$
is equivalent to the inequality $(Y(k)-rk)/(\theta k + Y(0)/\rho) \geq (\rho+\psi\lambda)(1-\lambda)$.
So let $Q(k) = (Y(k)-rk)/(\theta k + Y(0)/\rho)$, with $Q(0)=\rho$, and let $q(\lambda) = (\rho+\psi\lambda)(1-\lambda)$.
The quotient $Q(k) = (Y(k)-rk)/(\theta k + Y(0)/\rho)$ is the ruler's rate of revenue per unit of expropriatable wealth when capitalist investment is k. The quadratic $q(\lambda) = (\rho+\psi\lambda)(1-\lambda)$ is the ruler's required rate of return on expropriatable assets when liberalization is λ.
Then we can rewrite the incentive constraint as $Q(k) \geq q(\lambda)$.
Given any k, $V(k,\lambda)$ is decreasing in λ, so the rulers would prefer the smallest feasible λ.
So for any k such that $Y(k)-rk\geq 0$, let $\Lambda(k)$ denote the smallest $\lambda\geq 0$ such that $Q(k) \geq q(\lambda)$.
Notice $q(0) = \rho$. So if $Q(k) \geq \rho$ then $\Lambda(k) = 0$.
That is, investment k is compatible with the ruler's ideal of nonliberalization ($\lambda=0$) iff $Q(k) \geq \rho$. or equivalently $Y(k)- (r+\rho\theta)k \geq Y(0)$. With Y() concave, the set of k that satisfy this inequality is an interval $[0,K_0]$ for some $K_0 \geq 0$ which satisfies the binding constraint equation
$Y(K_0)-(r+\rho\theta)K_0=Y(0)$, and so $Q(K_0) = \rho$. With the nontriviality assumption, we have $K_0 < K^*$, and so $V(k,0)$ is increasing over k in $[0,K_0]$, and so the best investment without liberalization is K_0.

Now consider what can be achieved with positive liberalization $\lambda>0$.
An optimal investment k that requires positive liberalization must have $0 < Q(k) < \rho$.
($Q(k)\leq 0$ would imply $F(k)\leq rk$ and so $V(k,\lambda)\leq 0$ for all λ, and so such k could not be optimal.)
For any feasible investment k, if the incentive constraint were not binding then the ruler could increase his objective $V(k,\lambda)$ by decreasing λ slightly.

So any optimal regime (k,λ) with $\lambda>0$ must have $V(k,\lambda)=W(k,\lambda)$ and so $0 < Q(k) = q(\lambda) < \rho$.
With $0 < Q(k) < \rho$, the unique $\lambda>0$ that satisfies $Q(k) = q(\lambda) = \rho+(\psi-\rho)\lambda-\psi\lambda^2$ is $\lambda=\Lambda(k)$
where $\Lambda(k) = \{\psi-\rho + [(\psi-\rho)^2 + 4\psi(\rho-Q(k))]^{0.5}\}/(2\psi) = \{\psi-\rho + [(\psi+\rho)^2 - 4\psi Q(k)]^{0.5}\}/(2\psi)$.
Notice that this formula yields $(\psi-\rho)/\psi < \Lambda(k) < 1$ when $0 < Q(k) < \rho$.
[The quadratic $q(\lambda)$ is maximal at $\lambda=0.5(1-\rho/\psi)$, and $q(0) = \rho = q(1-\rho/\psi)$.]
Then the optimal investment is the k that maximizes $V(k,\Lambda(k))$ over all k between 0 and K^*.

Further analysis The objective function $V(k,\Lambda(k))$ may not be concave in k.
Notice that the quadratic $q(\lambda)$ is maximal at $\lambda = 0.5(1-\rho/\psi)$, and $q(1-\rho/\psi) = \rho = q(0)$.
So if $\rho<\psi$ then $\Lambda(k)$ has a discontinuity at $k=K_0$.
By definition of K_0 as the largest feasible k with $\lambda=0$, we have $Q(K_0)=\rho$ and $\Lambda(K_0) = 0$.
But with any $k>K_0$, we get $Q(k)<\rho$, and then the formula for $\Lambda(k)$ yields $\lambda > (\psi-\rho)/\psi = 1-\rho/\psi$.
At $k>K_0$, however $\Lambda(k)$ is continuously differentiable, and we have derivatives
$Q'(k) = (Y'(k) - r - \theta Q(k))/(\theta k+Y(0)/\rho)$, $q'(\lambda) = \psi-\rho-2\psi\lambda$,
$\Lambda'(k) = Q'(k)/q'(\Lambda(k)) = -[Y'(k) - r - \theta q(\Lambda(k))]/[(\theta k+Y(0)/\rho)(2\psi\Lambda(k) + \rho - \psi)]$.
Here $(2\psi\Lambda(k)+\rho-\psi)$ is positive, as $\lambda>\max\{0,1-\rho/\psi\} \Rightarrow 2\psi\lambda+\rho-\psi>\rho-\psi$ and $2\psi\lambda+\rho-\psi>\psi-\rho$.
The ruler's marginal value of additional investment (with the necessary liberalization) is then
$d/dk\ V(k,\Lambda(k)) = d/dk\ W(k,\Lambda(k)) = (1-\Lambda(k))\theta - (\theta k+Y(0)/\rho)\Lambda'(k)$
$= (1-\Lambda(k))\theta + [Y'(k) - r - \theta(1-\Lambda(k))(\rho+\psi\Lambda(k))]/(2\psi\Lambda(k) + \rho - \psi)$
$= [Y'(k) - r - \theta\psi(1-\Lambda(k))^2]/(2\psi\Lambda(k) + \rho - \psi)$.
So a locally optimal capital with $d/d\ V(k,\Lambda(k)) = 0$ must have $Y'(k) = r+\theta\psi(1-\Lambda(k))^2$.
When k satisfies the local-optimality condition, we have
$0 = d/dk\ W(k,\Lambda(k)) = (1-\Lambda(k))\theta - (\theta k+Y(0)/\rho)\Lambda'(k) = 0$,
and so $\Lambda'(k) = (1-\Lambda(k))\theta/(\theta k+Y(0)/\rho) > 0$.
Summarizing, we get the following characterization of an optimal liberalized regime.

Fact If the optimal regime (k,λ) has $\lambda=0$ then $k=K_0$ where $Y(K_0) - (r + \rho\theta)K_0 = Y(0)$.
On the other hand, if the optimal regime (k,λ) has $\lambda>0$ then it must satisfy the equations
$(Y(k)-rk)/(\theta k + Y(0)/\rho) = (\rho+\psi\lambda)(1-\lambda)$ and $Y'(k) = r+\theta\psi(1-\Lambda(k))^2$.
It must also satisfy the inequalities $\max\{0,1-\rho/\psi\} < \lambda < 1$ and $k > K_0$.

Example 0 Let our basic parameters be $r = \rho = 0.05$, $\psi = 0.1$, $\theta = 1$, and $Y(k) = 0.2k^{0.8}$.
The ideal without incentive constraints is $K^*=335.5$, $Y(K^*)=20.97$, $Y'(K^*)=0.05$.
With incentive constraints, the optimal regime is $k = 32$, $\lambda = 0$, $Y(k) = 3.2$, $V(k,\lambda) = 32$.
Reducing ψ to $\psi=0.09$ would change the optimum to $k = 303.9$, $\lambda = 0.895$, $Y=19.38$, $V=32.02$.
(Further reductions of the scandal rate ψ would reduce λ a bit, as the $V\geq W$ constraint is relaxed.)

Example 1 Let our basic parameters be $r = \rho = 0.05$, $\psi = 0.1$, $\theta = 1$.
Suppose that output is produced by three factors, which we may call labor (L), capital (K), and land (D) according to the function $L^{0.4}K^{0.5}D^{0.1}$.
Each island is endowed with 1 unit of fixed labor (L=1), one unit of fixed land (D=1).
If there were no capital on an island, the ideal would be feasible: $K^*=100$, $\lambda=0$, $Y=10$, $V= 100$.
But suppose instead that each island is endowed with 25 units of fixed capital (F=25), which can be augmented by capitalist investment k to yield output
$Y(k) = L^{0.4} (F+k)^{0.5} D^{0.1} = (25+k)^{0.5}$ with the given endowments L=1, F=25, D=1.
With this production function, the optimal regime for the ruler of each island is $k=0$, $\lambda=0$.
Although $Y'(0) = 0.1 > r$, any positive investment would require a liberalization $\lambda>0.5$, so that

rulers prefer not to encourage any investment. (This is the curse of natural resources.)
Production on each island is then $Y(0) = 5$, and the ruler's value is $V(0,0)=100$.
We have ignored to cost of wages, assuming that most laborers are bound serfs who are forced to work for negligible wages. But the the marginal product of labor in this solution is $.4Y(0)/L = 2$, which would be the wage rate for any free labor on each island.

Example 2 (Tiebout effects)
Now suppose that one island considers the possibility of attracting additional free laborers from other islands, at the given wage rate $w=2$. So with investment k and new free immigrant labor n on this island, the product net of wage costs would be $\hat{Y}(k,n) = (1+n)^{0.4}\,(25+k)^{0.5}(1)^{0.1} - 2n$.
For any given k, the maximum of this net product over all $n\geq 0$ is achieved when
$2 = 0.4(25+k)^{0.5}/(1+n)^{0.6}$, $2(n+1) = .4(1+n)^{0.4}(25+k)^{0.5}$, and $(1+n)^{0.4} = (.2)^{2/3}\,(25+k)^{1/3}$.
So $Y(k) = \max_{n\geq 0} \hat{Y}(k,n) = (1-.4)(.2)^{2/3}(25+k)^{5/6} + 2 = 0.2052(25+k)^{5/6} + 2$.
With this production function, the optimal regime on this island is $k=1431.6$, $\lambda=0.911$, so that production is $Y(k)=90.77$, with new labor $n=28.6$, and the ruler's value is $V(k,\lambda)=136.0$.
So mobility of labor (or other resources that are complementary to capital) can encourage a ruler to liberalize in a way that increases production and income for others in society.

Federalism and Incentives for Success of Democracy
R. Myerson, Quarterly Journal of Political Science, 1:3-23 (2006).
http://home.uchicago.edu/~rmyerson/research/federal.pdf
"Countries in transition that have aimed for national elections as a first step (Bosnia for example) have bogged down and generally handed power over to avatars of the old regime. By contrast, Kosovo and East Timor began with local elections, with a far better result of bringing forward new talents and capabilities, and giving people a sense of empowerment."
Final Report on the Transition to Democracy in Iraq (Nov, 2002), page 24.

1. How may the chances of success for a new democracy depend on its consitutional separation of powers? Constitution as rules of the game.
A new democracy can't guarantee success by copying constitution of an established democracy.
There are multiple equilibria, so culture matters.
What could make a nation culturally unready for democracy?
New and established democracies may systematically differ in the kinds of reputations that people attribute to their political leaders.
Any institution is sustained by individuals (officials) who expect to enjoy privileged status as long as they act according to the institution's rules. Such a reputational eqm is necessarily one of many eqms in the game, because loss of status does not change the intrinsic nature of any individual.
When democracy is new in a nation, no politician has an established reputation for responsibly using political power to serve the general population.
Reputational incentives in old regime: to serve superiors and reward supporters.
Voters may expect the first leader to suppress opposition, abuse power to benefit himself and his active supporters; any replacement may be expected to do same.
Structures that have been found to improve chances for success of democracy: parliamentarism with PR, federalism. Both increase opportunities for independent leaders to cultivate reputations for responsible use of power.
(Note: leaders may dislike structures that increase political competition.)
In a unitary democracy, we find multiple equilibria in the dynamic political game: equilibria where democracy succeeds, and where democracy is frustrated. But we show that democracy cannot be consistently frustrated in equilibrium at both levels in a federal structure with separation of powers, nor in a transition process where local democracy precedes national elections.

2. Basic model of unitary democracy
In each period, there is an election, then leader serves responsibly or corruptly:
b = the leader's benefit (each period) when he serves responsibly,
$b+c$ = leader's benefit from serving corruptly,
0 = politicians payoff out of office,
w = expected welfare for voters when leader serves responsibly,
0 = expected welfare for voters when leader serves corruptly,
x = expected transition cost for voters when changing to a new leader,
ρ = discount factor per period. All actions observable.
ε = probability that any new politician is always-responsible virtuous type;
$1-\varepsilon$ = probability of being normal, maximizing expected payoff as above.
Voters agree, so assume election determined by any representative voter.
Transition cost x may be due to new leader learning on job, or to thefts by outgoing leader, or to activists' costs of opposing an incumbent.

At any point in any equilibrium of this game, we may say that democracy
<u>succeeds</u> if the leader is expected to serve responsibly always (with prob'y 1);
is <u>frustrated</u> if the leader would be reelected always even after acting corruptly.
(Success is optimal for voters. Frustration is optimal for the incumbent leader.)
In eqm, frustration implies that only a virtuous leader would serve responsibly (= <u>failure</u> of democracy).

<u>Theorem 1.</u> Suppose $\varepsilon < x(1-\rho)/w < 1$ and $b+c < b/(1-\rho)$. Then there exists a good equilibrium where unitary democracy succeeds, but there also exists a bad equilibrium where unitary democracy is frustrated.

By first condition, $\varepsilon w/(1-\rho) < x < w/(1-\rho)$, so voters would replace a corrupt leader if replacements always serve responsibly, but not if only virtuous do so.
Second: politicians prefer serving responsibly forever over corruptly one period.
Here $x(1-\rho)/w$ is the lowest probability of a new leader serving responsibly such that national voters would be willing to replace a corrupt leader.

2.1. Variations on the basic model (Section 5 in paper)

Variation A:
With probability δ of an incompetent type who'd generate costs $-x/\delta$, voters get an expected cost $-x$ of trying new leadership. Taking $\delta \to 0$ yields the basic model.
(But in federal extension, there will be no cost of promoting a governor who has proven that he is not incompetent, making our positive results easier to prove.)

Variation B:
Each period's transition cost x is set by the incumbent from the previous period, subject to a constraint $0 \le x \le X$, where X is given maximal oppression level.
We may suppose that a virtuous leader would always choose $x = 0$.
Then in the conditions of Thm 1, we replace x by its upper bound X.
With $\varepsilon w/(1-\rho) < x \le X < w/(1-\rho)$, voters would resist corrupt oppression if they expect challengers to serve responsibly, but not if they expect normal challengers to become corrupt.
Either can happen in equilibrium.

Variation C:
Voters do not observe leader's action, but observe their welfare which is a uniform random variable over $[0-\Delta, 0+\Delta]$ or $[w-\Delta, w+\Delta]$, depending on whether the leader serves corruptly or responsibly.
For interest, suppose $0+\Delta > w-\Delta$.
To have an equilibrium where democracy succeeds, "$b+c < b/(1-\rho)$" in Thm 1 must be changed to $(b+c)/(1-\rho(1-0.5w/\Delta)) < b/(1-\rho)$.
Then success can be supported by voters reelecting a leader iff he has always generated welfare above the cutoff $w-\Delta$.
But higher standards may be incompatible with success of democracy in eqm:
Example: $w=\Delta=1$, $b=1$, $c=4$, $\rho=0.9$.
With cutoff $w-\Delta = 0$: $(1+4)/(1-0.9\times0.5) = 9.091 < 10 = 1/(1-0.9\times1)$.
When cutoff for reelection is 1: $(1+4)/(1-0.9\times0) = 5 > 1.818 = 1/(1-0.9\times0.5)$.
When cutoff for reelection is -1: $(1+4)/(1-0.9) = 50 > 10 = 1/(1-0.9)$.
(See Banks and Sundaram, 1993.)

3. Federal democracy.
N = number of provinces.
In each period, elect national president, then governor in each province,
each serves corruptly or responsibly.
b_1 = president's benefit (each period) when he serves responsibly,
b_1+c_1 = president's benefit from serving corruptly,
b_0 = governor's benefit when he serves responsibly,
b_0+c_0 = governor's benefit from serving corruptly,
0 = politician's payoff out of office.
w_1 = welfare for national voters with president serving responsibly,
0 = welfare for national voters with president serving corruptly,
x_1 = expected transition cost for voters when changing to a new president,
w_0 = welfare for provincial voters with governor serving responsibly,
0 = welfare for provincial voters with governor serving corruptly,
x_0 = expected transition cost for voters when changing to a new governor,
(but no cost when replacing a governor who's been promoted to president).
ρ = discount factor per period.
ε = probability that any new politician is always-responsible <u>virtuous</u> type.
Elections at each level are determined by voters' expected payoffs from this level of government, ignoring any effects from the other level of government.
(Spse national elections are not influenced by local effects in any one province of its governors becoming president; and provincial elections are not influenced by the national benefits of searching for better presidential candidates.)

<u>Basic assumptions:</u> $\varepsilon < x_0(1-\rho)/w_0 < 1$, $b_0+c_0 < b_0/(1-\rho)$,
$\varepsilon < x_1(1-\rho)/w_1 < 1$, $b_1+c_1 < b_1/(1-\rho)$, and $b_1 > b_0 + c_0$.
So multiple equilibria would exist at each level if it existed alone, and
politicians want promotion from governor to president.

With N large, P(no province has a virtuous governor) = $(1-\varepsilon)^N \le e^{-\varepsilon N}$ is small,
so there are likely to be some provinces where politicians have good reputations
(assuming candidates are recruited independently from pop'n in each province).

3.1 Equilibria of federal democracy
At either level (national or provincial), we may say that democracy:
<u>succeeds</u> if voters expect leader to serve responsibly always with prob'y 1;
is <u>frustrated</u> if the leader would always get re-elected even after acting corruptly.
National frustration implies that a normal president will act corruptly (failure).

∃ eqm where provincial democracy succeeds but national democracy is frustrated
(corrupt governors would not be re-elected, so all governors act responsibly; national voters understand that any governor would become corrupt with prob'y $1-\varepsilon$ after election to the presidency, so corrupt presidents are re-elected).

∃ eqm where provincial democracy is frustrated but national democracy succeeds
(a rare governor who serves responsibly can be identified as virtuous, but that doesn't make him more attractive to national voters, who expect any president to act responsibly for re-election; so governors have no motive to be responsible).

But such mixed equilibria require voters to have inconsistent expectations about functioning of democracy at different levels, and so seem less likely to be focal.
$\exists$ eqm where provincial and national democracy both succeed (presidents and governors always act responsibly, else they would not be re-elected).
But no eqm has sure frustration at both levels.

Theorem 2. In a sequential equilibrium of the federal game, as long as some province has a governor who has not yet acted corruptly, democracy cannot be frustrated both at the national level and at all provincial levels.

Proof. If democracy is frustrated at the national level, then the current president can get his optimal outcome by always serving corruptly, given that the frustrated voters will never replace him. (frustration => failure at national level)
So if he acts corruptly this period, then he is normal and should be expected to always act corruptly thereafter. Frustration of national democracy also imples that governors have no hope of promotion to president. So with frustration of provincial democracy, a normal governor would have no incentive to serve responsibly. So if a governor continued serving responsibly, then voters would infer that he must be virtuous, but then (with $x_1 < w_1/(1-\rho)$) they could do better by choosing him to replace the current president. (=><= nat'l frustration)

4. Provisional decentralization in a process of transition to unitary democracy.
Consider a process of transition to democracy with an initial phase of T periods when there will be only local democracy in the N provinces in confederation, after which a unitary national democracy will be established in period T+1.
Each politician initially has a small prob'y ε of being the virtuous type.

Theorem 3. Suppose $\rho^T(b_0+c_0) > (1-\rho^T)c_0$, $b_1+c_1 \geq N(b_0+c_0)$, and $w_0 > x_0$.
In any equilibrium where national democracy is expected to be frustrated after period T, decentralized democracy must succeed until period T, and any corrupt governor would be replaced by provincial voters. So there cannot be consistent frustration of democracy in any equilibrium of this transitional process. But there is an eqm in which democracy consistently succeeds at all periods.
(First inequality holds if $\rho^T \geq 0.5$, so $T \leq 13$ with ρ=0.95. Second says that the unitary national leader gets all power held initially by the N provincial leaders.)

Proof. Assuming normal presidents will be corrupt after T, the national voters at T+1 will elect a president with highest prob'y of being virtuous, given his record.
(Corrupt governor's prob'y of virtue = 0 < ε = any layman's prob'y of virtue.)
If some governors had any positive prob'y of acting corruptly, then by acting responsibly they could make voters believe that their probability of being virtuous was more than ε, and so one of them would be elected president.
There can be at most N such governors alive with good reputations at T+1,
and so some of them must expect prob'y at least $1/N$ of being elected president.
A governor's expected cost of governing responsibly for T periods is
$c_0(1 + \rho + ... + \rho^{T-1}) = c_0(1-\rho^T)/(1-\rho)$, but his expected gain from being a candidate for president after T periods is at least $\rho^T(1/N)(b_1+c_1)/(1-\rho)$.
The inequalities in the thm imply that this gain is strictly greater than the cost, and so no governor would choose to behave corruptly in the first T periods.
A governor with a corrupt record would have no incentive to be responsible at T, so (with $w_0 > x_0$)

provincial voters would replace him at T;
similarly, by backwards induction, a corrupt governor would be replaced earlier.
Eqm where democracy consistently succeeds: provincial voters at periods 2,...T and national voters after T+1 would reject an incumbent who has acted corruptly, and national voters at period T+1 will select a president at random from among the governors (if any) who served responsibly at period T.
Governors are responsible at any period $t \leq T$ because the basic assumptions and inequalities in thm imply $b_0(1-\rho^{T+1-t})/(1-\rho) + (1/N)\rho^{T+1-t} b_1/(1-\rho) \geq b_0+c_0$.

6. Discussion
In unitary democracy, success or frustration of democracy depends on eqm beliefs of voters and politicians in a dynamic political game.
In a new democracy, where no politicians have reputations responsibly serving citizens (worse, they've been building reputations for rewarding superiors and supporters in ruling elite), it's particularly likely that bad eqm will be focal.
Under federalism, an anticipated frustration of democracy at the national level would increase incentives for local politicians to make democracy succeed.
So a federal system can offer an insurance policy against total frustration of democracy: voters will see benefits of democracy at some level of government.

We didn't predict local government would be more or less corrupt than national. If long-run survival of democracy depends on its success at the national level, then survival selection should generate a population of democracies where federal countries have statistically more corruption than unitary (Treisman).
But if local democratic success can teach voters to expect national democratic success, then federalism should yield statistically higher survival rates (Boix).
Strong regional identities could undermine our argument: if the most likely behavioral-type were not generally virtuous but only locally chauvinistic, then responsible local service may not be effective for appealing to national voters.

To refocus the analysis of democratic failure, we have been assuming that voters' constitutional power to replace an incumbent leader is not in question.
By classic Madisonian arguments, constitutional constraints on leaders must be enforceable by other leaders with appropriate power and motivation, and so the protection of constitutional limits requires a system of separation of powers. The argument here may be viewed as an extension or modification of this classic argument for how federal separation of powers can support democratic survival. (Local leaders do not force national elections, but give reason to demand them.)

Under any system of separation of powers, agency problems can increase political corruption at the boundaries where mixed effects of different branches of government make responsibility unclear (Treisman, 1999). But regional separation of powers in federalism may yield clearer boundaries than other functional ways to separate powers.

Our analysis can be understood with parallels to oligopoly theory, where profit-taking is reduced by higher elasticity of demand and lower barriers to entry.
Political corruption may be seen as an analogue of oligopolistic profit.
In our argument, federalism lowers entry-barriers into national politics when it gives local leaders an opportunity to prove qualifications for national leadership.

The possibility of advancement to greater national office gives local leaders a higher elasticity of demand for leadership with respect their corruption-price.
Such political elasticity can also be created in a federal system by Tiebout effects: with national mobility of people and resources, local corruption erodes its own tax base.

Our conclusion that federalism sharpens political competition offers insights into tensions of the federal bargain between national and provincial leaders (Riker).
Seeing governors as potential rivals for power, national leaders prefer criminal punishment of provincial corruption, to prevent governors from building virtuous reputations without habituating voters to reject corrupt incumbents.
If national leader can influence selection of governors, he'd prefer governors who have been corrupt, so they cannot use the office to cultivate a virtuous reputation.
The appeal of secession for governors (especially when corrupt) is increased when local rivals' national ambitions make local politics more competitive.
Bi-level political parties serve politicians by moderating this competitive tension.

Our analysis should have implications for current efforts to cultivate democracy.
From our perspective, the danger of the democratic-failure equilibrium is greatest where historical experience suggests that citizens should expect little from their leaders, as in American-occupied Iraq. If local elections were held first in a decentralized provisional government, then local leaders with national ambitions would have a positive incentive to begin cultivating a reputation for responsible democratic leadership. Although the Democratic Principles Work Group (2002) suggested just such a plan, the most influential leaders had no incentive to recommend a decentralized system designed to lower the entry-barriers against new political rivals.

The effects of successful democracy, which should make voters want to defend a democratic system, could also make politicians want to undermine it.
Political leaders should prefer constitutional structures that have equilibria where democratic competition is frustrated, in our sense. If the voters do not understand how different constitutional structures would affect the quality of political competition, then political leaders are likely to get the less competitive constitution that they prefer.

Democracy is worth cultivating because the structure of political institutions matters. Thus we should actively search for political structures that can maximize the chances of success for new democracies. At a time when great armies have been sent across the world with an announced goal of building new democracies, the finer points of comparative institutional analysis may have a practical importance that should not be overlooked.

NOTES FOR DAY 3

The **selectorate model** is from Bueno de Mesquita, Siverson, Smith, & Morrow's paper in the American Political Science Review (1999) and book Logic of Political Survival (see pp 104-126).
The "selectorate" is some politically active group of S people (the selectors).
In each period (1,2,3,....), the incumbent ruler (in power last period) faces a new challenger.
The incumbent can hold power by forming a winning coalition with W supporters whom he recruits from the selectorate, but the challenger wins if he can take away at least one of the incumbent's supporters and get at least $W-1$ other supporters.
The ruler in each period has a resource budget R that he can spend on public goods, on private consumption for each supporter whom he designates, and on his own private consumption.
In a period when public-good spending is g, the current utility of an individual who gets private consumption x is $Ag^\gamma+x$. Future utility is discounted by the discount factor δ per period.
These parameters satisfy $S>W>0$, $R>0$, $A>0$, $1>\gamma>0$, $1>\delta>0$.

The incumbent and challenger compete by making promises of how much they will spend on public goods and on members of their coalition, but all coalition members must get the same amount.
First the incumbent designates his coalition of W supporters, and announces how much g_1 he will spend on public goods and how much private consumption x_1 each of his supporters will get
Then the challenger invites his coalition of W supporters and promises how much g_2 he will spend on public goods and how much x_2 he will pay each of his supporters.
Paying less to self is not credible, so plans must satisfy $g_i \geq 0$, $x_i \geq 0$, and $g_i+(W+1)x_i \leq R$, for i=1,2.
As this model makes the incumbent indifferent between any coalition of supporters, BdM et al. assume that the incumbent will recruit the W selectors with whom he feels the closest "affinity," but he will not know this affinity until he becomes the incumbent sitting in the presidential palace.
We look for stationary equilibria such that, in each period, the incumbent's equilibrium offer is always expected to be to spend the same amount amount G on public goods and X on private consumption for each of W favorite supporters (spending the rest $R-G-WX \geq 0$ on himself), and such that the incumbent is actually expected to defeat the challenger each period.
It is assumed that the challenger cannot commit to behave differently from the equilibrium expectation in the future, and cannot commit to retain supporters who do not have highest affinity.
So each current supporter of the challenger thinks that her probability of being retained by the current challenger after he becomes incumbent is W/S, and so each current supporter's expected utility from supporting the challenger is $(Ag_2^\gamma + x_2) + \delta(AG^\gamma + (W/S)X)/(1-\delta)$.
Current supporters of the incumbent know that they will always be in his favorite coalition, and so their expected utility from supporting the incumbent is $U_1 = (Ag_1^\gamma + x_1) + \delta(AG^\gamma + X)/(1-\delta)$.
Let u_2 be the maximum of $Ag_2^\gamma+x_2$ over (g_2,x_2) subject to $g_2 \geq 0$, $x_2 \geq 0$, $g_2+(W+1)x_2 \leq R$.
Then the incumbent can deter challengers with current promise (g_1,x_1) if
$u_2 + \delta(AG^\gamma + (W/S)X)/(1-\delta) \leq (Ag_1^\gamma + x_1) + \delta(AG^\gamma + X)/(1-\delta)$
or, equivalently $Ag_1^\gamma + x_1 + \delta(1-W/S)X/(1-\delta) \geq u_2$.
So the incumbent's optimal winning strategy is to choose (g_1,x_1) that maximize $Ag_1^\gamma+(R-g_1-Wx_1)$ subject to $Ag_1^\gamma + x_1 + \delta(1-W/S)X/(1-\delta) \geq u_2$, $g_1 \geq 0$, $x_1 \geq 0$, $g_1+(W+1)x_1 \leq R$.
In equilibrium, this optimal solution must satisfy the rational-expectations conditions: $g_1=G$, $x_1=X$.

Examples: Suppose $A=1$, $\gamma=0.5$, $R=1000$, $\delta=0.9$.
With $S=100$, $W=20$, we get $g_2=110.25$, $x_2=42.37$, $u_2=52.87$, $G=110.25$, $X=5.17$, $U_1=15.67$.
With $S=100$, $W=50$, we get $g_2=650.25$, $x_2=6.86$, $u_2=32.87$, $G=650.25$, $X=1.25$, $U_1=26.75$.
With $S=200$, $W=100$, we get $g_2=1000$, $x_2=0$, $u_2=31.62$, $G=100$, $X=0$, $U_1=31.62$.
Thus, even members of the incumbent's favored coalition may benefit from extending the franchise and requiring larger coalitions to win, so that political leaders will compete more in public goods.

[See also A. Lizzeri and N. Persico, "Why did the elites extend the suffrage?" Quarterly J of Economics 119:707-765 (2004), for a similiar argument about why the voting population was expanded in England in 1830s, but with a different model.]

How to solve these examples:
A challenger's most generous offer has bribes $x_2 = (R-g_2)/(W+1)$, and public goods g_2 maximizing $Ag_2^{\gamma}+(R-g_2)/(W+1)$, which has first-order condition $0 = \gamma A g_2^{\gamma-1} - 1/(W+1)$.
But we must have $g_2 \leq R$ to get $x_2 \geq 0$.
So we get $g_2 = \min\{[(W+1)\gamma A]^{1/(1-\gamma)}, R\}$, $x_2 = (R-g_2)/(W+1)$, and $u_2 = Ag_2^{\gamma}+x_2$.
g_1 and x_1 maximize $Ag_1^{\gamma}+(R-g_1-Wx_1)$ subject to $Ag_1^{\gamma} + x_1 + \delta(1-W/S)X/(1-\delta) \geq u_2$.
First-order optimality conditions with Lagrange multiplier λ are
$0 = \gamma A g_1^{\gamma-1} - 1 + \lambda\gamma A g_1^{\gamma-1}$ and $0 = -W + \lambda$, which imply $0 = \gamma A g_2^{\gamma-1} - 1/(W+1)$.
So we get $g_1 = \min\{[(W+1)\gamma A]^{1/(1-\gamma)}, R\} = g_2$,
and so $x_1 = x_2 - \delta(1-W/S)X/(1-\delta)$ satisfies the constraint equation.
The rational expectations condition $X = x_1$ gives us $x_1 = x_2/[1+\delta(1-W/S)/(1-\delta)]$.

An impossibility theorem of social choice
http://home.uchicago.edu/~rmyerson/research/schch1.pdf
Can a political institution abolish multiple equilibria?
A variant of Arrow's impossibility theorem says No.

Let N denote a given set of individual voters.
Let Y denote a given set of social-choice options, of which the voters must select one.
We assume that N and Y are both nonempty finite sets.
Let L(Y) denote the set of strict transitive orderings of the alternatives in Y.
Let $L(Y)^N$ denote the set of profiles of preference orderings, one for each voter.
We may denote such a preference profile by a profile of utility functions $u = (u_i)_{i\in N}$, where each u_i is in L(Y). So if the voters' preference profile is u, then the inequality $u_i(x) > u_i(y)$ means that voter i prefers alternative x over alternative y.
($u_i(x) = \#\{y\in T|\ x$ is preferred to y under i's preference in u$\}$.)

A <u>social choice function</u> is any function $F{:}L(Y)^N \to Y$, where F(u) denotes the alternative in Y to be chosen if the voters' preferences were as in u.
Let $F(L(Y)^N) = \{F(u)|\ u\in L(Y)^N\}$.
Given any game form $H{:}\times_{i\in N} S_i\to Y$ (where each S_i is a nonempty strategy set for i),
let E(H,u) be the pure Nash equilibrium outcomes of H with preferences u. That is,

$$E(H,u) = \{H(s)|\ s \in \times_{i\in N} S_i,\ \text{and, } \forall i\in N, \forall r_i\in S_i,\ u_i(H(s)) \geq u_i(H(s_{-i},r_i))\}.$$

<u>Theorem (Muller-Satterthwaite)</u> Suppose that a social choice function $F{:}L(Y)^N\to Y$ and a game form $H{:}\times_{i\in N} S_i\to Y$ satisfy

$$\#F(L(Y)^N) > 2 \ \text{ and } \ E(H,u) = \{F(u)\}\ \forall u\in L(Y)^N.$$

Then there is some h in N such that $u_h(F(u)) = \max_{x\in F(L(Y)^N)} u_h(x),\ \forall u\in L(Y)^N$.

That is, if an institution H admits more than two possible outcomes and always yields a unique pure-strategy Nash equilibrium, then H must be a dictatorship.
Different democratic institutions may have very different sets of equilibria, but we cannot expect any to abolish multiplicity or randomization of Nash equilibria,
and so democratic outcomes may depend on more than just the voters' preferences.
<u>Lemma (monotonicity)</u> Suppose $E(H,u) = \{F(u)\}\ \forall u\in L(Y)^N$. Then for any u and v,
if $\{(i,y)\in N\times Y|\ v_i(y) > v_i(F(u))\} \subseteq \{(i,y)\in N\times Y|\ u_i(y) > u_i(F(u))\}$, then $F(v) = F(u)$.

<u>Example: the Condorcet cycle.</u> Social options are Y = {a,b,c}, voters are N = {1,2,3}.
$u_1(a)=2 >u_1(b)=1 >u_1(c)=0$; $u_2(b)=2 >u_2(c)=1 >u_2(a)=0$; $u_3(c)=2 >u_3(a)=1 >u_3(b)=0$.
If H is symmetric with respect to social options (<u>neutrality</u>) and voters (<u>anonymity</u>) then its pure-strategy equilibrium outcomes are either E(H,u)=Y (multiple equilibria) or E(H,u)=Ø (only randomized equilibria).

We introduce here a simple formulation of the widely-used **probabilistic voting model**.
[For sophisticated probabilistic voting models that also include campaign contributions, see Persson and Tabellini Political Economics (2000) chapters 3 and 5. See also G. Grossman and E. Helpman, "Electoral competition and special interest politics," Review of Economic Studies 63:265-286 (1996), for a model that includes a version of probabilistic voting and campaign contributions.]

Let Y denote the set of social-choice alternatives or policy options for the government.
There are two parties, and each party k in {1,2} can simultaneously choose a policy x_k in Y.
Let us also allow that a party could promise to choose its policy according to any probability distribution σ_k in $\Delta(Y)$.
Each voter has a policy-type i that is independently drawn from a set of types I, getting type i with probability r_i. Each policy y in Y gives some utility $u_i(y)$ to every type-i voter.
In addition, each voter has a net personal bias toward party 1 that is drawn independently from a uniform distribution on the interval $[-\delta,\delta]$. A voter of type i with policy-type i and bias β gets payoff $\beta+u_i(x_1)$ if party 1 wins, but gets payoff $u_i(x_2)$ if party 2 wins.
After the parties have chosen their policy positions x_1 and x_2, each voter votes for the party that offers him the higher payoff, given his policy-type and his bias.
Each party wants to maximize its probability of winning the majority-rule election.

Fact. If both parties choosing the same policy $x_1 = x_2 = x \in Y$ is an equilibrium,
then x maximizes the expected sum of the voters' utility $x \in \operatorname{argmax}_{y\in Y} \sum_{i\in I} r_i\, u_i(y) = Eu_i(y)$.
Proof. When they both choose x for sure, a voter of any policy-type is equally likely to vote for either party, and so each party has an equal probability of winning a majority of the vote.
Now, keeping party 2 at x for sure, suppose that party 1 deviated and promised to choose x with probability $1-\varepsilon$ and some other y with probability ε, given $\varepsilon>0$ and $y\in Y$.
The possibility of changing policy from electing 1 instead of 2 would change type-i's expected utility by the amount $\varepsilon(u_i(y)-u_i(x))$, and so a type-i voter will vote for 1 if his bias β satisfies $\beta+\varepsilon(u_i(y)-u_i(x)) > 0$, that is $\beta > -\varepsilon(u_i(y)-u_i(x))$, which has probability $1/2+\varepsilon(u_i(y)-u_i(x))/(2\delta)$, if ε is small enough so that this formula is between 0 and 1.
So when ε is small, the probability of any randomly-sampled voter voting for party 1 is $1/2 + \varepsilon\sum_{i\in I} r_i\, (u_i(y)-u_i(x))$.
Thus, if $\sum_{i\in I} r_i\, u_i(y) > \sum_{i\in I} r_i\, u_i(x)$ then the ε-probabilistic deviation from x to y would make any randomly sampled voter more likely to vote for party 1 than for party 2, and so (be different voters' votes are independent) the deviating party 1 would get a greater than 1/2 chance of winning the election. But in equilibrium, such deviations from x cannot increase a party's chances of winning, and so we must have $\sum_{i\in I} r_i\, u_i(x) \geq \sum_{i\in I} r_i\, u_i(y)$ for all y in Y.

This result tells us that a convergent pure equilibrium must choose a policy that is a utilitarian optimum, maximizing the expected total utility of all voters.
But this result is somewhat misleading, because such convergent pure equilibria do not generally exist. They exist only when δ is very large, that is, when the effect of policy is small relative to the effect of individuals' biases toward one party or the other.

For example, suppose that Y={a,b,c}, I = {1,2,3}, and $u_i(y)$ is as follows

Type i	$u_i(a)$	$u_i(b)$	$u_i(c)$	r_i
1	2	1	0	0.4
2	0	2	1	0.3
3	1	0	2	0.3
Eu_i:	1.1	1.0	0.9	

So the utilitarian-optimum result is that, if there is a convergent equilibrium where both parties choose the same policy x, it must be the policy x=a, which maximizes voters' expected utility.

But when δ is small, if there are many voters, then policy a almost-surely beats policy b, policy b almost-surely beats policy c, and policy c almost surely beats policy [a], and either party could find a promise that would win with probability greater than 1/2 if it knew what the other party's (possibly probabilistic) promise would be. (Any surely promised randomization in $\Delta(Y)$ could be beaten by another promised randomization that shifts probability from b to a or from c to b or from a to c.)
In the limiting case of δ=0, this case reduces to the Condorcet cycle [ABC cycle] which a unique equilibrium where both parties choose policies randomly in the bipartisan set {a,b,c} as defined by Laffond, Laslier, and Le Breton (1993); see also my Fundamentals of Social Choice Theory survey paper at http://home.uchicago.edu/~rmyerson/research/schch1.pdf
So for small δ, there is no convergent equilibrium where both parties make the same predictable promise. To have a pure convergent equilibrium at policy a here, we must have $\delta > 1.5$.

To see that convergent equilibrium at a requires $\delta>1.5$, consider party 1 deviating to put probability ε on policy c, while party 2 remains at policy a for sure.
60% of the voters prefer c over a, but the 40% type-1s who prefer a care twice as much, and so the fraction of voters whom party 1 gains $0.6(1\varepsilon)/\delta$ is less than the fraction $0.4(2\varepsilon)/\delta$ that party 1 loses by the deviating. But this calculation goes wrong when ε becomes large enough that $(2\varepsilon)/\delta > 1/2$, because then party 1 will have lost all of the type-1 voters, and then further increases in ε can win more voters without losing any more voters. So the equilibrium might be overturned by ε=1 (all probability on c). That is, consider party 1 deviating to c for sure.
The least net pro-1 bias for a type-i voter to support party 1 is then $u_i(a)-u_i(c)$, and so a type-i voter's probability of voting for party 1 is $\max\{0, \min\{1, (\delta - (u_i(a)-u_i(c)))/(2\delta)\}\}$.
So in the whole population, the probability of a voter voting for the deviating party 1 is
$0.4\max\{0,\min\{1,(\delta-2)/(2\delta)\}\} + 0.3\max\{0,\min\{1,(\delta+1)/(2\delta)\}\} + 0.3\max\{0,\min\{1,(\delta +1)/(2\delta)\}\}$
$= 0.4\max\{0,(\delta-2)/(2\delta)\} + 0.3\min\{1,(\delta+1)/(2\delta)\} + 0.3\min\{1,(\delta +1)/(2\delta)\}$.
When $\delta < 1$, this probability of voting for 1 becomes $0+(0.3+0.3)(1) = 0.6 > 1/2$, and so the equilibrium fails.
When $\delta > 2$, this probability of voting for 1 is $1/2-0.4(1/\delta)+(0.3+0.3)(0.5/\delta) = 1/2-0.1/\delta <1/2$, and so the equilibrium does not fail.
When $1\leq\delta\leq2$, this probability of voting for 1 is $(0.3+0.3)(1/2+1/(2\delta))$ which is $>1/2$ when $\delta<1.5$.
So the equilibrium fails when $\delta<1.5$.

Costly voting ["Population Uncertainty and Poisson Games," Int. J. Game Theory 27 (1998).]
Let's analyze the question of whether large turnout can be rational when voting is costly.
Consider a population that consists of 2,000,000 leftist voters and 1,000,000 rightist voters.
There are two candidates. Candidate 1 is a leftist, and candidate 2 is a rightist.
Each voter gains \$1 when the winner is like him, but voting costs \$0.05.
Let $\tilde{S}_i$ = [number of votes for candidate i], for i = 1,2.
The candidate with the most votes wins. In case of a tie, the winner is selected by a coin toss.
We say that a voter pivots in the election if adding his vote in (instead of abstaining) would change the outcome of the election.
Let piv_i denote the event that one more vote for candidate i would change the outcome.
So piv_1 occurs in two ways:
if $\tilde{S}_1=\tilde{S}_2$ with probability 0.5 (when i would lose the toss),
if $\tilde{S}_1+1=\tilde{S}_2$ with probability 0.5 (when i would win the toss).

A randomized equilibrium. First, we can find an equilibrium in which
each leftist votes with a small probability p, and each rightist votes with small probability q.
Let $\lambda = 2000000p$. Then the probability of k votes for the leftist candidate is

$$\left(\frac{2000000!}{k!\,(2000000-k)!}\right) p^{k}(1-p)^{2000000-k} \approx \frac{\lambda^k}{k!}\left(1-\frac{\lambda}{2000000}\right)^{2000000} \approx e^{-\lambda}\frac{\lambda^k}{k!}.$$

So $\tilde{S}_1$ has a probability distribution that is approximately Poisson with mean λ.
Let $\mu = 1000000q$. Then $\tilde{S}_2$ has a prob'y distrib'n that is approximately Poisson with mean μ.
Fact When $(\tilde{S}_1,\tilde{S}_2)$ are independent Poisson random variables with with means (λ,μ),

$$P(piv_1) = 0.5\,P(\tilde{S}_1=\tilde{S}_2) + 0.5\,P(\tilde{S}_1+1=\tilde{S}_2)$$

$$= \sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!}\, e^{-\mu}\frac{\mu^k}{k!}\left(0.5+0.5\frac{\mu}{k+1}\right) \approx \frac{e^{2\sqrt{\lambda\mu}-\lambda-\mu}}{4\sqrt{\pi\sqrt{\lambda\mu}}}\left(\frac{\sqrt{\lambda}+\sqrt{\mu}}{\sqrt{\lambda}}\right).$$

Similarly, $$P(piv_2) \approx \frac{e^{2\sqrt{\lambda\mu}-\lambda-\mu}}{4\sqrt{\pi\sqrt{\lambda\mu}}}\left(\frac{\sqrt{\lambda}+\sqrt{\mu}}{\sqrt{\mu}}\right).$$

It can be verified that the most likely way that either pivot event can occur is when both parties' vote totals are near the geometric mean of their expected values, so that $k \approx \sqrt{\lambda\mu}$ in the above sum.
These approximations also rely on Stirling's formula: $k! \approx (k/e)^k\sqrt{2\pi k}$.

Notice $P(piv_1)/P(piv_2) = \sqrt{\mu/\lambda} = \sqrt{E(\tilde{S}_2)/E(\tilde{S}_1)}$.
Solving $P(piv_1) = P(piv_2) = 0.05$, we get $\lambda = \mu \approx 32$. So there is an equilibrium in which
each of 2000000 leftists votes with probability $p \approx 32/2000000$,
and each of 1000000 rightists votes with probability $q \approx 32/1000000$.

Are there other equilibria? Yes, Palfrey and Rosenthal Public Choice (1983) found many, some with large turnout. Here is an example of such a large-turnout equilibrium of this game.
Among 2000000 leftists, 1000000 leftists are expected to vote (the in-leftists), and another 1000000 are expected to abstain (the out-leftists).
Each of 1000000 rightists randomizes, abstaining with probability $r \approx 2.3/1000000$.
So $\tilde{X}$ = [number of abstaining rightist voters] is approximately Poisson with mean $\theta=2.3$, and
$P(\tilde{X}=0) = e^{-\theta} = 0.100$,
$P(\tilde{X}=1) = e^{-\theta}\,\theta/1 = 0.230$,
$P(\tilde{X}=2) = e^{-\theta}\,\theta^2/2 = 0.265$,
$P(\tilde{X}=3) = e^{-\theta}\,\theta^3/6 = 0.203$,...
P(in-leftist pivots) = $0.5\,P(\tilde{X}=0)+0.5\,P(\tilde{X}=1) = 0.5(0.100+0.230) > 0.05$.
P(out-leftist pivots) = $0.5\,P(\tilde{X}=0) = 0.5(0.100) \le 0.05$.
P(rightist pivots) = $0.5\,P(\tilde{X}=0) = 0.5(0.100) = 0.05$.
[Actually, P(out-leftist pivots) is slightly smaller than P(rightist pivots)=0.05 because
P(out-leftist pivots) = P(no rightists abstain)
< P(no rightists other than last abstain) = P(rightist pivots).]
In this equilibrium, the different behavior of in-leftists and out-leftists depends on the fact that, when a leftist voter looks at the other voters in his environment, an out-leftist sees in his environment one more leftist who is expected to vote than an in-leftist sees in his environment.
So this perverse equilibrium depends critically on every voter knowing his personal role and knowing exactly how many other voters are expected to vote in each way (or abstain).
Palfrey and Rosenthal (APSR 1985) showed that adding population uncertainty eliminates such perverse equilibria.

The mathematically simplest way of adding population uncertainty is to assume that the numbers of each group in the population are independent Poisson random variables (stdev = mean^0.5).
So suppose #Leftists is Poisson mean=2000000, #Rightists is Poisson mean=1000000, independent, each voter applies same type-dependent randomized strategy independently.
Then number of votes for each candidate are independent Poisson random variables.
We can get all pivot probabilities equal to 0.05 only if these means are 32 (approximately).

Fact Suppose the number of voters is a Poisson random variable with mean n and each voter has an independent probability τ_i of voting for candidate i, for each i in $\{1,2\}$. Then the magnitude of the pivot probability for each i is $\lim_{n\to\infty} \mathrm{LN}(P(\mathrm{piv}_i))/n = 2\sqrt{\tau_1\tau_2}-\tau_1-\tau_2 = -(\sqrt{\tau_1}-\sqrt{\tau_2})^2$.
Furthemore, $\lim_{n\to\infty} P(\mathrm{piv}_1)/P(\mathrm{piv}_2) = \sqrt{\tau_2/\tau_1}$.

The **Condorcet jury theorem** asserts that, with large numbers of independent voters, majority rule achieves correct decisions with asymptotic efficiency.
Austen-Smith and Banks (APSR, 1996) and Feddersen and Pesendorfer (APSR, 1996) have transformed this 200-year-old literature by assuming rational strategic voting.
(Myerson GEB 1998 shows a general Poisson version of the Condorcet jury theorem.)
We do an example here. (See also the Jury pages of the course spreadsheet, pekingu.xls).

There are two possible states: Bad or Good (= quality of Citizen Capet).
Voters must choose Yes (acquit Capet) or No (condemn Capet).
Voters all get payoff 0 from No, +1 from Yes if Good, -1 from Yes if Bad.
Common-knowledge prior: P(Good) = 0.50 .
#Voters is Poisson random variable with mean n.
Each voter independently gets a signal which will be Positive or Negative:
$P(\text{Positive}|\text{Good}) = 0.20$, $P(\text{Positive}|\text{Bad}) = 0.10$.
(Drop old assumption of "$P(\text{Positive}|\text{Good}) > 0.5$, $P(\text{Positive}|\text{Bad}) < 0.5$".)
Notice $P(\text{Good}|\text{Positive}) = 2/3 = 0.667$, $P(\text{Good}|\text{Negative}) = 8/17 = 0.471$.

Sincere scenario: "Positives vote Yes, Negatives vote No, and so No wins with high probability, even in Good state, by expected margin of 80-20 when n = 100.
Not an equilibrium, because pivotal votes would be much more likely in Good state than in Bad state. In fact, this scenario gives
$P(\text{N-pivotal}|\text{Bad}) \approx 1.46 * 10^{-19}$, $P(\text{N-pivotal}|\text{Good}) \approx 6.89 * 10^{-11}$,
and so $P(\text{Good}|\text{N-pivotal}) > .99999999$.
Knowing this, even Negatives would prefer to vote Yes.
...But everybody voting Yes is not an equilibrium either!

An equilibrium exists between these two scenarios: all Positives vote Yes;
Negatives randomize between No with prob'y ≈ 0.588, Yes with prob'y ≈ 0.412,
when n is large. Then expected vote shares are approximately
53% Yes and 47% No in Good state, 47% Yes and 53% No in Bad state.
So the voting outcome is probably correct in each state.
For large n, the Negative voters voting No with probability p = 0.588 equates the two states' pivot magnitudes: $-[(0.9p)^{1/2} - (0.1+0.9(1-p))^{1/2}]^2 = -[(0.2+0.8(1-p))^{1/2} - (0.8p)^{1/2}]^2 = -0.00173$.

For case of $n = 100$, the equilibrium is slightly different from $n \to \infty$ limit:
Negatives vote No with probability 0.594, Yes with probability 0.406.
Then the expected vote totals are
47.5 No and 52.5 Yes if state = Good,
53.5 No and 46.5 Yes if state = Bad.
Pivot probabilities are then
Pr(Y-pivot|Good) = 0.0345, Pr(N-pivot|Good) = 0.0362,
Pr(Y-pivot|Bad) = 0.0325, Pr(N-pivot|Bad) = 0.0303.
Notice that the ratios of pivot probabilities in each state are:
P(N-pivot|Good)/ P(Y-pivot|Good) = 0.0362/0.0345 = 1.05 ≈ $(52.5/47.5)^{0.5}$,
P(N-pivot|Bad)/ P(Y-pivot|Bad) = 0.0303/0.0325 = 0.933 ≈ $(46.5/53.5)^{0.5}$.
So a voter with a Negative signal should assess the expected gains
of adding a Yes vote to be $0.471 * 0.0345 - 0.529 * 0.0325 = -0.001$,
of adding a No vote to be $-0.471 * 0.0362 + 0.529 * 0.0303 = -0.001$.
Thus, voters with Negative signals are willing to randomize, as the claimed equilibrium requires.
But they would prefer to abstain!

This is the <u>swing voter's curse</u>, discovered by Feddersen and Pesendorfer.
(Given that some vote is pivotal, learning that your vote is pivotal makes it more likely that fewer others voted on your side, which suggests that the quality of your side may have been worse than you thought.)

If abstention is allowed then the equilibrium with $n = 100$ is as follows:
Negative voters vote No with prob'y 0.173, and abstain otherwise, while Positive voters vote Yes.
Then the expected vote totals are
13.9 No and 20 Yes if state = Good,
15.6 No and 10 Yes if state = Bad.
Pivot probabilities are then
Pr(Y-pivot|Good) = 0.036, Pr(N-pivot|Good) = 0.043,
Pr(Y-pivot|Bad) = 0.048, Pr(N-pivot|Bad) = 0.039 .
Notice that the ratios of pivot probabilities in each state are:
P(N-pivot|Good)/ P(Y-pivot|Good) = 0.043/0.036 = 1.20 ≈ $(20/13.9)^{0.5}$,
P(N-pivot|Bad)/ P(Y-pivot|Bad) = 0.039/0.048 = 0.80 ≈ $(10/15.6)^{0.5}$.
So a voter with a Negative signal should assess the expected gains
of adding a No vote to be $-0.471 * 0.043 + 0.529 * 0.039 = 0$,
but of adding a Yes vote to be $0.471 * 0.036 - 0.529 * 0.048 < 0$.
A voter with a Positive signal should assess the expected gains
of adding a Yes vote to be $0.667 * 0.036 - 0.333 * 0.048 > 0$,
but of adding a No vote to be $-0.667 * 0.043 + 0.333 * 0.039 < 0$.
So Negative voters randomize between No and Abstain, Positives vote Yes.

For large n, Negative voters voting No with probability $p = 0.172$ equates the two state's
pivot magnitudes $-[(0.9p)^{1/2} - 0.1^{1/2}]^2 = -[0.2^{1/2} - (0.8p)^{1/2}]^2 = -0.00589$.

NOTES FOR DAY 4 Citizen-Candidate Model T. Besley and S. Coate, "An economic model of representative democracy," Quarterly J of Economics 112:85-114 (1997).

N = {citizens}, Y = {policy space}. For each $i \in N$, $u_i: Y \to \mathbb{R}$ is i's utility for policies.
δ = cost of becoming a candidate. Let θ_i be i's ideal point $\theta_i = \mathrm{argmax}_x\, u_i(x)$.
First, each citizen decides independently whether to become a candidate.
Then all citizens learn $K = \{\text{candidates}\} \subseteq N$, and each votes for one candidate.
The candidate with the most votes is the winner (ties resolved by randomization) and the government policy is the ideal point of the winner.
So if j is winner then each citizen i gets payoff $u_i(\theta_j)$ if $i \notin K$, or $u_i(\theta_j) - \delta$ if $i \in K$.
If $K = \emptyset$ then the outcome is some given x_0 in Y, and i gets $u_i(x_0)$.
The game is analyzed by looking at subgame-perfect equilibria in pure (nonrandom) strategies, after eliminating dominated strategies (voting for the least-preferred candidate) in each subgame after K is determined. The existence of such equilibria can be proven.
We consider cost δ to be small, taking limit as $\delta \to 0$.

An equilibrium in which exactly one candidate enters can only be near or at (as $\delta \to 0$) a Condorcet-winning policy position. Equilibria where exactly two candidates i and j enter (Duverger's equilibria) can exist for any {i,j} such that the number of citizens who prefer i's ideal policy over j's is equal to the number who prefer j's ideal over i's.

Equilibria with three or more tied winners are hard to sustain (for the same reason as in Feddersen AJPS 1992): If a pure-strategy equilibrium generates a tie among k candidates, then no voter can strictly prefer any two of the tied candidates over the k-way randomization, because he could break the tie in favor of whichever candidate he was not expected to vote for (in the eqm).

But we can construct equilibria where three or more candidates enter even though most are expected to lose, because the presence of these spoilers can change the focal equilibrium in the subgame after candidates' entry. Remember, for any pair of candidates, there exists an equilibrium in the plurality-voting election where this pair is considered to be the only serious race, and so everybody votes for the one in this pair whom he prefers.
Consider a simple Hotelling example where Y = [0,100], citizens have ideal points that are distributed uniformly over the interval 0 to 100, and each citizen's policy-payoff is minus the distance of policy from his ideal point.
Pick any x such that $2 < x < 98$. We can construct an equilibrium in which seven candidates enter with ideal points {0,1,2,x,98,99,100}.
On the equilibrium path, the only serious race is 0 versus x, and x wins.
But if any candidate other than x dropped out, then the post-entry subgame equilibrium would switch to one where the only serious race is between two extreme candidates, the least moderate remaining on the side of the unexpected dropout, and the most moderate on the other side.
(E.g.: if 0 dropped out, then the serious race would be between 1 and 98, and 98 would win).
An unexpected extra entrant could be ignored (or could lead to an eqm where the 2 or 98 wins, whichever is worse for the unexpected entrant).

Theoretical comparison of electoral systems (European Economic Review 43:671-697, 1999)
http://home.uchicago.edu/~rmyerson/research/schump.pdf

Variety of electoral systems
We have a wide variety of ballot types in different voting rules.
Let K denote the number of candidates (or party-lists) in the election.
In a scoring rule, the ballot is a K-vector, giving points to each candidate, which are summed across all voters to determine outcome.
Non-scoring rules include single-transferable vote (STV).

Determination of winners in scoring rules.
Single winner: Candidate with highest total point score wins.
M winners: M highest scorers are winners.
Proportional representation: M seats are allocated to party lists in proportion to their scores (using some rounding formula).
Single winner with quota: Candidate with highest total point score wins if her score exceeds some quota (50%), otherwise a second election is called.

Ballot types in scoring rules.
In a rank-scoring rule, permissible vote-vectors are the permutations of one vector ($s_1, s_2, \ldots, s_{K-1}, s_K$) where (w.l.o.g) $1 = s_1 \geq s_2 \geq \ldots \geq s_{K-1} \geq s_K = 0$.
Single-positive vote: (1, 0, ..., 0, 0) (with single-winner = plurality,
with M winners = SNTV, with proportional representation = list PR)
Single-negative vote: (1, 1,..., 1, 0).
V-positive votes: $1 = s_1 = \ldots = s_V$, $s_{V+1} = \ldots = s_K = 0$.
V-negative votes: $1 = s_1 = \ldots = s_{K-V}$, $s_{1+K-V} = \ldots = s_K = 0$.
One-positive-and-one-negative vote: $s_1 = 1$, $s_2 = \ldots = s_{K-1} = 1/2$, $s_K = 0$.
Borda voting: $s_j = (j-2)/(K-1)$ for j = 1,2,...,K.

Nonrank scoring rules may permit the permutations of several vectors.
Approval voting allows any number of positive votes (the union of permissible ballots under all V-positive-vote rules, for V = 1,...,K−1).

Ballots like single-positive voting ask voters to reward the best candidate; ballots like single-negative voting ask voters to punish the worst candidate.
With large K, $\sum_j s_j/K \approx 0$ if best-rewarding, $\sum_j s_j/K \approx 1$ if worst-punishing.

Symmetric equilibria and incentives for diversity.
Hotelling (1929) posed the question of rational competitive diversity both in oligopolistic markets and in politics. But in politics, each voter's utility is from the policy position of the winner, not the candidate that he supported. No costs of "excessive sameness."

Gary Cox (*Am.J.Pol.Sci.*, 1987, 1990) showed that candidates' incentives for diversity depend systematically on the voting rule.

Example: single winner, K candidates, {policy alternatives}={Left, Right}. Q denotes the fraction of voters expected to prefer Left; $1-Q$ prefer Right.
First, each candidate independently chooses a policy position in {Left,Right}.
Assume symmetry: candidates at same position are treated same by voters.
Given K, Cox's threshold of diversity Q* is the greatest value of Q such that there is a symmetric equilibrium in which all K candidates choose Right.

Fact. $Q^* = \sum_j s_j/K$ for a rank-scoring rule with points $1 = s_1 \geq s_2 \geq ... \geq s_K = 0$.

Single-positive voting yields $Q^* = 1/K$, and so small minority positions can win when K is large.
Best-rewarding creates incentives to diversify, low Q*.
Single-negative voting yields $Q^* = (K-1)/K$, so a majority can be neglected when K is large!
Worst-punishing creates incentives to cluster, high Q*.
Approval voting and Borda voting yield $Q^* = 1/2$ for any K. Majoritarian.

A related model: The winner of the election will get a budget of \$1 per voter to distribute as cash or to spend on a public good worth \$B to every voter.
An equilibrium where the public good is guaranteed exists only if $B \geq 1/Q^*$.
When $B < 1/Q^*$, if all other candidates promised the public good, then a candidate could win by promising \$1/Q* to a Q* minority of the voters.
(Myerson, APSR 1993; Lizzeri and Persico, AER 2001.)

These results depend only on ballot type; extend to multi-winner elections.
Consider open list PR, where supporters of a party vote to order its list.
Single-positive vote for candidates in Brazil induces separate constituencies
(B. Ames, J Politics, 1995). Approval voting would induce broader appeals.

Nonsymmetric equilibria and Duverger's law.
Preceding analysis assumed symmetry, all candidates taken equally seriously.
Duverger's law: Plurality elections (single-vote single-winner) tend to have only two serious parties.
More parties can persist under PR.
(Maurice Duverger, Political Parties, 1954. William Riker, APSR 1982. Gary Cox, Making Votes Count, 1997.)

A candidate is in a close race when a small number of votes could change her from winner to loser or from loser to winner. A rational voter knows that his vote matters only in the event of a close race, and so should vote in equilibrium to maximize expected utility in this event.
A candidate is a serious contender if, conditional on a close race existing, there is a substantial positive probability of her being in the close race.
A stronger candidate has greater probability of winning; weaker, less.
A likely winner has probability of winning close to 1; likely loser, close to 0.
In an election with K candidates for M seats, serious candidates normally include the strongest likely loser and the weakest likely winner.
With best-rewarding votes, being a serious contender strengthens a candidate.

With worst-punishing votes, being a serious contender weakens a candidate.

These propositions yield generalizations of Duverger's law. Consider an election with M winners, single-positive voting for candidates (SNTV).
Becoming weaker than the strongest likely loser makes a candidate less serious, which weakens her further until she gets almost no votes.
Among likely winners however, being perceived stronger tends to weaken.
Such a tendency towards M+1 serious candidates, with relatively level scores for top M candidates was found by Steven Reed in Japan (BJPS, 1991).
Unstable nonDuvergerian equilibria have ties for strongest likely loser. (Cox)

In list-PR elections, any party likely to win a seat can seriously contend for more or less; but only strongest of likely losers can be a serious contender. So list PR in M-seat districts should yield at most M+1 serious parties.

Electoral barriers to entry and incentives to reduce corruption.
Interpretation of Duverger's law:
Plurality voting creates a <u>barrier to entry</u> against new "third" parties.
Economists should recognize importance of barriers to entry, which may be crucial determinants of long-run profit. Politicians' profits = corruption.
So Duverger's law suggests that electoral systems may differ in their effectiveness at deterring political abuse of power, a basic goal of democracy.

Simple theoretical model (Myerson, *G.E.B.*, 1993). Each party publicly chooses a corruption level in $\mathbb{R}_+$ and a policy position in {Left, Right}.
A party's profit is its seats multiplied by its corruption level; max E(profit).
A voter gets his utility of legislative majority's policy, minus parties' profits.
Will multiparty democratic competition force party profits to 0?

4-party example: L1 and L2 are leftist parties, R1 and R2 are rightist parties.
L1 and R1 are similarly corrupt, L2 and R2 are noncorrupt. (Fixed, known.)
Half of voters prefer left policy, half prefer right, all prefer noncorrupt.

Plurality voting admits a Duvergerian equilibrium where only L1 and R1 are serious contenders. A leftist voter deviating from L1 to L2 is more likely to affect a close L1-R1 race unfavorably, than a close L2 race favorably. This equilibrium may be focal when L1 and R1 are established parties.
Coordination game. Corrupt parties hold left/right policy question as hostage.
Shift to less corrupt equilibrium would require coordinating leaders before the election; but elections are our institution for choosing our leaders!
(Single-positive voting for candidates in multi-seat districts demands even more coordination of voters to maximize seats for a bloc; reforms in Japan.)

With approval voting, such a corrupt equilibrium does not exist.
Even if a close L1-R1 race is most likely, voters approve L2 or R2 as well as their preferred serious

contender. So a perception of weakness cannot become a self-fulfilling prophecy against the noncorrupt parties.

List PR (pure, assuming large M) also has no corrupt equilibrium in this model, because small parties can contend seats as seriously as large parties.

Worst-punishing ballots creates <u>barriers to exit</u> that protect corrupt parties.
Under single-negative voting, or even Borda voting, a perception that "only L2 or R2 can win" would encourage voting against the less-preferred among L2 and R2, which strengthens L1 and R1 until they are serious contenders.

Other perspectives on democratic incentives to reduce corruption.
Barriers to entry can depend on many other factors in political systems.
A list-PR system with a national 5% quota to win seats (Germany) may create higher barriers to entry than a list-PR system with 20-seat districts.
Freedom of the press, freedom of assembly, and restrictions on patronage are important in democratic theory because they lower political barriers to entry.

Recent Italian electoral reforms to reduce corruption went from PR to a system where most seats are allocated by plurality voting.
In my model, such a reform decreases effectiveness against corruption!
Sharpened question: what assumption of my model may be wrong, causing it to overestimate the effectiveness of multiparty PR against corruption?
(Difficulties identifying corruption should not affect electoral comparisons.)

Critical assumption: government policy (left/right) is determined by a majority coalition in which noncorrupt parties can participate as well as corrupt parties that advocate the same left/right policies. But if most parties are corrupt, a small anticorruption party would be very unlikely to be invited into the governing coalition. So a vote that wins an extra seat for a small anticorruption party is as wasted as if it got no seats.
This argument suggest that only a leader of a major party that is a serious contender for a legislative majority can gain support by pledging to fight corruption.
This shows advantage of a political system dominated by two large parties.
With corruption as politicians' profit, we must be concerned about collusion among the leaders of the two big parties on this question, but their independent incentive is to undercut each other on the corruption dimension.

A related argument for presidentialism: Voters know that a committed anti-corrupt winner of a presidential election would have power to reduce corruption,
So presidential candidates can effectively compete against corruption.

The **M+1 law** for multi-seat legislative elections under the single nontransferable vote (SNTV) system, as was used in Japanese legislative elections, can be found in section 4 of the paper "Theoretical Comparison of Scoring Rules". See the papers by Reed in British J. of Political Science, 1991 and by Cox in APSR 1994.

Short proof of the weak M+1 law for plurality voting with a large Poisson electorate:
Suppose that there are K candidates, numbered 1,2,...,K, in an election with single nontransferable vote where the top M candidates win. In case of a tie for M'th and M+1'th place, a random ordering of the candidates is generated, and the set of M winners is completed by selecting from the borderline-winning candidate in this order.
Here $M < K$.
Each voter has a type which is drawn independently from some finite set according to some fixed probability distribution.
Each type of voter has a strict utility ranking of the candidates, with $u_i(t)$ denoting the utility of candidate i winning for a voter of type t.
A voter's payoff from the election is the sum of his utility from each of the M winners.
In the n'th voting game, the number of voters is a Poisson random variable with mean n.
A large equilibrium is a convergent sequence of equilibria of these voting games as $n\rightarrow\infty$.
By convergent, we mean that the expected fraction of the electorate who vote for each candidate i is converging to some limit τ_i. Choosing a subsequence if necessary, we may assume that other probabilities are also convergent to well-defined limits as $n\rightarrow\infty$.

Now consider a large equilibrium. Without loss of generality, we may assume that the candidates are numbered so that $\tau_1 \geq \tau_2 \geq ... \geq \tau_K$.
The {i,j} race is close when adding one vote for i or j could make one of them replace the other in the set of winners.
Let $\tilde{x}_i$ denote the number of votes for candidate i (a random variable).

If there is no close race, then adding one more vote in cannot matter to anybody.
In equilibrium, each voter must vote cast the ballot that would maximize his conditional expected utility gain, relative to not voting, given that there is at least one close race.

A race between two candidates is <u>serious</u> iff its conditional probability of being close, given that there is some close race, is strictly positive in the limit as $n\rightarrow\infty$.
A candidate is <u>serious</u> iff he is involved in at least one close race.
A voter's conditional expected gain, given that there is a close race, from voting for his favorite serious candidate would be strictly positive in the limit.
So each voter's ballot in equilibrium must give him a strictly positive conditional expected gain, given that there is a close race. Thus we get:

Lemma 1. In the large equilibrium, nobody votes for candidates who are not serious. That is, if h is not a serious candidate then $\tau_h = 0$.

From any standard paper on Poisson voting games, we get

Fact For any two candidates i and j such that $\tau_i > \tau_j$, the magnitude of the event that $1+\tilde{x}_j \geq \tilde{x}_i$ is $-(\sqrt{\tau_i} - \sqrt{\tau_j})^2$.

A close race between candidates M and M+1 can occur when their vote-totals are within one of each other and all other candidates' vote-totals are near their expected values. Thus:

Lemma 2. The magnitude of a close race involving candidates M and M+1 is $-(\sqrt{\tau_M} - \sqrt{\tau_{M+1}})^2$.

Consider now some candidate j > M+1.

If candidates 1,..,M all got strictly more votes than $1+\tilde{x}_j$, then candidate j would not be in a close race. Thus, when candidate j is in a close race, there at least one candidate i in {1,...,M} such that $1+\tilde{x}_j \geq \tilde{x}_i$. The magnitude of this event is $-(\sqrt{\tau_i} - \sqrt{\tau_j})^2 \leq -(\sqrt{\tau_M} - \sqrt{\tau_j})^2$.

So the magnitude of the event that j is in a close race is not more than $-(\sqrt{\tau_M} - \sqrt{\tau_j})^2$.

But if $\tau j < \tau_{M+1}$ then this magnitude is strictly less than the magnitude of a close race between candidates M and M+1. Thus we get

Lemma 3 For any j in {M+2,...,K}, if $\tau_j < \tau_{M+1}$ then candidate j is not serious.

Consider now some candidate i < M.

If candidates M+1,...,K all got strictly less votes than $\tilde{x}_i - 1$, then candidate i would not be in a close race, because he would be a guarenteed winner even with one more vote. Thus, when candidate i is in a close race, there at least one candidate j in {M+1,...,K} such that $1+\tilde{x}_j \geq \tilde{x}_i$. The magnitude of this event is $-(\sqrt{\tau_i} - \sqrt{\tau_j})^2 \leq -(\sqrt{\tau_i} - \sqrt{\tau_{M+1}})^2$.

So the magnitude of the event that j is in a close race is not more than $-(\sqrt{\tau_i} - \sqrt{\tau_{M+1}})^2$.

But if $\tau_i > \tau_M$ then this magnitude is strictly less than the magnitude of a close race between candidates M and M+1. Thus we get

Lemma 4 For any i in {1,...,M−1}, if $\tau_i > \tau_M$ then candidate i is not serious.

We obviously cannot have $\tau_i > \tau_M$, because then i would not be serious and so τ_i would be 0, contradicting $\tau_i > \tau_M \geq 0$. Thus:

Theorem For each i in {1,2,...,M}, τ_i must be equal to τ_M. For each j in {M+1,...,K}, τ_j must be either τ_{M+1} or 0.

References: W. Riker, "The two-party system and Duverger's law," APSR 76:753-766 (1982). S. Reed, "Structure and behavior: extending Duverger's law to the Japanese case," British J of Political Science 20:335-356 (1990). G. Cox, "Strategic voting equilibria under single non-transferable vote," APSR 88:608-621 (1994).

Bipolar multicandidate elections with corruption
Myerson, Scandinavian Journal of Economics (2006) 108:727-742.

Suppose the set of candidates K is partitioned into K_1={leftists} and K_2={rightists}.
Each candidate k has corruption level $f(k) \geq 0$. k is clean if f(k)=0, corrupt if f(k)>0.
In game Γ_n, the number of voters is a Poisson random variable with mean n.
Each voter has a type t drawn independently from a probability distribution r that has a continuous positive density on the whole real line $\mathbb{R}$. $r(S) = \text{Prob}(\tilde{t} \in S)$ $\forall S \subseteq \mathbb{R}$.
A voter's type t measures his net preference for rightist candidates in K_2,
so t's utility payoff if k wins is $u_k(t) = t - f(k)$ if $k \in K_2$, $u_k(t) = 0 - f(k)$ if $k \in K_1$.
Suppose that, $\forall i \in \{1,2\}$, there exists a clean candidate k in K_i with f(k)=0. (wlog)
To complete the game, we must specify an electoral system (ties broken at random).

An equilibrium in the game Γ_n specifies a (weakly undominated) optimal strategy $\sigma_n(t)$ for each type t, and generates expected fractions $\tau_n(c)$ for each ballot c that is allowed in this electoral system, and win-probabilities $q_n(k)$ for each candidate k.
A large equilibrium (σ,τ,q) is a limit of (σ_n,τ_n,q_n) equilibria of Γ_n as $n \to \infty$.
A pair of candidates {i,j} is distinct iff $u_i(t) \neq u_j(t)$ for some t in T.
The {i,j}-race is close when adding one vote could change winner from i to j, or j to i.
The {i,j}-race is serious in a large equilibrium iff {i,j} is a distinct pair and there is a strictly positive limit($n \to \infty$) of the conditional probability of a close {i,j}-race given that some pair of distinct candidates are in a close race.
A candidate is serious iff he is involved in at least one serious race.
A candidate i is strong in a large equilibrium (σ,τ,q) iff $q(k) > 0$ (positive win-proby).

Theorem 1 (effectiveness against corruption). In a large equilibrium under approval voting, no corrupt candidates can be strong or serious.
Theorem 2 (majoritarianism). In a large equilibrium under approval voting, with probability 1, the winner will be a candidate who is considered best by at least half of the voters.

Failures of effective majoritarianism for other electoral systems:
In three-candidate elections, consider rank-scoring rules where ballots are vectors that are permutations of (1,A,0), for some number A such that $0 \leq A \leq 1$.
Suppose K_1={1}, K_2={2,3}, 1 and 2 are clean, 3 is corrupt.
If A<1/2 then there is an equilibrium where {1,3} is the only serious race. In this equilibrium, everybody votes (1,A,0) or (0,A,1), and so the winner will be either 1 or 3.
If $A \geq 1/2$ then 3 must be serious in all equilibria, because if people thought 3 was not serious then everybody would vote (1,0,A) or (0,1,A), but then 3 would always be in first place when 1 and 2 tie!

Now consider more general scoring rules where ballots are vectors that are permutations of (1,A,0) and (1,B,0), where $0 \le A \le B \le 1$.
Approval voting is (A,B)=(0,1), plurality voting is (0,0),
Borda voting is (1/2,1/2), negative voting is (1,1).
Suppose now $K_1=\{1\}$, $K_2=\{2,3\}$, all three candidates are clean.
There is a symmetric equilibrium where leftist voters randomize equally among (1,A,0) and (1,0,A), while rightist voters randomize equally among (0,B,1) and (0,1,B).
Notice $r < r(0+A)/2 + (1-r)(1+B)/2$ (1 loses) iff $r < (1+B)/(3+B-A)$.
$(1+B)/(3+B-A)$ is Cox's threshold of diversity here.
Also $1/2 < (1+B)/(3+B-A)$ iff $1 < A+B$.
When $1 < A+B$ and $1/2 < r(\mathbb{R}_-) < (1+B)/(3+B-A)$, then almost-surely leftists are a majority, but a rightist candidate wins (duplication helps righists).
When $1 > A+B$ and $1/2 > r(\mathbb{R}_-) > (1+B)/(3+B-A)$ then almost-surely rightists are a majority, but the leftist candidate wins (duplication hurts rightists).

The magnitude of any event M is $\mu(M) = \lim_{n\to\infty} \log_e(P_n(M))/n$.
Fact If $\{S_0, S_1, S_2, S_3\}$ is a partition of all voter-types, then the event "equal numbers in S_1 and S_2 but none in S_0" has magnitude $2\sqrt{r(S_1)r(S_2)}+r(S_3)-1$.

Proof of Theorem 1. All leftist voters with types in $\mathbb{R}_-$ approve clean candidates in K_1, as best among all candidates, because approving-best weakly dominates not-approving.
Similarly, all rightist voters in $\mathbb{R}_+ = [0,+\infty]$ approve clean candidates in K_2.
If type t approves candidate i in K_1 and $s < t$ then type s also approves i
(because $u_i(s) - u_k(s) \ge u_i(t) - u_k(t)$ $\forall k\in K$, with "=" if $k\in K_1$ and ">" if $k\in K_2$).
So for i in K_1, exists $\theta_n(i)$ such that t approves i in σ_n if $t < \theta_n(i)$ but not if $t > \theta_n(i)$.
For j in K_2, exists $\theta_n(j)$ such that t approves j in σ_n if $t > \theta_n(j)$ but not if $t < \theta_n(j)$.
Let $\theta(k) = \lim_{n\to\infty} \theta_n(k)$.
Let $h_1\in H_1 = \operatorname{argmax}_{i\in K_1} \theta(i)$, $h_2\in H_2 = \operatorname{argmin}_{j\in K_2} \theta(j)$ (highest E scores on each side).
A clean candidate in K_1 has $\theta\ge 0$; a clean candidate in K_2 has $\theta\le 0$. So $\theta(h_2)\le 0\le\theta(h_1)$.
Let $r_1 = r([-\infty, \theta(h_2)])$, $r_2 = r([\theta(h_1), +\infty])$, $r_3 = r([\theta(h_2), \theta(h_1)])$.
The event of a close $\{h_1,h_2\}$-race has magnitude $2\sqrt{r_1 r_2}+r_3-1$ (> -1).
Let i and j be any other candidates in K_1 and K_2 respectively.
Let $s_0 = r([\theta(h_2),\theta(j)] \cup [\theta(i),\theta(h_1)])$ (E fractn for-h_2-but-not-j or for-h_1-but-not-i),
$s_1 = r([-\infty, \min\{\theta(i),\theta(h_2)\}])$ (E fraction for-i-but-not-h_2),
$s_2 = r([\max\{\theta(j),\theta(h_1)\}, +\infty])$ (E fraction for-j-but-not-h_1),
$s_3 = r([\theta(j), \theta(i)]$ (E fraction for-i-and-j). Here $s_3 = 0$ if $\theta(j)\ge\theta(i)$.
The event of a close $\{i,j\}$-race has magnitude $2\sqrt{s_1 s_2}+s_3-1$.
If $\theta(i) < \theta(h_1)$ or $\theta(h_2) < \theta(j)$ then $s_1 \le r_1$, $s_2 \le r_2$, $s_3 < r_3$, and

so a close $\{i,j\}$-race has strictly lower magnitude than a close $\{h_1,h_2\}$-race.

So a serious race between a leftist and rightist candidate can only involve candidates in H_1 and H_2 (those with highest expected scores on each side as $n\to\infty$).

Now suppose, contrary to the theorem, that some corrupt candidate is serious.

Let i denote the most corrupt serious candidate. Suppose w.l.o.g. that $i\in K_1$.

There must exist some j in H_2 such that the $\{i,j\}$ race is serious, because nobody would vote for i if i's serious races were all with other less-corrupt candidates in K_1.

Candidate i is the worst serious candidate for all voters in $\mathbb{R}_+$, so $\theta_n(i) < 0 \;\; \forall n$.

Let g be a clean candidate in K_1, who is approved by all voters in $\mathbb{R}_-$, so $\theta_n(g) \geq 0 \;\; \forall n$.

So the set of voters approving i is a subset of those approving g.

Candidate i can win only when all voters for-g-but-not-for-i vanish, leaving g in a tie with i. So whenever an additional vote for i could make i win, there is a positive limiting conditional probability that the winner would be g otherwise.

But for type-0 voters, g is strictly better than i, and no serious candidate is worse than i.

So in the limit, there are strictly negative conditional expected gains for type-0 voters from approving i, given the event that some serious race is close.

So $\theta(i) < 0 \leq \theta(g)$. Thus, i is not in H_1.

But then a close $\{i,j\}$-race must have lower magnitude than some other close race involving a higher-expected-scoring candidate in H_1.

So the $\{i,j\}$ race cannot be serious.

This contradiction shows that no corrupt candidate i can be serious.

Thus, all serious candidates must be clean.

A pair of clean candidates who are both in K_1 (or both in K_2) would not be distinct, so every serious race involves a clean candidate in K_1 and a clean candidate in K_2.

In a single-winner election, a strong candidate must be serious, and so all strong candidates must be clean.

Proof of Theorem 2. From Theorem 1 all serious races are between clean candidates in K_1 and K_2.

So leftist voters in $\mathbb{R}_-$ will all approve the clean candidates in K_1 but not in K_2, while rightist voters in $\mathbb{R}_+$ will all approve the clean candidates in K_2 but not in K_1.

Corrupt candidates may get some votes, but only from an expected-strict subset of the voters on their same side of the political spectrum (so not serious contenders).

So with probability 1, the winner will be a clean candidate from the side of the political spectrum that has a majority (or at least half) of the electorate,

and so the winner will be an optimal candidate for at least half of the voters.

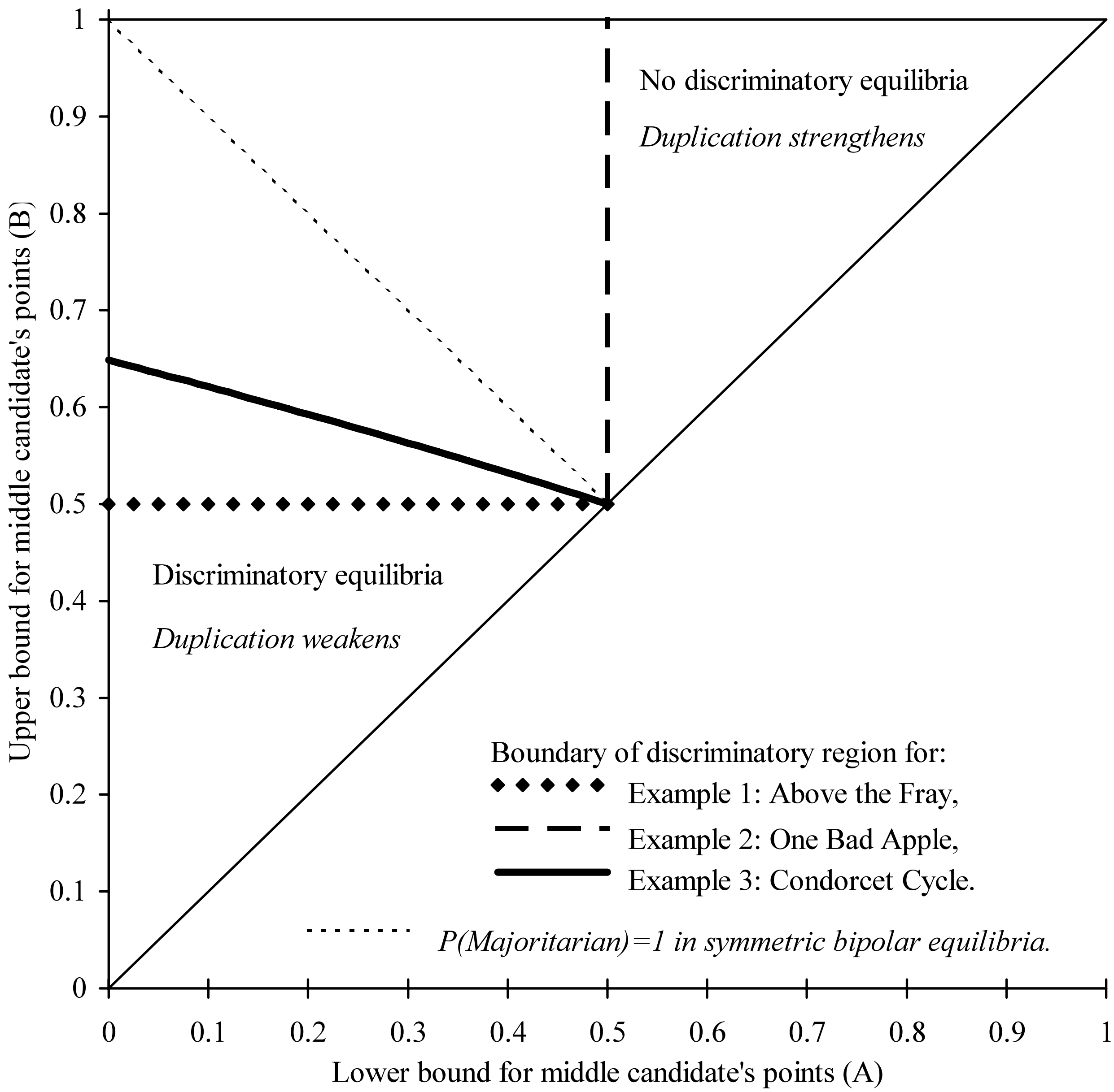

Figure 1. Characterizing Equilibria of (A,B)-Scoring Rules.

Ex 1: u(**1**) = (6,0,9), r(**1**) = 1/2; u(**2**) = (0,6,9), r(**2**) = 1/2.

Ex 2: u(**1**) = (9,3,0), r(**1**) = 1/2; u(**2**) = (3,9,0), r(**2**) = 1/2.

Ex 3: u(**1**) = (9,6,0), r(**1**) = 1/3; u(**2**) = (0,9,6), r(**2**) = 1/3; u(**3**) = (6,0,9), r(**3**) = 1/3.

R. Myerson, "Comparison of scoring rules in Poisson voting games," Journal of Economic Theory 103:219-251 (2002).

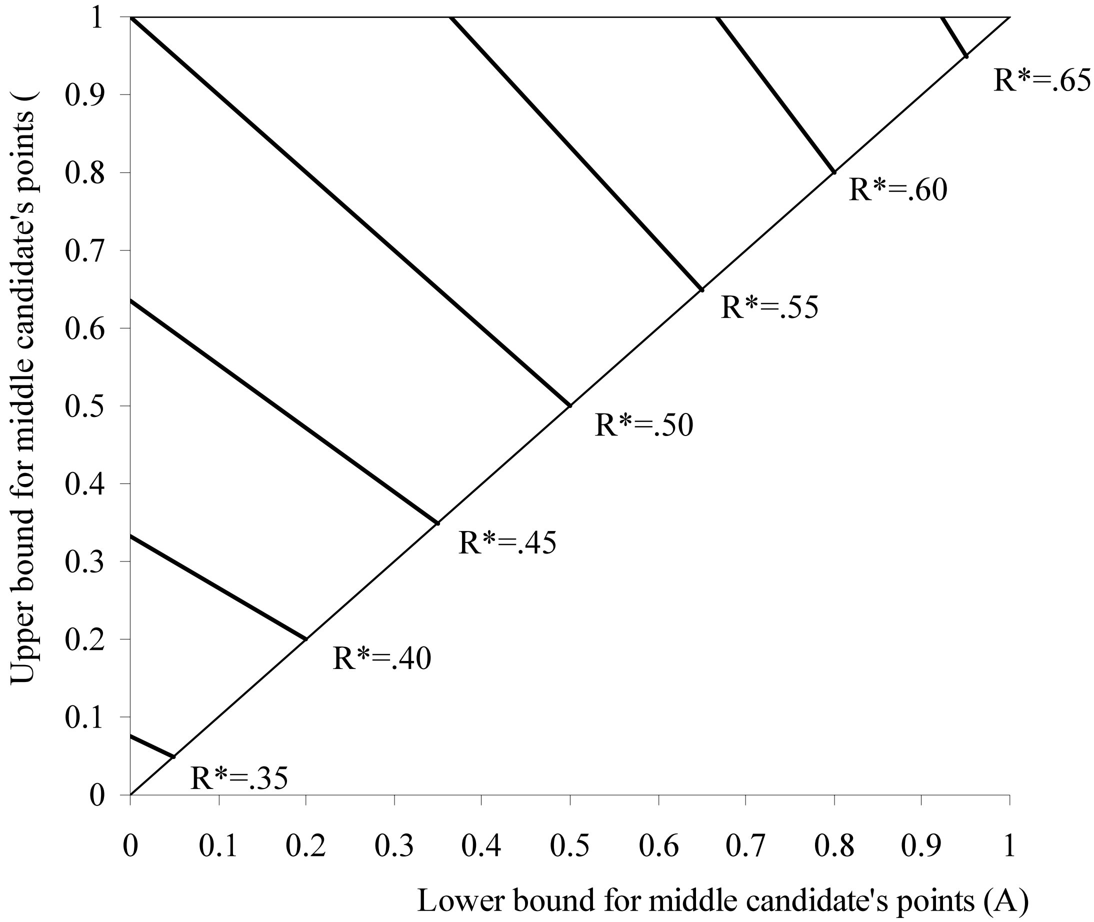

Figure 2. Cox's threshold of diversity (R*) for (A,B)-scoring rules.

R* = (1+B)/(3+B−A) = [largest fraction that can lose with one candidate against two in a symmetric eqm]

NOTES FOR DAY 5

Binary agendas and the top cycle from "Fundamentals of Social Choice Theory" http://home.uchicago.edu/~rmyerson/research/schch1.pdf

When there are more than two alternatives, we might still try to apply the principle of majority voting by dividing the decision problem into a sequence of binary questions. For example, one simple binary agenda for choosing among three alternatives {a,b,c} is as follows. At the first stage, there is a vote on the question of whether to eliminate alternative a or alternative b from further consideration. Then, at the second stage, there is a vote between alternative c and the alternative among {a,b} that survived the first vote. The winner of this second vote is the implemented social choice.

This binary agenda is represented graphically in Figure 1.1. The agenda begins at the top, and at each stage the voters must choose to move down the agenda tree along the branch to the left or to the right. The labels at the bottom of the agenda tree indicate the social choice for each possible outcome at the end of the agenda. Thus, at the top of Figure 1.1, the left branch represents eliminating b at the first vote, and the right branch represents eliminating a. Then at each of the lower nodes, the right branch represents choosing c and the left branch represents choosing the other alternative that was not eliminated at the first stage.

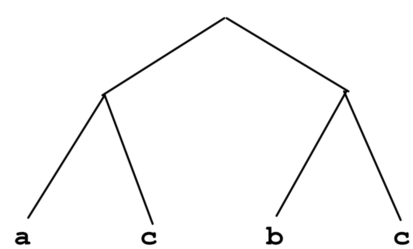

Figure 1.1

Now suppose that the voters have preferences as in the Condorcet (ABC) paradox example (described in the preceding section). Then there is a majority (voters 1 and 3) who prefer alternative a over b, there is a majority (voters 1 and 2) who prefer alternative b over c, and there is a majority (voters 2 and 3) who prefer alternative c over a. Let us use the notation x >> y (or equivalently y << x) to denote the statement that a majority of the voters prefer x over y. Then we may summarize the majority preference for this example as follows:

a >> b, b >> c, c >> a.

(This cycle, of course, is what makes this example paradoxical.)

Given these voters' preferences, what will be the outcome of the binary agenda in Figure 1.1? At the second stage, a majority would choose alternative b against c if alternative a were eliminated at the first stage, but a majority would choose alternative c against a if alternative b were eliminated at the first stage. So a majority of voters should vote to eliminate alternative a at the first stage (even though a majority prefers a over b), because they should anticipate that the ultimate result will be to implement b rather than c, and a majority prefers alternative b over c. This backwards analysis is shown in Figure 1.2 which displays, in parentheses above each decision node, the ultimate outcome that would be chosen by sophisticated majority voting if the process reached this node.

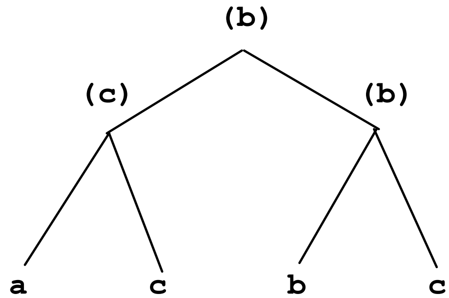

Figure 1.2

In general, given any finite set of alternatives Y, a <u>binary agenda on Y</u> is a rooted tree that has two branches coming out of each nonterminal node, together with a labelling that assigns an outcome in Y to every terminal node, such that each alternative in Y appears as the outcome for at least one terminal node. Given a binary agenda, a <u>sophisticated solution</u> extends the labelling to all nodes so that, for every nonterminal node θ, the label at θ is the alternative in Y that would be chosen by a majority vote among the alternatives listed at the two nodes that directly follow θ. A <u>sophisticated outcome</u> of a binary agenda is the outcome assigned to the initial node (or root) of the agenda tree in a sophisticated solution.

Given any preference profile for an odd number of voters, each binary agenda on Y has a unique sophisticated solution, which can be easily calculated by backward induction. (See Farquharson, 1969, and Sloth, 1993.) Thus, once a binary agenda has been specified, it is straightforward to predict the outcome that will be chosen under majority rule, assuming that the voters have a sophisticated understanding of each others' preferences and of the agenda.

But different agendas may lead to different majority-rule outcomes for the same voters' preferences. Thus the chairman who sets the agenda may have substantial power to influence the sophisticated majority-rule outcome. To quantify the extent of such agenda-setting power, we want to characterize the set of alternatives that can be achieved under binary agendas, for any given preference profile.

To compute sophisticated solutions, it is only necessary to know, in each pair of alternatives, which one would be preferred by a majority of the voters. That is, we only need to know, for each pair of distinct alternatives x and y in Y, whether x >> y or y >> x. (Read ">>" here as "would be preferred by a majority over.")

The Condorcet paradox shows the majority-preference relation >> is not necessarily transitive, even though each individual voter's preferences are assumed to be transitive. In fact, McGarvey (1953) showed that a relation >> can be generated as the majority-preference relation for an odd number of voters whose individual preferences are transitive if and only if it satisfies the following completeness and antisymmetry condition:

$$x \gg y \text{ or } y \gg x, \text{ but not both}, \ \forall x \in Y, \ \forall y \in Y \setminus \{x\}.$$

Any such relation >> on Y may be called a <u>tournament</u>.

The top cycle

Let >> be any fixed tournament (complete and antisymmetric) on the given set of alternatives Y. We now consider three definitions that each characterize a subset of Y.

Let Y*(1) denote the set of all alternatives x such that there exists a binary agenda on Y for which x is the sophisticated outcome. That is, Y*(1) is the set of outcomes that could be achieved by agenda-manipulation, when the agenda-setter can plan any series of binary questions, subject only to the constraint that every alternative in Y must be admitted as a possibility under the agenda, and all questions will be resolved by sophisticated (forward-looking) majority votes.

Let Y*(2) denote the set of alternatives y such that, for every alternative x in Y\{y}, there must exist some chain $(z_0,z_1,...,z_m)$ such that $x = z_0$, $z_m = y$, and $z_{k-1} << z_k$ for every k = 1,...,m. That is, an alternative y is in Y*(2) iff, starting from any given status quo x, the voters could be manipulated to give up x for y through a sequence of replacements such that, at each stage, the majority would always prefer to give up the previously chosen alternative for the manipulator's proposed replacement if they believed that this replacement would be the last. In contrast to Y*(1) which assumed sophisticated forward-looking voters, Y*(2) is based on an assumption that voters are very naive or myopic.

Let Y*(3) be defined as the smallest (in the set-inclusion sense) nonempty subset of Y that has the following property:

> for any pair of alternatives x and y, if y is in the subset and x is not in the subset then y >> x.

An argument is needed to verify that this set Y*(3) is well defined. Notice first that Y is itself a "subset" that has this property (because the property is trivially satisfied if no x outside of the subset can be found). Notice next that, if W and Z are any two subsets that have this property then either $W \subseteq Z$ or $Z \subseteq W$. (Otherwise we could find w and z such that $w \in W$, $w \notin Z$, $z \in Z$, and $z \notin W$; but then we would get w >> z and z >> w, which is impossible in a tournament.) Thus, assuming that Y is finite, there exists a smallest nonempty set Y*(3) that has this property, and which is a subset of all other sets that have this property.

A fundamental result in tournament theory, due to Miller (1977), is that the above three definitions all characterize the same set Y*. This set Y* is called the <u>top cycle</u>.

<u>Theorem 1.2.</u> Y*(1) = Y*(2) = Y*(3).

<u>Proof</u>. We first show that $Y*(1) \subseteq Y*(2)$. Suppose that y is in Y*(1). Then there is some binary agenda tree such that the sophisticated solution is y. Given any x, we can find a terminal node in the tree where the outcome is x. Now trace the path from this terminal node back up through the tree to the initial node. A chain satisfying the definition of Y*(2) for x and y can be constructed simply by taking the sophisticated solution at each node on this path, ignoring repetitions. (This chain begins at x, ends at y, and only changes from one alternative to another that beats it in the tournament. For example, Figure 1.2 shows that alternative b is in Y*(2), with chains a << c << b and c << b.)

We next show that $Y*(2) \subseteq Y*(3)$. If not, then there would be some y such that $y \in Y*(2)$ but $y \notin Y*(3)$. Let x be in Y*(3). To satisfy the definition of Y*(2), there must be some chain such that $x = z_0 << z_1 << ... << z_m = y$. This chain begins in Y*(3) and ends outside of Y*(3), and so there must exist some k such that $z_{k-1} \in Y^*(3)$ and $z_k \notin Y^*(3)$, but then $z_{k-1} << z_k$ contradicts the definition of Y*(3).

Finally we show that $Y^*(3) \subseteq Y^*(1)$. Notice first that Y*(1) is nonempty (because binary agendas and their sophisticated outcome always exist). Now let y be any alternative in Y*(1) and let x be any alternative not in Y*(1). We claim that y >> x. If not then we would have x >> y; but then x would be the sophisticated outcome for a binary agenda, in which the first choice is between x and a subtree that is itself a binary agenda for which the sophisticated outcome would be y, and this conclusion would contradict the assumption $x \notin Y^*(1)$. So every y in Y*(1) beats every x outside of Y*(1); and so Y*(1) includes Y*(3), which is the smallest nonempty set that has this property.

Thus we have $Y^*(1) \subseteq Y^*(2) \subseteq Y^*(3) \subseteq Y^*(1)$. Q.E.D.

In the Condorcet paradox from the previous section, the top cycle includes all three alternatives {a,b,c}. If we add a fourth alternative d that appears immediately below c in each individual's preference ranking (so that b >> d and c >> d but d >> a), then d is also included in the top cycle for this example, even though d is Pareto-dominated by c.

When the top cycle consists of a single alternative, this unique alternative is called a Condorcet winner. That is, a Condorcet winner is an alternative y such that y >> x for every other alternative x in Y\{y}. The existence of a Condorcet winner requires very special configurations of individual preferences. For example, suppose that each voter's preferences is selected at random from the (#Y)! possible rank orderings in L(Y), independently of all other voters' preferences. R. May (1971) has proven that, if the number of voters is odd and more than 2, then the probability of a Condorcet winner existing among the alternatives in Y goes to zero as #Y goes to infinity. (See also Fishburn, 1973.)

McKelvey (1976, 1979) has shown that, under some common assumptions about voters' preferences, if a Condorcet winner does not exist then the top cycle is generally very large. We now state and prove a simple result similar to McKelvey's.

We assume a given finite set of alternatives Y, and a given odd finite set of voters N, each of whom has strict preferences over Y. Let Δ(Y) denote the set of probability distributions over the set Y. We may identify Δ(Y) with the set of lotteries or randomized procedures for choosing among the pure alternatives in Y. Suppose that each individual i has a von Neumann-Morgenstern utility function $U_i:Y \rightarrow \mathbb{R}$ such that, for any pair of lotteries, individual i always prefers the lottery that gives him higher expected utility. So if we extend the set of alternatives by adding some lotteries from Δ(Y), then U_i defines individual i's preferences on this extended alternative set. With this framework, we can prove the following theorem.

Theorem 1.3. If the top cycle contains more than one alternative then, for any alternative z and any positive number ε, then we can construct an extended alternative set, composed of Y and a finite subset of Δ(Y), such that the extended top cycle includes a lottery in which the probability of z is at least 1-ε.

Proof. If the top cycle is not a single alternative, then the top cycle must include a set of three or more alternatives $\{w_1, w_2, ..., w_K\}$ such that $w_1 << w_2 << .. << w_K << w_1$.

Von Neumann-Morgenstern utility theory guarantees that every individual who strictly prefers w_{j+1} over w_j, will also strictly prefer $(1-\varepsilon)q + \varepsilon[w_{j+1}]$ over $(1-\varepsilon)q + \varepsilon[w_j]$ for any lottery q. (Here $(1-\varepsilon)q + \varepsilon[w_j]$ denotes the lottery that gives outcome w_j with probability ε, and otherwise implements the outcome randomly selected by lottery q.) Thus, by continuity, there must exist some large integer M such that every individual who strictly prefers w_{j+1} over w_j, will also strictly prefer

$$(1-\varepsilon)\big((1 - (m+1)/M)[w_1] + ((m+1)/M)[z]\big) + \varepsilon[w_{j+1}]$$

$$\text{over} \quad (1-\varepsilon)\big((1 - m/M)[w_1] + (m/M)[z]\big) + \varepsilon[w_j]$$

for any m between 0 and M-1. That is, the same majority that would vote to change from w_j to w_{j+1} would also vote to change from w_j to w_{j+1} with probability ε even when this decision also entails a probability $(1-\varepsilon)/M$ of changing from w_1 to z.

We now prove the theorem using the Y*(2) characterization of the top cycle. Because w_1 is in the top cycle, we can construct a naive chain from any alternative x to w_1 ($x <<...<< w_1$). This naive chain can be continued from w_1 to z as follows:

$$w_1 = (1-\varepsilon)[w_1] + \varepsilon[w_1] \;<<\; (1-\varepsilon)\big((1\text{-}1/M)[w_1] + (1/M)[z]\big) + \varepsilon[w_2]$$

$$<< (1-\varepsilon)\big((1\text{-}2/M)[w_1] + (2/M)[z]\big) + \varepsilon[w_3]$$

$$... << (1-\varepsilon)[z] + \varepsilon[w_j] \quad \text{(for some j)}.$$

So including all the lotteries of this chain as alternatives gives us an extension of Y in which a lottery $(1-\varepsilon)[z] + \varepsilon[w_j]$ can be reached by a naive chain from any alternative x. Q.E.D.

The proof of Theorem 1.3 uses the naive-chain characterization of the top cycle (Y*(2)), but the equivalence theorem tells us that this result also applies to agenda manipulation with sophisticated voters. That is, if the chairman can include randomized social-choice plans among the possible outcomes of an agenda then, either a Condorcet winner exists, or else the chairman can design a binary agenda that selects any arbitrary alternative (even one that may be worst for all voters) with arbitrarily high probability in the majority-rule sophisticated outcome.

Lobbying and incentives for legislative organization

(Diermeier & Myerson, AER 1999; Groseclose & Snyder, APSR 1996)
Legislatures develop different internal structures in different countries.
USA: Many independent committees, each with blocking power only.
UK: One committee (cabinet) normally determines passage of bills into law.
Why? "Party discipline in UK prevents independent committees."
But might this remark reverse cause and effect?
Strength of party discipline is not exogenous.

There is good reason to expect that party discipline can be affected by the electoral system (Carey & Shugart, Electoral Studies,1995).
Closed-list PR should make stronger parties than single-member districts.
But USA and UK both use plurality voting in single-member districts.
For structural explanations, we must look to legislative structures.

In his history of the development of British party discipline, Cox (Efficient Secret, 1987) finds that legislative party discipline developed ahead of electoral party discipline.
Shugart & Carey (Presidents & Assemblies, 1992) find correlation between president's legislative powers and weak parties. Independent gatekeeping committees in legislatures tend to correlate with weak party structure.

We haves many theories to explain why committees are powerful in USA:
Committee power as commitment device, to enforce and maintain distributive agreements of a majority coalition (Shepsle AJPS 1979, Weingast & Marshall JPE 1988).
Committee power as incentive for information-gathering (Gilligan&Krehbiel, JLawEconOrg 1987).
Committee proposal power as a seniority system that encourages reelection of incumbents (McKelvey & Riezman APSR 1992, based on Baron & Ferejohn APSR 1989).
But none of these models apply more to USA than UK.

Groseclose & Snyder's (APSR 1996) model of vote-buying by lobbyists:
Two lobbyists: agent 1 for new bill, agent 0 for status quo.
V = (agent 0's value for blocking a bill)
W = (agent 1's value for passing a bill)
Perfect information assumed. Agents learn V and W,
then agent 1 can offer bribes to legislators to pass bill,
and then agent 0 can offer bribes to kill bill.
Each legislator acts for the agent who has offered more
(for 1 if positive tie, for 0 if tie at $0).

If agent 1 makes bribes, they should be $(x(i))_{i\in L}$ that solve:

minimize $\sum_{i\in L} x(i)$ subject to $x(i) \geq 0$, $\forall i \in L = \{\text{legislators}\}$

and $\sum_{i\in C} x(i) \geq V$ for each coalition C that can block a bill.

The optimal value is $\sum_i x(i) = rV$ for some <u>hurdle factor</u> r.
Legislators get total payoffs rV if $W \geq rV$, get 0 otherwise.

Diermeier & Myerson (AER 1999) note that the hurdle factor depends on legislative organization:
r = 1 when a disciplined majority obeys single leader.
r = 2 with simple independent majority voting (no discipline)
r = 3 when a gatekeeper is added to an undisciplined majority-rule legislature
$r = 1/(1-Q)$ when a supermajority Q is required (without gatekeeper).

In a serial multicameral legislature, the overall hurdle factor is the sum of the hurdle factors in the separate chambers.
Let s = (hurdle factor in the "House"),
let t = (total hurdle factors in other chambers). So $r = s + t$.

Suppose chambers fix internal structures independently, before learning V & W.
Let $D(r) = P(\text{bill can pass})\, E(V|\text{bill can pass}) = P(W > rV)\, E(V|\ W > rV)$
Then E(payments to House) = $s\, D(s + t)$.
If $s + t = r_0$ where r_0 maximizes E(total payments to legislators) = $r\, D(r)$,
then $0 = (s+t)D'(s+t) + D(s+t)$ and so $\partial/\partial s\ \ s\, D(s + t) = -t\, D'(s + t) > 0$.
So bicameral separation creates incentive to raise hurdle factors.

Suppose V and W are independent exponential random variables, mean 1.
Then $s\, D(s + t) = s \int_0^\infty \int_{(s+t)v}^\infty v\, e^{-w}\, dw\, e^{-v}\, dv = s/(s + t + 1)^2$.
So $\operatorname{argmax}_{s\geq 1} s\, D(s+t) = t + 1$.
Simple unicameral case: $t = 0$ so $s = 1$ is optimal (strong majority leader).
With presidential veto: $t = 1$ so $s = 2$ optimal (no discipline).
Bicameral legislature: competition to have slightly higher hurdles...

An integrated model of elections and legislative bargaining. Austen-Smith&Banks APSR 1988 develop an integrated model of both electoral and legislative politics, to analyze the question: "Does PR truly create a legislature that is a proportional mirror of the people's interests groups."

The space of possible government policies is [0,1]. Benefits of power: $$ worth $G \geq 1$.
There are many voters, who have ideal points θ uniformly distributed in [0,1].
Three parties in {1,2,3} announce policies (x_1,x_2,x_3) in [0,1] simultaneously and independently.
Then voters each vote for a party, and seats are allocated by proportional representation (PR) with $\alpha = 0.05$ minimum.
After election, parties bargain to form a governing majority coalition.
An offer specifies a policy y in [0,1] and a division of spoils (g_1,g_2,g_3) with all $g_i \geq 0$, $g_1+g_2+g_3=G$.
The largest party offers first, if it fails to get a majority then second-largest offers, and if it fails to get a majority then smallest offers.
If all three fail to get a majority, then the outcome is caretaker government and new elections, which gives very negative payoffs to all parties.
Otherwise, each party i gets payoff $g_i - (y - x_i)^2$ (share of spoils minus policy embarassment).
(Getting no seats is also very negative outcome.)
Voter with ideal θ gets payoff $-(y - \theta)^2$ if government policy is y.

In equilibrium, if the largest party does not have a majority, then it offers to form coalition with smallest; if rejected then second largest would offer to largest; if both rejected then smallest would offer to closest; but first offer is accepted in equilibrium.
Government policy is the average of coalition partners' positions.
There is an equilibrium in which parties 1,2,3 respectively choose $x_1 = 0.20$, $x_2 = 0.80$, $x_3 = 0.50$; and each voter with ideal point θ votes for party 1 if $\theta \in [0,0.475)$, for party 3 if $\theta \in [0.475, 0.525]$, for party 2 if $\theta \in (0.525, 1]$.
So government policy outcome is equally likely to be 0.35 or 0.65 .
(Voting for most-preferred among this (x_1,x_2,x_3) would not be an equilibrium for the voters.)

If voters in $[0.475, 0.475+\varepsilon)$ voted for 1, the policy outcome would be 0.20, because party 2 would fall below the threshold α and would get no seats.
If voters in $[0.475-\varepsilon, 0.475)$ sincerely voted for 3, the policy outcome would be 0.80 for sure, because 2 would surely be the largest party.
If party 1 moved closer to center then, in this off-path subgame, voters could focus on an equilibrium in which everyone votes for party 2 or party 3 (so 3 gets majority, but 1 gets nothing).

The model has other equilibria, but this one suffices to show that extremists could be systematically overrepresented in a legislature that has been elected by proportional representation.
A reform that removes this advantage for being the largest party would decrease voters' incentive to vote for a big party.