CAPITALIST INVESTMENT AND POLITICAL LIBERALIZATION

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Abstract. We consider a simple political-economic model where capitalist investment is constrained by the government's temptation to expropriate. Political liberalization can relax this constraint, increasing the government's revenue, but also increasing the ruler's political risks. We analyze the ruler's optimal liberalization, where our measure of political liberalization is the probability of the ruler being replaced if he tried to expropriate private investments. Poorer endowments can support reputational equilibria with more investment, even without liberalization. So we find a resources curse, where larger resource endowments can decrease investment and reduce the ruler's revenue. The ruler's incentive to liberalize can be greatest with intermediate resource endowments. Strong liberalization becomes optimal in cases where capital investment yields approximately constant returns to scale. Adding independent revenue decreases optimal liberalization and investment. Mobility of productive factors that complement capital can increase incentives to liberalize, but equilibrium prices may adjust so that liberal and authoritarian regimes co-exist.

1. Introduction

Democratic political liberalization depends on incentives for the ruling elite. Even a popular revolution could not create sustained democracy if any leader, once installed in power, would act to make himself an authoritarian ruler. This paper analyzes a simple model to show how fundamental economic forces can motivate a ruler to liberalize his regime, even though such liberalization increases his political risks and shortens his expected term of office.

The key is that liberalization can encourage private investment which enlarges the government's tax base. In a tightly controlled authoritarian state, the ruler would incur little or no political risk from expropriating investors' assets, and so people may fear to invest in a country where a ruler governs without any potential checks on his power. (For other related models of authoritarian polities, see Wintrobe, 2007; and for other related models of insecure property rights, see Besley and Ghatak, 2009.) But in a liberal state where people have freedom to speak and organize politically, expropriation of private investments could cause a political scandal or crisis that could provoke a change of political leadership. Thus, a ruler may benefit from political liberalization when it enables him to credibly encourage more investment that enlarges

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his tax base, even if it also increases his political risks.

This relationship between political liberalization and capitalist investment is complicated by interactions with other factors that can also affect political incentives for protect private investment. Tiebout (1956) argued that mobility of people and resources can motivate a ruler to provide good government services even without democracy, to attract tax-payers and investors into his domain. This effect has been analyzed in a rigorous model by Epple and Zelenitz (1981). Following them, we find that democracy and resource mobility can be substitutes to some extent.

The political incentive problem interacts with natural resource endowments in a way that can cause a resource curse, as noted by Sachs and Warner (1995). Resources that are substitutes for invested capital can exacerbate the temptation to expropriate investments, and fixed resource revenue can reduce a ruler's willingness to accept the political risks of liberalization (see also Boix, 2003; Robinson, Torvik, Verdier, 2006; Al-Ubaydli, 2008; and Paltseva, 2008). We will see, however, that the liberalization and investment that results from these forces can be nonmonotone and discontinuous functions of the underlying parameters. Tightening the ruler's incentive constraint can cause a discontinuous increase in liberalization. As Paltseva (2008) found in a closely related model with more complex dynamics, the greatest incentives for liberalization may occur in nations with intermediate resource endowments.

In section 2, we introduce a simple model of investment that is constrained by the threat of expropriation under an authoritarian regime. In section 3, we extend the model by allowing the regime to liberalize politically, so that capitalists can be granted some political ability to protect their investments. The probability of political change if government officials wrongfully expropriated private investments is taken as the basic measure of political liberalization, and we analyze the benefits and costs of such liberalization for the ruling elite. Sections 4 through 6 consider special parametric cases and examples, to illustrate the subtle interactions in this model. Our final example, following Epple and Zelenitz (1981), considers a general equilibrium where mobile labor flows between local polities, which may adopt different levels of liberalization.

2. A model of investment without liberalization

Imagine an island that has some fixed productive resources F (such as land), which can be augmented by additional capital investment. With any capital investment $k \geq 0$, $Y(F+k)$ denotes the net output production flow per unit time. The invested capital k is assumed to be mobile and durable, and it can be used productively only when it is controlled by individuals outside the ruling elite, whom we may call capitalists. Capitalists' control over the invested capital k would enable them to take it abroad at any time. The fixed durable resources F can be controlled by the ruling elite, and cannot be removed from the island. For simplicity, we assume that the production function $Y(\bullet)$ is differentiable and strictly concave, with

$$Y(0) \geq 0, \;\; \lim_{\kappa \to 0} Y'(\kappa) = +\infty, \;\; \text{and} \;\; \lim_{\kappa \to +\infty} Y'(\kappa) = 0.$$

The capitalists' rate of time discounting is r. So to deter capital flight, the capitalists must always get an income flow worth rk from their invested capital. We may assume that the output $Y(F+k)$ is net of labor and resource costs, and so the authoritarian rulers of the government can take (in taxes and fixed resource rents) the remaining flow $Y(F+k)-rk$.

Let ρ denote the rate of time discounting for an authoritarian ruler who has not liberalized. This discount rate ρ may be different from r because, for example, the ruler might face some exogenous risk of losing power even without liberalization, which would increase ρ above r. So the present discounted value for the ruler of this island is

$$(Y(F+k)-rk)/\rho.$$

On the other hand, depreciation of invested capital could be included in our model by adding the depreciation rate into the capitalists' required rate of return r, which could make r greater than ρ.

At any time, the ruler could try to expropriate the invested capital. We assume that, if the ruler tried to seize the capital, the capitalists could not flee fast enough to prevent the ruler from profiting from some or all of their investments. To be specific, we let θk denote the fraction of the invested capital stock that the ruler could expropriate, for some θ between 0 and 1. So $1-\theta$ may represent the fraction of capital that would be taken abroad by fleeing capitalists or destroyed in their struggle with expropriating officials. (For numerical examples, we can just consider cases where $\theta=1$.) These parameters are assumed to satisfy

$$r > 0, \;\; \rho > 0, \;\; \text{and} \;\; 1 \geq \theta > 0.$$

Now to characterize the strongest possible deterrent against such expropriation, we should consider the worst possible subsequent outcome for the ruler if his regime survives. In the worst case, the ruler thereafter would have lost any reputation for protecting capitalists' investments, and so the value of his continuation in power would be $Y(F)/\rho$ after a successful expropriation. The ruler would be unable to productively use the expropriated capital θk on his island (as doing so would require him to give control of it back to distrustful capitalists), but the ruler could sell the capital abroad for its value θk (to be used productively in some country where capitalists can still trust the government). So the authoritarian ruler's expected value of expropriation would be

$$\theta k + Y(F)/\rho.$$

Thus, capital k can be safely invested in an authoritarian regime iff k satisfies the ruler's nonexpropriation incentive constraint

$$(Y(F+k)-rk)/\rho \geq \theta k + Y(F)/\rho. \tag{1}$$

An investment k is <u>feasible without liberalization</u> iff it satisfies this constraint, which is equivalent to

$$Y(F+k)-(r+\rho\theta)(F+k) \geq Y(F)-(r+\rho\theta)F. \tag{2}$$

Let K_r denote the ideal total value of resources and investments that would maximize the island's net output less the cost of capital if there were no incentive constraint. That is, let

$$K_r = \text{argmax}_{\kappa\geq 0}\ Y(\kappa)-r\kappa, \text{ and so } Y'(K_r) = r.$$

If the fixed-resource endowment F were greater than K_r then no investments in the island could ever be profitable with the capital cost r, and so we assume henceforth that F is less than K_r,

$$0 \leq F < K_r.$$

The unconstrained ideal investment k is K_r-F, which is feasible iff

$$Y(K_r)-(r+\rho\theta)K_r \geq Y(F)-(r+\rho\theta)F.$$

Analogously, let $K_{r+\rho\theta}$ denote the endowment κ that maximizes $Y(\kappa)-(r+\rho\theta)\kappa$, satisfying

$$K_{r+\rho\theta} = \text{argmax}_{\kappa\geq 0}\ Y(\kappa)-(r+\rho\theta)\kappa,$$

$$Y'(K_{r+\rho\theta}) = r+\rho\theta, \text{ and } 0 < K_{r+\rho\theta} < K_r.$$

Inequality (2) and concavity of Y together imply that positive investment is feasible without liberalization only if the fixed endowment F is less than this critical level $K_{r+\rho\theta}$.

Theorem 1. The set of investments that are feasible without liberalization is an interval $\{k \mid 0 \le k \le h_0\}$, for some h_0 which denotes the maximum feasible investment without liberalization. In this feasible set, the ruler's expected value $(Y(F+k)-rk)/\rho$ is maximized letting

$$k = \min\{h_0, K_r - F\}.$$

If $F \ge K_{r+\rho\theta}$, then $h_0 = 0$. But if $F < K_{r+\rho\theta}$, then h_0 satisfies $F + h_0 > K_{r+\rho\theta}$ and

$$Y(F+h_0)-(r+\rho\theta)(F+h_0) = Y(F)-(r+\rho\theta)F.$$

In this case, when $F < K_{r+\rho\theta}$, a small increase in the fixed-resource endowment F would cause both h_0 and $F+h_0$ to decrease.

Proof. The set of investments k that are feasible without liberalization is

$$\{k \ge 0 \mid Y(F+k)-(r+\rho\theta)(F+k) \ge Y(F)-(r+\rho\theta)F\}$$

Because Y is concave, this set is a closed interval that includes k=0. The interval has a finite upper bound, which we denote by h_0, because $Y'(\kappa)-(r+\rho\theta)$ goes to $-(r+\rho\theta)$ as $\kappa \to \infty$. If $F \ge K_{r+\rho\theta}$ then no positive investment k can be feasible without liberalization, because $Y'(F+k)-(r+\rho\theta)<0$ when $F+k > K_{r+\rho\theta}$, and so we get $h_0=0$ in this case.

On the other hand, if $F < K_{r+\rho\theta}$, then the investment $K_{r+\rho\theta}-F$ is in this feasible interval, and so its supremum h_0 must satisfy $F+h_0 > K_{r+\rho\theta}$ and

$$Y(F+h_0)-(r+\rho\theta)(F+h_0) = Y(F)-(r+\rho\theta)F.$$

In this case, with $F < K_{r+\rho\theta}$ and $F+h_0 > K_{r+\rho\theta}$, we have $Y'(F) > r+\rho\theta$ and $Y'(F+h_0) < r+\rho\theta$. So a small increase of F would cause $Y(F)-(r+\rho\theta)F$ to increase, but then maintaining the equation must cause $F+h_0$ to decrease.

In any case, any feasible increase of investment k could increase the ruler's expected payoff value as long as $F+k<K_r$, because $Y'(F+k) > r$ when $k<K_r-F$. So if the ideal investment K_r-F is not in the feasible interval, then the ruler wants the largest feasible investment h_0. QED.

By Theorem 1, when the unconstrained ideal investment K_r-F is not feasible, the authoritarian ruler wants to encourage investment up to the maximum feasible value h_0, where the incentive constraint is binding. Theorem 1 also tells us that, if an authoritarian ruler can get any positive investment in his island, then a small parametric increase in the island's fixed resources F would actually cause the maximum feasible output $Y(F+h_0)$ to decrease, as h_0 implicitly depends on F. Such a "resources curse" can occur because adding fixed resources makes the ruler's incentive constraint harder to satisfy.

[Insert Figure 1 about here]

But Theorem 1 also implies that no positive investment is feasible without liberalization unless the fixed-resource endowment is smaller than the critical value $K_{r+\rho\theta}$, which is strictly less than the unconstrained ideal capital stock K_r. Thus, the ideal investment $K_r - F$ cannot be feasible without liberalization unless the fixed-resource endowment F is much smaller than K_r.

Corollary 1. Given the production function Y and the parameters (r,ρ,θ), if $Y(K_r) - (r+\rho\theta)K_r < Y(0)$ then the ideal investment $K_r - F$ is not feasible without liberalization for any fixed endowment $F < K_r$. But if $Y(K_r) - (r+\rho\theta)K_r \geq Y(0)$, then there exists some f_0 strictly less than K_r such that the ideal investment $K_r - F$ is feasible without liberalization iff $F \leq f_0$. This bound f_0 satisfies $Y(f_0) - (r+\rho\theta)f_0 = Y(K_r) - (r+\rho\theta)K_r$ and $f_0 < K_{r+\rho\theta}$.

Proof. From Theorem 1, the unconstrained ideal investment is feasible without liberalization if and only if $F+h_0 \geq K_r$, where h_0 implicitly depends on F. But $h_0=0$ when $F \geq K_{r+\rho\theta}$, and so the ideal cannot be feasible without liberalization for any F between $K_{r+\rho\theta}$ and K_r. Furthermore, $F+h_0$ is a decreasing function of F when F is less than $K_{r+\rho\theta}$. So the unconstrained ideal cannot be feasible for any F between 0 and $K_{r+\rho\theta}$ unless it is feasible for F=0, which means that $Y(K_r) - (r+\rho\theta)K_r \geq Y(0)$. In that case, by inequality (2), the set of possible F such that the ideal is feasible without liberalization is

$$\{F \mid Y(K_r) - (r+\rho\theta)(K_r) \geq Y(F) - (r+\rho\theta)F,\ 0 \leq F \leq K_{r+\rho\theta}\}.$$

But $Y'(F) - (r+\rho\theta) > 0$ when $F < K_{r+\rho\theta}$, and $Y(K_r) - (r+\rho\theta)K_r < Y(K_{r+\rho\theta}) + (r+\rho\theta)K_{r+\rho\theta}$, and so this set is an interval from 0 to some upper bound $f_0 < K_{r+\rho\theta}$ where, by continuity, we have $Y(f_0) - (r+\rho\theta)f_0 = Y(K_r) - (r+\rho\theta)K_r$. QED.

By Corollary 1, to test whether the unconstrained ideal investment could be feasible without liberalization for any fixed-resource endowment F, it suffices to consider the case of F=0. For a Cobb-Douglas production function, if the exponent of capital is large enough then the unconstrained ideal investment cannot be feasible without liberalization, even with F=0.

Corollary 2. If the production function is $Y(\kappa) = A\kappa^{\alpha}$, for some $A>0$ and $0<\alpha<1$, then the unconstrained ideal investment $K_r - F$ is feasible without liberalization for $F=0$ iff $\alpha \leq r/(r+\rho\theta)$.

Proof. Given $Y(\kappa)=A\kappa^{\alpha}$, we have $Y(0) = 0$. So the ideal investment is feasible without liberalization for $F=0$ iff $AK_r^{\alpha}-(r+\rho\theta)K_r \geq 0$. But the ideal K_r satisfies $Y'(K_r) = \alpha AK_r^{\alpha-1} = r$, and so $AK_r^{\alpha} = K_r\, r/\alpha$. Thus $AK_r^{\alpha} - (r+\rho\theta)K_r = (r/\alpha - r - \rho\theta)K_r$, which is nonnegative iff $r/\alpha - r - \rho\theta \geq 0$. This inequality is equivalent to $\alpha \leq r/(r+\rho\theta)$. QED.

3. Encouraging investment with political liberalization

When the unconstrained ideal investment $K_r - F$ is not feasible without liberalization, the nonexpropriation incentive constraint strictly reduces the authoritarian ruler's revenue, and so the ruler should be willing to pay a positive cost to relax this constraint. The incentive constraint can be relaxed by giving capitalists some power to protect their investments from the ruler. As long as the regime remained absolutely authoritarian, there could be no question of instituting processes by which citizens outside the government could punish their ruler for violating laws. Distributing power over the government to individuals outside the government constitutes political liberalization, and it must entail some possibility that the ruler may be replaced as a result of actions by these individuals. This possibility that the ruler might lose office if he tried to expropriate capitalist investments is key to both the benefits and costs of such political liberalization for the ruler.

Abstracting from the details of political institutions, let us here define the basic measure of political liberalization to be this probability that the ruler would fall from power if he ever tried to wrongfully expropriate investments that were owned by individuals outside the government. That is, saying that a regime has liberalization λ can be taken to mean that, if the ruler ever tried to expropriate capitalist investments, then he would face a probability λ of being deposed. For such liberalization to be credible, we should also stipulate that the ruler would face (at least) the same probability λ of falling from power if he ever tried to reverse the liberalization itself.

The benefit of liberalization is that it relaxes the ruler's incentive constraint by reducing the temptation to expropriate. If capitalists have invested k in an island that is governed by a regime with liberalization λ, then the ruler's expected payoff value from trying to expropriate their investments would be

$$W(k,\lambda) = (1-\lambda)(\theta k+Y(F)/\rho).$$

This expected payoff is just the ruler's value of expropriation, from the previous section, multiplied by the $(1-\lambda)$ probability of the expropriation being successful. (We assume here that. in the λ-probability event that the ruler falls from power, his subsequent payoff would be 0.)

Liberalization also has a political cost for the ruler. Any liberalization that could cause the ruler's downfall in the event of expropriation could also cause his downfall in other events. When the ruler is subject to the political judgment of the people, there will be some chance that they may judge him guilty even when he is innocent. Suspicious events or scandals may make it appear that he is trying to expropriate even when he is not.

To be specific, let us assume that there are false-alarm scandals that occur at some Poisson rate ψ, and people react to such scandals exactly as they would to a genuine attempt to expropriate capital. That is, for a regime with liberalization λ, when the government is actually not trying to expropriate capital, in any short time interval of length ε there will be an approximate probability $\psi\varepsilon$ of a scandal, and there will be an approximate probability $\psi\varepsilon\lambda$ of the ruler's being replaced because of such a scandal. This scandal rate ψ is assumed to be positive,

$$\psi > 0.$$

So in a regime with liberalization λ, the current ruler should discount future revenue at rate $\rho+\psi\lambda$. Thus, when a regime with liberalization λ gets capital investment k, the ruler's expected present discounted value of all future net revenue is

$$V(k,\lambda) = (Y(F+k)-rk)/(\rho+\psi\lambda).$$

We are assuming that, as long as the ruler remains in power, political liberalization does not empower any other political groups to expropriate capitalists or take part of the regime's profits.

Capital k can be safely invested in a regime with liberalization λ iff k and λ satisfy the ruler's incentive constraint

$$V(k,\lambda) \geq W(k,\lambda). \tag{3}$$

Of course capital must be nonnegative $k \geq 0$. As our measure of liberalization λ is a probability, it must satisfy the probability constraints

$$0 \le \lambda \le 1.$$

The ruler's optimal regime (k,λ) should maximize $V(k,\lambda)$ subject to these constraints.

With the definitions of V and W, the incentive constraint (3) is equivalent to

(4) $$(Y(F+k)-rk)/(\theta k + Y(F)/\rho) \ge (1-\lambda)(\rho+\psi\lambda).$$

So let

$$Q(k) = (Y(F+k)-rk)/(\theta k + Y(F)/\rho), \text{ with } Q(0) = \rho,$$

and let

$$q(\lambda) = (1-\lambda)(\rho+\psi\lambda).$$

The quotient $Q(k)$ here is the ruler's rate of revenue per unit of expropriatable wealth when capitalist investment is k. The quadratic $q(\lambda)$ is the ruler's required rate of return on expropriatable assets when liberalization is λ. Then we can rewrite the incentive constraint as

$$Q(k) \ge q(\lambda).$$

Given any k, $V(k,\lambda)$ is decreasing in λ, so the rulers would prefer the smallest feasible λ. So for any k such that $Y(k)-rk\ge 0$, let $\Lambda(k)$ denote the smallest $\lambda\ge 0$ such that $Q(k) \ge q(\lambda)$.

$$\Lambda(k) = \min\{\lambda \mid \lambda\ge 0, \ q(\lambda) \le Q(k)\}.$$

So the regime (k,λ) is optimal when k maximizes $V(k,\Lambda(k))$ and $\lambda=\Lambda(k)$.

An investment k is feasible with $\lambda=0$ (that is, without liberalization) iff $Q(k) \ge \rho$, because $q(0)=\rho$. Thus, if $Q(k) \ge \rho$ then $\Lambda(k) = 0$ and k is in the interval $[0,h_0]$ that is the feasible set without liberalization, as characterized in Theorem 1.

For any investment k that is not feasible without liberalization, we must have $Q(k) < \rho$. In this case, $\Lambda(k)$ is the liberalization $\lambda>0$ that satisfies

$$Q(k) = q(\lambda) = \rho+(\psi-\rho)\lambda-\psi\lambda^2,$$

and so, by the quadratic formula,

(5) $$\Lambda(k) = \{\psi-\rho + [(\psi-\rho)^2 + 4\psi(\rho-Q(k))]^{0.5}\}/(2\psi).$$

This formula implies

$$0 < \Lambda(k) \text{ and } 1-\rho/\psi < \Lambda(k) < 1 \text{ when } Q(k) < \rho,$$

because $[(\psi-\rho)^2 + 4\psi(\rho-Q(k))]^{0.5} > |\psi-\rho|$.

If $\psi>\rho$, then the liberalization function $\Lambda(k)$ is discontinuous at the investment k where

$Q(k)=\rho$, because then $\Lambda(k)=0$ but $\Lambda(k+\varepsilon)>1-\rho/\psi$ for any $\varepsilon>0$. Indeed, we will see examples where a small parametric change can cause the optimal liberalization to jump discontinuously.

With these results, we can characterize the regime (k,λ) that is optimal for the ruler subject to the general nonexpropriation incentive constraint (3).

Theorem 2. A regime with liberalization λ and investment k is optimal for the ruler when

$$k=\operatorname{argmax}_{\kappa\geq 0} V(\kappa,\Lambda(\kappa)) \text{ and } \lambda = \Lambda(k).$$

If an optimal regime has $\lambda=0$, then the optimal investment k is as in Theorem 1,

$$k = \min\{h_0, K_r-F\}.$$

If an optimal regime has $\lambda>0$, then it satisfies the equations

$$[Y(F+k)-rk]/[\theta k + Y(F)/\rho] = (\rho+\psi\lambda)(1-\lambda),$$

$$Y'(F+k) = r+\theta\psi(1-\lambda)^2,$$

and it satisfies the inequalities

$$\max\{0, 1-\rho/\psi\} < \lambda < 1, \text{ and } k > h_0.$$

Proof. The case of $\lambda=0$ reduces to the model without liberalization that we considered in section 2. The only reason to choose a positive liberalization would be to get more investment $k>h_0$, where $Q(k) < \rho$. For any such k that requires positive liberalization, if the incentive constraint were not binding then the ruler could increase $V(k,\lambda)$ by decreasing λ slightly, and so optimal liberalization is $\lambda=\Lambda(k)$ and satisfies the incentive constraint is as a binding equation $Q(k)=q(\lambda)$, which is the first equation listed in the theorem for the $\lambda>0$ case.

The second equation for the $\lambda>0$ case is a local optimality condition. With $k>h_0$ and $Q(k)<\rho$, $\Lambda(k)$ is continuously differentiable, by (5), and the derivatives satisfy:

$$\begin{aligned} Q'(k) &= [Y'(F+k) - r - \theta Q(k)]/(\theta k+Y(F)/\rho) \\ &= [(Y'(F+k)-r)(\theta k+Y(F)/\rho) - \theta(Y(F+k)-rk)]/(\theta k+Y(F)/\rho)^2 \\ &= [(Y'(F+k)-r)Y(F)/\rho - \theta(Y(F+k)-kY'(F+k))]/(\theta k+Y(F)/\rho)^2 \\ &\leq [(Y'(F+k)-r)/\rho - \theta]Y(F)/(\theta k+Y(F)/\rho)^2 < 0, \end{aligned}$$

$$q'(\Lambda(k)) = \psi-\rho-2\psi\Lambda(k) < -\psi\Lambda(k) < 0,$$

$$\begin{aligned} \Lambda'(k) &= Q'(k)/q'(\Lambda(k)) \\ &= [r + \theta q(\Lambda(k)) - Y'(F+k)]/[(\theta k+Y(F)/\rho)(2\psi\Lambda(k)+\rho-\psi)] > 0. \end{aligned}$$

Here the Q' inequalities use concavity of Y and get $Y'(F+k) < r+\rho\theta$ from $F+k > F+h_0 \geq K_{r+\rho\theta}$, and the q' inequalities use $\Lambda(k) > \max\{1-\rho/\psi, 0\}$. So when $k>h_0$, the ruler's marginal value of additional investment, with the necessary liberalization, is

(6) $$d/dk\ V(k,\Lambda(k)) = d/dk\ W(k,\Lambda(k)) = d/dk\ (1-\Lambda(k))(\theta k+Y(F)/\rho).$$
$$= (1-\Lambda(k))\theta - (\theta k+Y(F)/\rho)\Lambda'(k)$$
$$= (1-\Lambda(k))\theta - [r + \theta(1-\Lambda(k))(\rho+\psi\Lambda(k)) - Y'(F+k)]/(2\psi\Lambda(k)+\rho-\psi)$$
$$= [Y'(F+k) - r - \theta\psi(1-\Lambda(k))^2]/(2\psi\Lambda(k)+\rho-\psi).$$

So a locally optimal investment k where $d/d\ V(k,\Lambda(k)) = 0$ must have
$$Y'(F+k) = r+\theta\psi(1-\Lambda(k))^2.$$
All other points in the theorem have been argued above. QED.

4. Optimal liberalization in special parametric cases

When the unconstrained ideal is not feasible without liberalization, positive liberalization $\lambda>0$ will be optimal if the scandal rate ψ is small enough. Beyond this remark, it seems difficult to characterize when positive liberalization will be optimal for a general production function. There is, however, one instructive case which may be worth noting. The case of $F=K_{r+\rho\theta}$ is the worst endowment for an authoritarian regime, as this is the smallest endowment F for which no investment is feasible without liberalization.

Corollary 3. If $F=K_{r+\rho\theta}$ and $\psi<\rho$ then positive liberalization $\lambda>0$ is optimal.

Proof. With $F=K_{r+\rho\theta}$, the maximal investment with $\lambda=0$ is $h_0=0$. Equation (6) gives us a formula for $d/dk\ V(k,\Lambda(k))$ when $k>h_0$. With $\psi<\rho$, this formula can be extended continuously to $k=h_0$ (as Λ from (5) is continuously differentiable), and there $\Lambda(0)=0$ and $Y'(F)=r+\rho\theta$ imply $d/dk\ V(k,\Lambda(k)) = [r+\rho\theta-r-\theta\psi]/(\rho-\psi) = \theta > 0$. Thus, increasing k and $\Lambda(k)$ above 0 increases the value V. QED.

For Cobb-Douglas production functions that are approximately linear in capital, we can show that the optimal liberalization is close to 1 if the fixed endowment is substantially below the unconstrained ideal. Approximate linearity implies that, when the marginal product of capital Y' is greater than the cost of capital r, surplus returns can be gained from large investments, but then strong liberalization is required to credibly protect these large investments. This strong liberalization result may be applicable with endogenous growth theories (such as Romer 1987) that suggest an approximately linear dependence of national output on investment.

Theorem 3. Consider production functions of the form $Y(F+k) = A(F+k)^{\alpha}$, where $A>0$ and $0<\alpha<1$, so that the parameters of our models are $(r,\rho,\theta,\psi,F,A,\alpha)$. Consider a sequence of models where the parameters $(r,\rho,\theta,\psi,A,\alpha)$ converge to finite positive limits, and the fixed endowments F satisfy $\lim Y'(F)/r > 1$. If $\lim \alpha = 1$ then the optimal regimes (k,λ) satisfy $\lim Y'(F+k)/r = 1$ and $\lim \lambda = 1$.

Proof. For the optimal solution (k,λ), let $s = Y'(F+k)$. We know $s \geq r$, because the optimal solution always satisfies $F+k \leq K_r$. The Cobb-Douglas production function gives us

$$s = \alpha A(F+k)^{\alpha-1} = Y(F+k)\alpha/(F+k), \text{ and } F+k = (A\alpha/s)^{1/(1-\alpha)}.$$

Now suppose, contrary to the theorem, that $\limsup s/r > 1$. We have bounds

$$\begin{aligned} V(k,\lambda) &\leq Y(F+k)/\rho = (F+k)Y'(F+k)/(\rho\alpha) \\ &= (A\alpha/s)^{1/(1-\alpha)}\, s/(\rho\alpha), \\ V(K_r-F,1) &\geq [Y(K_r)-rK_r]/(\rho+\psi) = rK_r(1/\alpha-1)/(\rho+\psi) \\ &= r(A\alpha/r)^{1/(1-\alpha)}\,(1/\alpha-1)/(\rho+\psi), \\ V(K_r-F,1)/V(k,\lambda) &\geq (1-\alpha)(r/s)(s/r)^{1/(1-\alpha)}\,\rho/(\rho+\psi) \\ &= [(1-\alpha)^{(1/\alpha-1)}\,(s/r)]^{\alpha/(1-\alpha)}\,\rho/(\rho+\psi). \end{aligned}$$

But $(1-\alpha)^{(1/\alpha-1)} \to 1$ as $\alpha\to 1$ (take logs and apply l'Hospital's rule). So along the subsequence where $\limsup s/r > 1$, as $\alpha\to 1$ we would get $V(K_r-F,1)/V(k,\lambda) \to +\infty$, but this would contradict the assumption that $V(k,\lambda)$ is optimal.

Thus, we must have $\lim Y'(F+k)/r = \lim s/r = 1$. With $\lim Y'(F)/r > 1$, this implies $F/(F+k) = (Y'(F+k)/Y'(F))^{1/(1-\alpha)} \to 0$ and so $k/(F+k)\to 1$ as $\alpha\to 1$. Then we get

$$\begin{aligned} \lim q(\lambda) &\leq \lim Q(k) = \lim\, [Y(F+k)-rk]/[\theta k+Y(F)/\rho] \\ &= \lim\, [(F+k)s/\alpha - rk]/[\theta k + Y(F)/\rho] \\ &\leq \lim\, [s/\alpha - rk/(F+k)]/[\theta k/(F+k)] = 0. \end{aligned}$$

So $\lim q(\lambda) = 0$, and so $\lim \lambda = 1$. QED

We have been considering resources F that are substitutes for capital investment, but let us now consider the impact of adding revenue income for the government that is independent of capitalist investments. Adding a fixed revenue means adding a positive constant $z>0$ to the production function, changing it from $Y(\kappa)$ to $\hat{Y}(\kappa) = Y(\kappa)+z$ for all $\kappa \geq 0$, keeping all other parameters unchanged.

Theorem 4. Adding a fixed revenue $z>0$ would allow greater investments to be feasible with any given liberalization λ such that $\max\{0,1-\rho/\psi\} < \lambda < 1$. When the optimal regime has such a positive liberalization λ, however, adding a fixed revenue would decrease both the optimal liberalization and the optimal investment. When the optimal regime involves no liberalization ($\lambda=0$), adding a fixed revenue would not change the optimal regime.

Proof. An investment k is feasible with λ iff it satisfies the incentive constraint

$$V(k,\lambda) = (Y(F+k)-rk)/(\rho+\psi\lambda) \geq W(k,\lambda) = (1-\lambda)(\theta k+Y(F)/\rho).$$

Adding $z>0$ to Y would increase V by $z/(\rho+\psi\lambda)$ but would increase W by $z(1-\lambda)/\rho$. With $\lambda > \max\{0,1-\rho/\psi\}$, we have $(1-\lambda)(\rho+\psi\lambda) < \rho$, and so $z/(\rho+\psi\lambda) > z(1-\lambda)/\rho$. Thus, the additional revenue would strictly relax the incentive constraint, allowing strictly greater investments to become feasible, as long as liberalization is held constant. With $\lambda=0$, however, both sides of the incentive constraint would increase by z/ρ, and so the additional revenue z would not change the set of feasible investments.

Let $K(\lambda)$ be the largest investment feasible with liberalization λ, so $K = \Lambda^{-1}$. From the formula for Λ' that was derived in the proof of Theorem 2, we get

$$K'(\lambda) = 1/\Lambda'(K(\lambda)) = (\theta K(\lambda)+Y(F)/\rho)(2\psi\lambda+\rho-\psi)/[r+\theta q(\lambda)-Y'(F+K(\lambda))].$$

Now to show that adding fixed revenue would cause the optimal liberalization to decrease when it is positive, consider the function $v(\lambda) = LN(V(K(\lambda),\lambda))$. For any $\lambda > \max\{0,1-\rho/\psi\}$, the marginal (logarithmic) value of liberalization can be computed using formula (6) for dV/dk:

$$\begin{aligned} v'(\lambda) &= d/d\lambda\, [LN(V(K(\lambda),\lambda))] = (dV/dk)\, K'(\lambda)/V(K(\lambda),\lambda) \\ &= [(1-\lambda)\theta - (\theta K(\lambda)+Y(F)/\rho)\Lambda'(K(\lambda))]\, K'(\lambda)/[(\theta K(\lambda)+Y(F)/\rho)(1-\lambda)] \\ &= \theta(2\psi\lambda+\rho-\psi)/[r+\theta q(\lambda)-Y'(F+K(\lambda))] - 1/(1-\lambda). \end{aligned}$$

Adding revenue $z>0$ to the production function Y would increase $K(\lambda)$, and so would decrease $Y'(F+K(\lambda))$, and thus would decrease $v'(\lambda)$. In the case of $\psi>\rho$, we must also consider the jump from $\lambda=0$ to $\lambda=1-\rho/\psi$; but as both ends allow the same maximal investment h_0, we get

$$v(1-\rho/\psi)-v(0) = -LN(\rho+\psi(1-\rho/\psi)) + LN(\rho),$$

which would not be changed by additional revenue. Thus, the marginal value of liberalization is nonincreasing in additional revenue, and this marginal value $v'(\lambda)$ is strictly decreasing in additional revenue when λ is in the interval where positive optima can be found. Thus, additional revenue would decrease the optimal liberalization when it is positive. If the optimal liberalization decreased all the way to 0, then the optimal investment would decrease to h_0, the maximal investment without liberalization (which is not changed by additional revenue). Otherwise, if the optimal liberalization λ decreased within the positive range, then the optimality equation $Y'(F+k) = r+\theta\psi(1-\lambda)^2$ would imply that $Y'(F+k)$ must increase, and so the optimal investment k must decrease. QED.

The consequences of additional revenue in Theorem 4 require careful interpretation. While the old optimal equilibrium applies, decreasing liberalization would cause a politically risky crisis, and so the ruler would choose to maintain the given liberalization and invite additional investment. In the long run, however, when a ruler can renegotiate the equilibrium optimally, liberalization would decrease, and investment would decrease below its original level. Of course, adding fixed revenue could never make the ruler worse off, as the old regime would remain feasible, and so it could not cause the kind of resources curse that harms even the ruler, which a parametric increase in F can cause.

5. Examples

Consider an island where the production function is $Y(\kappa) = \kappa^{0.4}$, the capitalists' discount rate is $r=0.05$, the authoritarian discount rate is $\rho=0.1$, the expropriatable fraction of investments is $\theta=1$, and the democratic scandal rate is $\psi=0.1$.

With no incentive constraints, the unconstrained ideal capital stock for this Y would be

$$K_r = (0.4/r)^{1/(1-0.4)} = 32,$$

which would yield output $Y(K_r) = 4$. But this ideal is not feasible without liberalization for $F=0$ or any other endowment $F \geq 0$.

The given scandal rate $\psi=0.1$ is high enough that the ruler here would prefer not to liberalize with any fixed endowment F. With $F=0$, the optimal regime is

$$k = h_0 = (1/(r+\rho\theta))^{1/(1-0.4)} = 23.61, \text{ and } \lambda=0,$$

which yields output $Y(F+k) = 3.54$ and gives the ruler $V(k,\lambda) = 23.61$.

Increasing the endowment parameter F above 0 would tighten the incentive constraint and reduce investment. The critical endowment, above which no positive investment is feasible without liberalization, is

$$K_{r+\rho\theta} = (0.4/(r+\rho\theta))^{1/(1-0.4)} = 5.128.$$

When F=5.128, the optimal regime is k=0 and λ=0, and then output is $Y(F+0) = 1.92$, and the ruler's payoff value is $V(k,\lambda)=19.23$. Thus, we find a fixed-resources curse here, as increasing the fixed endowment F from 0 to 5.128 decreases output Y (from 3.54 to 1.92) and decreases the ruler's payoff V (from 23.61 to 19.23).

With F=5 (slightly less than $K_{r+\rho\theta}$), the optimal regime has very small investment k=0.258 and no liberalization λ=0, so that output is Y= 1.94, and the ruler's value is V = 19.29.

By Corollary 3, decreasing the scandal rate ψ would encourage the ruler to liberalize for at least some fixed endowments F near 5.128. For comparison, the case of ψ=0.05 is shown in Figure 2. With ψ=0.05, the optimal regime involves positive liberalization when the fixed endowment F is between 1.9 and 10.1 (up to a maximal liberalization of λ=0.26 at F=4.6), but there would still be no liberalization with F<1.9 or F>10.1. When F<1.9, the ruler does not liberalize because, even without liberalization, he can credibly promise to protect large capitalist investments in a reputational equilibrium, as his lack of resources would make the loss of reputation very costly for him. When F>10.1, the ruler does not liberalize because the large revenue from fixed resources makes him unwilling to take any risks of losing power.

Thus, Figure 2 shows that the total capital stock F+k and the net output Y(F+k) can be nonmonotone functions of the fixed-resource endowment F. In a politically unsophisticated society where the scandal rate ψ is relatively high, we find a U-shaped dependence of output on fixed endowments, with a minimum at $F=K_{r+\rho\theta}$. When greater political sophistication reduces the scandal rate ψ, liberalization on islands with intermediate endowments near $K_{r+\rho\theta}$ can create a W-shaped dependence of output on fixed endowments, with local minima above and below the interval of liberalization.

[Insert Figure 2 about here]

6. Examples continued: general equilibrium in an archipelago

Now consider an archipelago of many politically independent islands, on each of which output is produced by labor (L) and capital (F+k) according to the gross production function

$$(F+k)^{0.4}L^{0.5}.$$

The missing 0.1 exponent may be due to the effect of other fixed complementary factors on each island, such as government services. Each island is endowed with L=1 unit of labor and F=5 units of fixed capital, which can be augmented by capitalist investment k to yield total capital F+k. Let other parameters be as above: $r=0.05$, $\rho=0.1$, $\theta=1$, $\psi=0.1$.

If the laborers L=1 are held as unpaid serfs on each island, then the production function reduces to $(F+k)^{0.4}$, as in the preceding section. So with serfdom, as we saw above, the optimal regime on each island has small capitalist investment k=0.258 and no liberalization $\lambda=0$, and the ruler's payoff value is $V(k,\lambda) = 19.29$.

Now suppose that a small fraction of labor in the archipelago is actually free and mobile. In the above equilibrium, the marginal product of labor is $0.5(F+k)^{0.4}L^{0.5-1} = 0.971$, and so the equilibrium wage for such mobile labor should be $w = 0.971$. Assuming that the number of islands is large, however, the small fraction of mobile labor in the archipelago could be a virtually unbounded supply of labor for one island. Let us now consider what would happen if the ruler of one island decided to try to recruit such mobile free laborers in large numbers.

For simplicity, suppose that such a ruler would first free his own local serfs, before he could recruit extensively from the mobile labor supply at the wage w. Net of labor costs, the returns for capital and government for this island would be

$$Y(F+k) = \max_{L\geq 0}\,[(F+k)^{0.4}\,L^{0.5} - wL] = (F+k)^{0.8}/(4w), \tag{7}$$

which is achieved with the labor demand

$$L = (F+k)^{0.8}/(2w)^2. \tag{8}$$

Thus, mobility of the complementary factor effectively increases the exponent of capital in the net production function, from 0.4 to 0.8. Theorem 3 tells us increasing this exponent toward 1 can stimulate strong liberalization.

With the wage w=0.971, the net-output function from (7) becomes

$$Y(F+k) = 0.2574(F+k)^{0.8},$$

and then the ruler's optimal regime is $k = 1126$ and $\lambda = 0.931$. In this liberalized regime, the net output after wages is $Y = 71.36$, labor demand is $L = 73.48$, and the ruler's expected payoff is $V(k,\lambda) = 77.95$. Thus, the liberalizing ruler is much better off than the authoritarian rulers on the other islands, who get $V=19.29$. The possibility of matching capitalist investments with additional recruited labor has created a strong incentive for liberalization on this island.

But if one ruler can gain by liberalizing, then more should want to follow, and their increased demand for labor must ultimately drive up wages in the archipelago. To see what this new general equilibrium may be, let us go forward in history and suppose that demand for free labor in liberal islands has caused serfdom to break down throughout the archipelago. So now each island must recruit all its workers at some equilibrium wage w. This equilibrium wage must cause the average demand for labor across all islands to equal the given supply $L=1$ per island.

This equilibriating wage turns out to be $w = 1.777$. Substituting this wage into (7) yields

$$Y(F+k) = 0.1407(F+k)^{0.8}.$$

With this net-output function Y, we find two optimal regimes that maximize $V(k,\Lambda(k))$, as shown in Figure 3: a liberal regime $k=47.74$ and $\lambda=0.903$, and a nonliberal regime $k=0$ and $\lambda=0$. (For comparison, the unconstrained ideal investment with this Y would be $K_r-F = 57.84-5 = 52.84$.) Both of these optimal regimes yield the same optimal value $V(k,\lambda) = 5.10$ for the ruler. From (8), labor demand in the liberal regime is $L=1.89$, but labor demand in the nonliberal regime is $L=0.287$. When 44% use the liberal regime and 56% of the islands use the nonliberal regime, average demand for labor over all islands is equal to the supply of $L=1$ per island, and so labor markets clear (with 84% working in liberal islands, 16% in nonliberal islands).

[Insert Figure 3 about here]

Any higher wage ($w>1.777$) would yield a unique optimum with $\lambda=0$ and excess supply of labor; and any lower wage would yield a unique optimum with $\lambda>0.903$ and excess demand for labor. Thus, small parametric changes can imply large discontinuous changes of liberalization in the optimal regime. With $w=1.777$, if we dropped the assumption that every island has the same fixed endowment $F=5$ and instead assumed that the islands have different fixed endowments near 5, then those with smaller endowments would strictly prefer the liberal regime, and those with larger endowments would strictly prefer the nonliberal regime.

If the scandal rate ψ on one island were decreased to ψ=0.05 (keeping F=5 and all other parameters as above), then this island would have a uniquely optimal liberal regime with investment k=48.44 and liberalization λ=0.874. Notice that this liberalization λ is less than we got with ψ=0.1. If ψ were further decreased toward 0, then the optimal liberalization λ would approach the smallest liberalization that is compatible with the unconstrained ideal investment,

$$\lim_{\psi\to 0} \lambda = 1-[Y(K_r)-r(K_r-F)]/[Y(F)+\rho\theta(K_r-F)] = 0.832$$

So decreasing the scandal rate ψ can cause the optimal liberalization to decrease, at least in some cases. This result may seem surprising, as the scandal rate ψ represents the cost of liberalization in our model. But when the liberalization λ is positive, decreasing ψ increases the ruler's expected value V, which tends to relax the $V \geq W$ constraint, and so can reduce the liberalization that is required for any given level of investment.

7. Conclusions

Our analysis here has used an assumption that invested capital is mobile and can be used productively only by capitalists who are outside the ruling elite. Under these assumptions, investment can instantaneously reach any long-run capital stock, and the need to compensate capitalists for any given stock of invested capital does not depend on its past financing; and thus we have eliminated many dynamic issues from our model. Such dynamic issues must be studied in a more complex dynamic model like that of Paltseva (2008), and indeed this paper may be viewed as a simplification of her model. By simplifying away such dynamic issues, we have aimed for a clearer focus on the comparative statics of long-run steady states.

When capitalist investment is constrained by the government's temptation to expropriate, the ruling elite may benefit from political liberalization, as a broader social distribution of political power can encourage greater investments that increase the tax base. But even in the simple model that we have considered here, such incentives to liberalize may depend on the fundamental parameters in complex ways.

An increase in the endowment of fixed resources, which are substitutes for capitalist investment but can be managed by the government, causes the nonexpropriation constraint to tighten and so can cause total productive output to decrease. Thus, we find a curse of resources that can make even the ruling elite worse off.

The possibility of liberalization can add another nonmonotone twist to this relationship between fixed resources and investment, because the incentive to liberalize tends to be greatest for intermediate resource levels. States with very small resource endowments may be able to encourage large investment without liberalization, as the government would have so little without its reputation for protecting investors. On the other hand, the risk of losing power from liberalization becomes more costly for the ruler when resource endowments are very large. Thus, we find that the dependence of output on fixed-resource endowments can be W-shaped, with two local minima of production where rulers choose no liberalization, separated by a parametric interval where rulers choose to increase production by liberalizing.

When liberalization is positive, the addition of a fixed revenue income that is independent of capitalist investment would cause both liberalization and investment to decrease in the optimal regime. Adding such revenue could never make the ruler worse off, however, because the old (k,λ) regime would still be feasible.

Optimal liberalization and investment can be discontinuous functions of the underlying parameters. Such discontinuities regularly occur when the scandal rate ψ is greater than the authoritarian discount rate ρ, because the optimal liberalization must be either 0 or greater than $1-\rho/\psi$. Thus, a small parametric change may stimulate a sudden jump from absolute authoritarianism to strong liberalization and growth, caused not by a popular revolution but by changing incentives for the ruling elite.

Mobility of productive factors that complement capital can greatly increase incentives for political liberalization. Theories of market-preserving federalism (like Weingast 1995) may be derived from this effect, as labor is likely to be such an elastically supplied factor for autonomous local governments when workers can move freely throughout the larger nation. In a global equilibrium, however, we found that prices of these mobile factors may adjust so that liberal and authoritarian regimes can co-exist.

Knowledge spillovers and specialization have also been seen in endogenous growth theory as technological effects that can linearize the dependence of national output on aggregate investment, so that returns to scale become approximately constant for large investments. Our analysis suggests that such effects may be fundamental forces for democratic political liberalization in the modern world.

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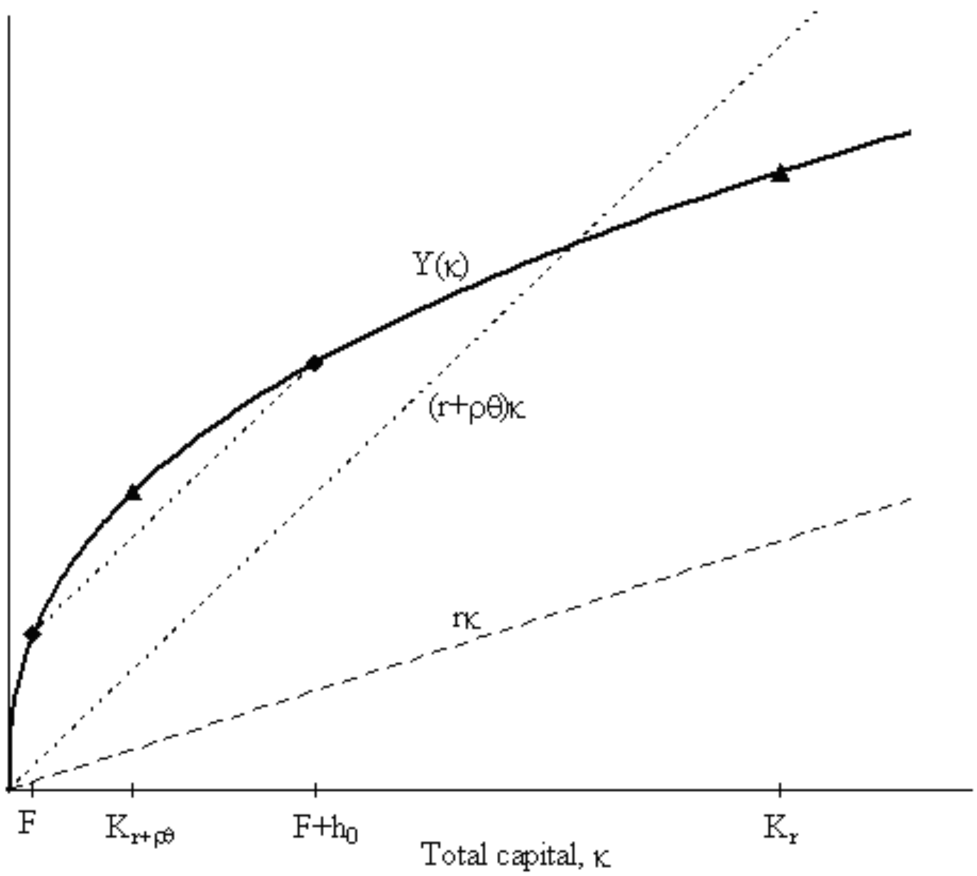

Figure 1. Dependence of h_0, the maximal investment without liberalization, on fixed resources F.

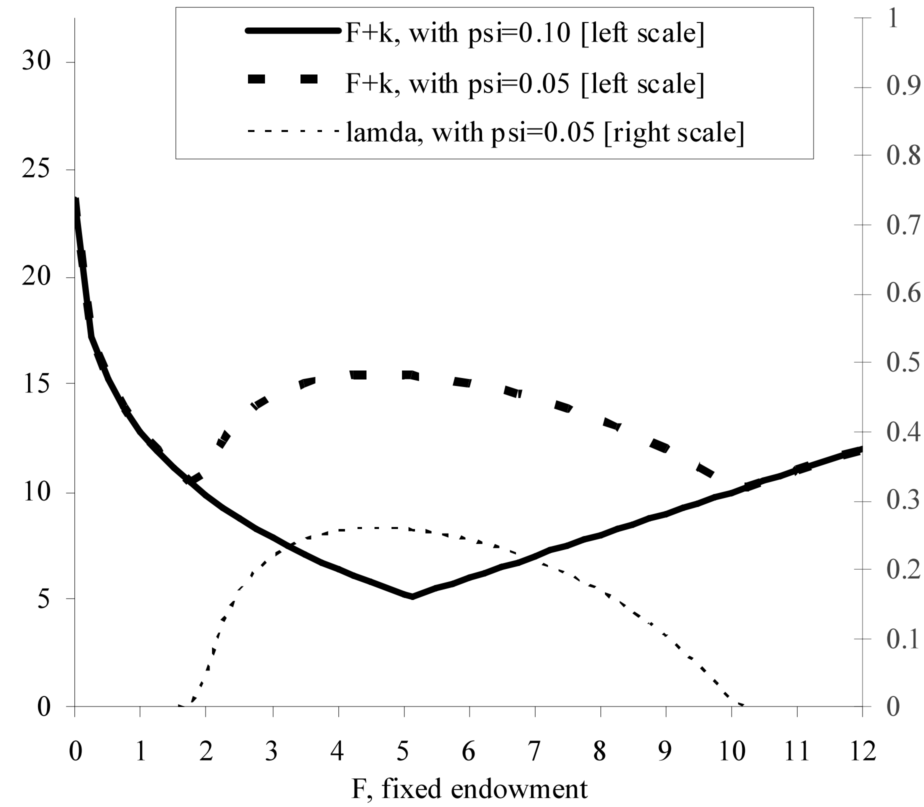

Figure 2. Total capital F+k and liberalization λ as functions of fixed endowment F, with $Y(F+k)=(F+k)^{0.4}$, r=.05, ρ=.1, θ=1, ψ=.10 or ψ=.05.

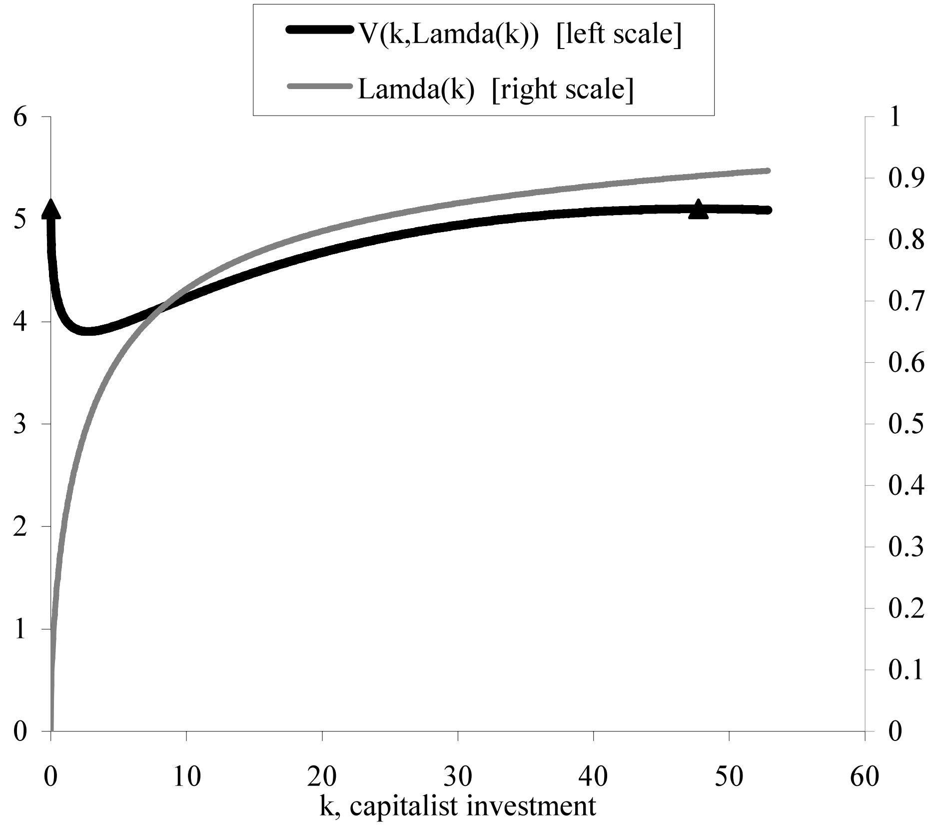

Figure 3. Required liberalization $\Lambda(k)$ and ruler's value $V(k,\Lambda(k))$, for $Y(F+k)=0.1407(F+k)^{0.8}$ with F=5, r=.05, ρ=.1, ψ=.1, θ=1; from w=1.777.

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