BIPOLAR MULTICANDIDATE ELECTIONS WITH CORRUPTION

by Roger B. Myerson

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Abstract. The goals of democratic competition are not only to give implement a majority's preference on policy questions, but also to provide a deterrent against corrupt abuse of power by political leaders. We consider a simple model of multicandidate elections in which different electoral systems can be compared according to these two criteria. Among a wide class of single-winner scoring rules, only approval voting is found to be satisfy both effectiveness against corruption and majoritarianism for this model.

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I. Introduction

There is a natural analogy between political competition in a democracy and economic competition in a market. Economists expect competition in the marketplace to reduce the profits that suppliers can take from consumers, in comparison to what a monopolistic supplier could take. Similarly, political scientists may expect that democratic competition in elections to reduce corrupt benefits of power that political leaders can take from the tax-paying public, in comparison to what an unelected dictator could take. Theoretical models in economics have shown, however, that the effectiveness of market competition for eliminating excess profits may depend on the details of market structure. A similar proposition can be shown in political science: The effectiveness of democratic competition for eliminating corrupt profits of power may depend on the details of the electoral system. The question of what kinds of democratic structures create the strongest competitive incentives against political corruption should be a central concern of political theorists, but it has received surprisingly little attention until recently.

This paper develops a simple model to probe the ways that voting rules may affect the effectiveness of democratic competition against corrupt political profit-taking. Other variations of the model in this paper have been considered previously by Myerson (1993a, 2002). These previous versions assumed only two types of voters. By admitting a continuum of voters' types here, we get results that are actually somewhat simpler and stronger. Other theoretical models that probe the competitive effectiveness of different constitutional structures have been considered by Persson and Tabellini (2000, chapters 8 and 9), Persson, Roland, and Tabellini (2000), Persson, Tabellini, and Trebbi (2003), Kunicova and Rose-Ackerman (2005), and Myerson (2006). Empirical analysis of how constitutional structures may affect political rents and corruption have been studied by Persson and Tabellini (2003, chapter 7). For broader introduction to the economic analysis of political corruption, see Bardhan (1997) and Rose-Ackerman (1999).

II. The model

Consider an election with a given collection of candidates who differ along a dimension that we will call corruption. If voters cannot observe any differences of corruption among politicians, then no democratic system can have any deterrent effect against corruption. So to compare the effectiveness of democratic competition against corruption, we must start with an assumption that the voters have information indicating that some politicians are more corrupt than others. We assume that the only beneficiaries of this corruption are the politicians (and their immediate families), so that virtually all voters would agree that corruption is cost of government that they would prefer to minimize.

If candidates only differed along such a corruption dimension, where all voters agree that less corruption is better, then it would be hard to see why voters would ever support a candidate who was known to be more corrupt than other available candidates. So to see the different effectiveness of different voting rules, we must admit that candidates differ along some other policy dimension where voters have different preferences. To keep the model as simple as possible, this policy dimension only needs to include two policy alternatives. So we may let {1,2} denote the set of policy alternatives, where 1 may be called the left policy and 2 may be called the right policy.

Let K denote the set of candidates, which is partitioned into two subsets: K_1 is the set of leftist candidates who advocate policy alternative 1, and K_2 is the set of rightist candidates who advocate policy alternative 2. Each candidate k in $K = K_1 \cup K_2$ also has a known corruption level f(k). We assume that all corruption levels are nonnegative numbers, so that a voter's favorite candidate should have corruption level 0. We may say that a candidate k is clean if $f(k)=0$. Otherwise, if $f(k) > 0$, we may say that candidate k is positively corrupt.

Voters may differ in their preferences for the left or right policy position, with different preferences corresponding to different types of voters. To be specific, we may denote the type of a voter by number t which measures the voter's net preference for the right policy alternative 2. Let $u(k|t)$ denote the payoff to a voter of type t when candidate k is the winner of the election. We assume that the voters' payoffs depend on their type and the winner's policy and corruption level according to the formula:

$u(k|t) = t - f(k)$ if $k \in K_2$,

$u(k|t) = 0 - f(k)$ if $k \in K_1$.

So the winner's corruption f(k) is a cost paid by all voters, and each voter's type t is the net payoff increment that he gets from the right policy 2, relative to the left policy 1. Thus, voters with positive types $t > 0$ are rightist voters who prefer policy 2, and their favorite candidate would be a rightist candidate in K_2 with f(k)=0. Voters with negative types $t < 0$ are leftist voters who prefer policy 1, and their favorite candidate would be a clean leftist candidate in K_1 with f(k)=0.

We will assume that there exists at least one clean leftist candidate and one clean rightist candidate. This assumption is essentially without loss of generality because, if it were violated, we could create an equivalent model with this property by redefining each voter's type to be the difference of his payoffs from the best rightists and best leftist candidates, and then redefining each candidate's corruption to be the difference between his corruption and the best candidate on his side of the policy question.

Uncertainty about the number and types of other voters in the population can be a critical aspect of a voter's optimal decision problem. Palfrey and Rosenthal (1983,1985) showed that voting games can often have perverse counter-intuitive equilibria when population uncertainty is ignored. Here we use the Poisson model of population uncertainty developed by Myerson (1998, 2000). That is, we assume that the number of voters is a Poisson random variable with mean n. For any nonnegative integer j, a Poisson random variable with mean n has probability $e^{-n} n^j / j!$ of being equal to j. The standard deviation of Poisson random variable is the square root of its mean. So for a population of expected size 1,000,000, the Poisson model of population uncertainty would yield a standard deviation of 10,000. One convenient property of the Poisson model is that any given voter also views the number of other voters as being a Poisson random variable with the same mean n. (This property is called environmental equivalence by Myerson, 1998.)

In this model of population uncertainty, we assume that each voter has a type t that is drawn independently from some given probability distribution r on the real number line $\mathbb{R}$, which is assumed to have a continuous positive probability density on all of $\mathbb{R}$. For example, r could be a Normal probability distribution with any given mean and standard deviation. For any set

$S \subseteq \mathbb{R}$, we let r(S) denote the probability of any voter's type being in the set S. Given that the total number of voters is a Poisson random variable with mean n, the number of voters who have types in any set S is a Poisson random variable with mean nr(S). Furthermore, the numbers of voters in disjoint sets are independent in the Poisson model. That is, for any two sets S_1 and S_2 such that $S_1 \cap S_2 = \emptyset$, the numbers of voters who have types in S_1 and S_2 are independent Poisson random variables with means $nr(S_1)$ and $nr(S_2)$ respectively.

To complete the specification of an election game, we must specify an voting rule. As the goal of this paper is to compare the game-theoretic incentive properties of different voting rules, we will consider many different ways of voting, but we will restrict our attention to scoring rules. In a scoring rule, the permissible ballots that voters must choose are a finite set C that is a subset of $\mathbb{R}^K$. That is, any permissible ballot c in C is a vector $c=(c_k)_{k \in K}$, where c_k denotes the number of points that the voter is giving to candidate k. These vectors are then summed over all voters, and the winner is the candidate who gets the most points. In the event of a tie, we assume that the winner is chosen by random selection among the candidates who get the most points, each tied candidate having equal probability. (A more complicated tie-breaking rule was assumed in Myerson 1993a.) That is, if x(c) denotes the number of voters who cast the ballot-vector c, then the set of tied winners is

$$W(x) = \text{argmax}_{k \in K} \sum_{c \in C} x(c) c_k,$$

and the probability of candidate k winning is

$$\omega(k|x) = 1/\#W(x) \text{ if } k \in W(x), \; \omega(k|x) = 0 \text{ if } k \notin W(x).$$

To give some specific examples, we may consider plurality voting, where each voter names one candidate, and the winner is the candidate who is named by the most voters. In this notation, the set of permissible ballots under plurality voting is

$$C = \{c \in \mathbb{R}^K | \; \exists k \in K \text{ such that } c_k=1 \text{ and } c_j=0 \text{ for all } j \neq k\}.$$

In approval voting, each voter names any subset of the candidates, and again winner is the candidate who is named by the most voters. So the permissible ballots under approval voting is

$$C = \{c \in \mathbb{R}^K | \; c_k \in \{0,1\} \text{ for all k in K}\}.$$

In negative voting, each voters names one candidate, but the winner is the candidate who is named by the fewest voters, because each voter is voting against the candidate whom he has named. We can represent negative voting by interpreting a ballot that names candidate k as a vector that gives one point to every candidate except k, so

$$C = \{c \in \mathbb{R}^K \mid \exists k \in K \text{ such that } c_k=0 \text{ and } c_j=1 \text{ for all } j \neq k\}.$$

We may also consider <u>Borda voting</u>, in which a voter must give each of the #K candidates a different point-value selected from the #K numbers that are equally spaced from 0 to 1, and so

$$C = \{c \in \mathbb{R}^K \mid \forall j \in \{0,1,2,...,\#K-1\}, \exists k \in K \text{ such that } c_k = j/(\#K-1)\}.$$

Given any voting rule, let Γ_n denote the voting game with population uncertainty where the number of voters playing the game is a Poisson random variable with mean n. Let C denote the set of permissible ballots that a voter can cast in the election. Then an equilibrium of this game Γ_n specifies an optimal mixed strategy $\sigma_n(t)$ for every type t, where $\sigma_n(t)$ is a probability distribution over the set of ballots C. That is, $\sigma_n(c|t)$ denotes the probability that a type-t voter would cast the ballot c in C, in this equilibrium of the game with n expected voters. In such an equilibrium, let $\tau_n(c)$ denote the expected fraction of voters who will cast the ballot c in the election, that is

$$\tau_n(c) = \int_{t \in \mathbb{R}} \sigma_n(c|t)\, dr(t).$$

With the Poisson model of population uncertainty, the number of voters casting each permissible ballot $c \in C$ is then a Poisson random variable with mean $n\tau_n(c)$, and it is independent of the number of voters casting any other ballot. So for any vector $x=(x(c))_{c \in C}$, where each x(c) is a nonnegative integer, the probability of each ballot c being chosen by x(c) voters is

$$P(x|n\tau_n) = \prod_{c \in C} e^{-n\tau_n(c)} (n\tau_n(c))^{x(c)}/x(c)!\,.$$

Let $Q_n(k)$ denote the probability that candidate k will win the election in this equilibrium. So

$$Q_n(x) = \sum_x P(x|n\tau_n)\omega(k|x).$$

We say that two candidates j and k are <u>distinct</u> if $u(j|t) \neq u(k|t)$ for at least one type t. That is, two different candidates are distinct unless they have both the same side of the policy question and exactly the same perceived corruption level. Let $D \subseteq K \times K$ denote the set of distinct pairs of candidates. We assume that voters are instrumentally motivated only by their effect on the outcome of the election. So each voter is concerned about his vote only in the event that it could change the winner from some candidate j to some distinct candidate k. When one more ballot d is added a profile of vote counts $(x(c))_{c \in C}$, the vote counts are changed to the vector x+[d], where

$$(x+[d])(c) = x(c) \text{ if } c\neq d, \quad (x+[d])(d) = x(d)+1.$$

So the probability that adding one more d ballot would change the winner from candidate j to candidate k, given x, is

$$\pi(j,k,d|x) = \omega(j|x)\omega(k|x+[d]) \text{ if } d_j < d_k, \text{ and } \pi(j,k,d|x) = 0 \text{ if } d_j \geq d_k.$$

We say that there exists a <u>close race</u> between candidates j and k at the vote-counts vector $(x(c))_{c\in C}$ iff j and k are a distinct pair and there exists some permissible ballot d such that $\pi(j,k,d|x)\neq 0$ or $\pi(k,j,d|x)\neq 0$. Let X denote the set of all possible vote-count vectors at which there exists a close race between some pair of distinct candidates

$$X = \{x|\ \exists(j,k)\in D, \exists d\in C \text{ such that } \pi(j,k,d|x) > 0\}.$$

A voter cares about how he votes only in the event that there is some close race where his vote could matter, that is, in the event that the vote-counts vector is in this set X. Let $q_n(j,k,c)$ denote the conditional probability, in the σ_n equilibrium, that adding one more c ballot would change the winner from j to k, given that there exists a close race between some pair of distinct candidates;

$$q_n(j,k,c) = \sum_{x\in X} P(x|n\tau_n)\pi(j,k,c|x)/[\sum_{x\in X} P(x|n\tau_n)].$$

Because these are conditional probabilities, we have

$$\sum_{\{j,k\}\in D} \sum_{c\in C} q_n(j,k,c) \geq 1.$$

A voter of type t should want to choose a ballot c that maximizes his expected gain from voting conditional on there being some close race where his vote could matter, and so an optimal c in C for type t should maximize

$$\sum_{\{j,k\}\in D} \sum_{c\in C} q_n(j,k,c)(u(k|t)-u(j|t)).$$

To study large populations, we will study here the limits of such equilibria as $n\rightarrow\infty$. To be precise, we will consider sequences of equilibria such that the probabilities $\tau_n(c)$, $Q_n(k)$, and $q_n(j,k,c)$ all converge as $n\rightarrow\infty$ to some limits $\tau(c)$, $Q(k)$, and $q(j,k,c)$, for all c in C, all k in K, and all j such that $\{j,k\}\in D$. Given that the sets of candidates K and permissible ballots C are both finite sets, any sequence of equilibria parameterized by $n\rightarrow\infty$ has a subsequence in which these probabilities all converge as $n\rightarrow\infty$. A <u>large equilibrium</u> (τ,Q,q) is defined here to be any such limit of equilibria as $n\rightarrow\infty$.

We may say that, in a large equilibrium, there is a <u>serious race</u> among two candidates j and k iff $(j,k) \in D$ and there exists some ballot c in C such that $q(j,k,c)+q(k,j,c) > 0$. That is, the {j,k} race is serious if j and k are distinct and the conditional probability of j and k being in a close race, given that some pair of distinct candidates is in a close race, has a positive limit as $n \to \infty$. We may say that a candidate k is serious in the large equilibrium iff there is some other candidate j such that the race among j and k is serious. In the limit, a voters optimal voting decision must be based on the effect that his vote may have on the serious races.

Notice that being a serious candidate is not the same as being a candidate who is likely to win. We may call such likely winners the strong candidates. That is, a candidate k is strong in a large equilibrium iff $Q(k) > 0$. In the single-winner elections that we will study, a candidate is generally serious if he is strong, but there may also be serious candidates who are not strong. For example, consider a large equilibrium in plurality voting where candidate 1 is expected to get 50% of the vote ($\tau(1) = 0.5$), candidate 2 is expected to get 30% of the vote ($\tau(2)=0.3$), and candidate 3 is expected to get 20% of the vote ($\tau(3)=0.2$). Then in the large-population limit, candidate 1 is the only strong candidate ($Q(1)=1$, $Q(2)=0=Q(3)$), but candidates 1 and 2 are both serious, because conditional on there being a close race between two candidates it would almost surely be between candidates 1 and 2.

We say that an voting rule is effective against corruption iff, for any large equilibrium, no positively corrupt candidate can be strong or serious. We say that an voting rule is majoritarian iff, in any large equilibrium, with probability 1, the winner will be a candidate who is considered best by at least half of the voters. Our main results, in Section 4, show that these good properties are satisfied by approval voting with any number of candidates. But first, in Section 3, we show that these properties cannot be satisfied by any of a wide range of other scoring rules for three-candidate elections, including plurality voting, negative voting, and Borda voting.

In the terminology of Riker, 1982, effectiveness against corruption is a liberal criterion for successful democracy, because it involves restraining leaders from abusing their power, whereas majoritarianism is a populist criterion for successful democracy, because it asks whether the preferences of different voters are aggregated in an democratically appropriate way. Riker has argued that the impossibility theorems of social choice theory imply that populist criteria can only be defined for very restricted social-choice environments. In this case, the simple binary structure of the policy space is what enables us to define such a populist formulation of the majority-rule principle, because there is always a clean candidate on one side or the other who is considered the best candidate by at least half of the voters.

The proofs of these results will depend on one basic result about large Poisson games, which we now state.

Lemma. Consider a partition of the set of all possible voters' types into four disjoint sets $\{S_0,S_1,S_2,S_3\}$. Let Λ denote the event that the number of voters with types in S_1 differs by at most 1 from the number of voters with types in S_2, and the number of voters with types in S_0 is equal to 0. Let $P(\Lambda|n,r)$ denote the probability of this event Λ in the Poisson model when expected number of voters is n and the voters' types are independently drawn from the distribution r. Then

$$\lim_{n\to\infty} LN(P(\Lambda|n,r))/n = 2\sqrt{r(S_1)r(s_2)} + r(S_3) - 1 .$$

This limit of the logarithm of the probability of Λ divided by the expected population size is called the magnitude of Λ. As $n\to\infty$, the probability of Λ goes to zero (unless $r(S_3)=1$), and so the logarithm of this probability goes to $-\infty$, but the logarithm of the probability divided by n converges to a finite negative number. In fact, the magnitude of Λ cannot be less than -1, because the event Λ includes as a subset the event that there are no voters at all, which has probability e^{-n} and so has magnitude $LN(e^{-n})/n = -1$.

This lemma can be proven as a consequence of Theorem 1 of Myerson (2000), which implies that the magnitude is the maximum over all $y\geq 0$ and $z\geq 0$ of

$$r(S_0)\ \psi(0) + r(S_1)\ \psi(y/r(S_1)) + r(S_2)\ \psi(y/r(S_2)) + r(S_3)\ \psi(z/r(S_3))$$

where the function ψ is defined by the formula

$$\psi(\theta) = \theta(1-LN(\theta))-1, \quad \psi(0) = -1.$$

By calculus, it can be shown that this maximum is achieved by $y = \sqrt{r(S_1)r(S_2)}$ and $z = r(S_3)$. Substituting this y and z back into the above formula yields the magnitude in the Lemma, when we use the fact that the partition $\{S_0,S_1,S_2,S_3\}$ must satisfy $r(S_0)+r(S_1)+r(S_2)+r(S_3)=1$. QED

III. Failures of effectiveness or majoritarianism in rules for three-candidate elections

Let us first consider a class of rank-scoring rules for three-candidate elections that is parameterized by a number A such that $0 \leq A \leq 1$. Given this number A, the set of permissible ballot vectors is C = {(1,0,A), (0,1,A), (1,A,0), (0,A,1), (A,1,0), (A,0,1)}. That is, a voter must give 1 point to one of the three candidates, 0 points to another of the three candidates, and A points to the remaining candidate. In the case of A=0, this system becomes plurality voting, with the permissible ballots C = {(1,0,0), (0,1,0), (0,0,1)}. In the case of A=1, this system becomes negative voting, with the permissible ballots C = {(0,1,1), (1,0,1), (1,1,0)}. The case of A=0.5 corresponds to Borda voting.

Proposition 1. In a rank-scoring rule parameterized by A as above, suppose that A<0.5. Consider a three-candidate election where there is one leftist candidate in K_1 = {1}, and there are two rightist candidates in K_2 = {2,3}. Suppose that candidates 1 and 2 are clean (f(1)=f(2)=0) but candidate 3 is positively corrupt (f(3)>0). With A < 0.5, we can construct a large equilibrium in which {1,3} is the only serious race, and so the corrupt candidate is serious. In this equilibrium, each voter is expected to vote either (1,A,0) or (0,A,1), depending on whether the voter's type t is less than f(3) or greater than f(3). If $r(\{t|t>f(3)\}) > 0.5$ then the corrupt candidate 3 is the strong likely winner.

Proof. When the event of a close race between candidates 1 and 3 is considered much more likely than a close race between any other pair of candidates, then all voters will want to maximally separate the point that they give these two serious candidates, in favor of one that they prefer among these two candidates. A voter of type t prefers the corrupt rightist candidate 3 over the clean leftist candidate 1 when $t-f(3)>0$, and so such a voter should vote (0,A,1). Even though he also prefers candidate 2 over candidate 3, if he changed to voting (0,1,A) then, conditionally on this change making any difference, it would almost surely be making a difference by letting candidate 1 win rather than candidate 3. Now let $\tilde{\rho}$ denote the random fraction who vote (1,A,0) in the election. Given that everyone is voting either (1,A,0) or (0,A,1) in this scenario, the candidates' points per voter will be $\tilde{\rho}$ for candidate 1, A for candidate 2, and $1-\tilde{\rho}$ for candidate 3. Notice that at least one of $\tilde{\rho}$ and $1-\tilde{\rho}$ is always strictly greater than A when A<0.5. So with

A<0.5, candidate 2 cannot be in a close race when there is any positive turnout, and so the magnitude of candidate 2 being in a close race is -1. But when we apply the Lemma with $S_0=\emptyset$, $S_1=\{t|t<f(3)\}$, $S_2=\{t|t>f(3)\}$, and $S_3 = \{f(3)\}$, we find that the magnitude of a close race between candidates 1 and 3 is $2\sqrt{r(S_1)r(s_2)}-1$, which is strictly greater than -1. This strict inequality of magnitudes implies that a close race between candidates 1 and 3 is indeed infinitely more likely than any close race involving candidate 2 in this scenario. So our initial assumption that only candidates 1 and 3 are serious is justified in this equilibrium. QED

Notice that Proposition 1 is only about the existence of bad equilibria where the corrupt candidate is a serious contender. Proposition 1 allows that there may be other equilibria that do not have this bad property. In fact, when A<0.5, this example also has a good equilibrium where only the two clean candidates are serious, and the winner is in this equilibrium always the clean candidate who is preferred by a majority (as everyone is voting either (1,0,A) or (0,1,A)). But things become worse when $A\geq 0.5$, because Proposition 2 asserts that the corrupt candidate 3 must then be a serious contender in all large equilibria.

<u>Proposition 2.</u> In a rank-scoring rule parameterized by A as above, suppose that $A\geq 0.5$. Consider again a three-candidate election where $K_1 = \{1\}$, $K_2 = \{2,3\}$, f(1)=f(2)=0, and f(3)>0. With $A \geq 0.5$, candidate 3 must be serious in all large equilibria.

<u>Proof.</u> Suppose, contrary to the theorem, we had an equilibrium in which candidate 3 was not serious. In this equilibrium, every voter would want to maximize the impact of his vote on the only serious race, among candidates 1 and 2. So every voter would vote either (1,0,A) or (0,1,A), depending on whether the voters type t is negative or positive. Now let $\tilde{\rho}$ denote the random fraction who vote (1,0,A) in the election. With everyone is voting either (1,A,0) or (0,A,1) in this scenario, the candidates' points per voter would be $\tilde{\rho}$ for candidate 1, $1-\tilde{\rho}$ for candidate 2, and A for candidate 3. But with $A\geq 0.5$, $\tilde{\rho}$ and $1-\tilde{\rho}$ could not be equal without A being at least as large as them both. So with $A\geq 0.5$, a close race involving candidates 1 and 2 but not candidate 3 would be impossible, which contradicts the initial hypothesis that candidate 3 was not serious. QED

Intuitively, Proposition 1 is about voting rules like plurality voting, where the main effect of a voter's choice is to reward the candidate at the top of the voter's ballot (as $1-A > A-0$). With such top-rewarding voting rules, putting a nonserious candidate at the top of a ballot would be a wasted vote, and so a perception that any candidate k is not serious would tend to make k a weaker candidate; and thus the perception that k is not serious can become a self-fulfilling prophecy in equilibrium (even if all voters prefer k to the likely winner). On the other hand, Proposition 2 is about voting rules like negative voting, where the main effect of a voter's choice is to punish the candidate at the bottom of the voter's ballot (as $1-A < A-0$). With such bottom-punishing voting rules, putting a nonserious candidate at the bottom of a ballot would be a wasted vote, and so a perception that k is not serious would tend to make k a stronger candidate; and so in equilibrium all candidates must be serious (even those who are disliked by all voters).

The one-parameter family of voting rules that we considered above did not include approval voting. To include approval voting in a natural way, let us consider a more general family of scoring rules for three-candidate elections that are parameterized by two parameters (A,B) such that $0 \leq A \leq B \leq 1$. The set of permissible ballots is the set of all permutations of the vectors (1,A,0) and (1,B,0):

$$C = \{(1,0,A), (0,1,A), (1,A,0), (0,A,1), (A,1,0), (A,0,1), \\ (1,0,B), (0,1,B), (1,B,0), (0,B,1), (B,1,0), (B,0,1)\}.$$

That is, a voter must give 1 point to one of the three candidates, 0 points to another of the three candidates, and either A or B points to the remaining candidate. The one-parameter family that was considered above corresponds to the special case of A=B. But this two-parameter family also includes approval voting, for the case where A=0 and B=1. We now show that majoritarianism can fail in equilibrium for any voting rule in this family other than approval voting. (This result has coincides with Proposition 3 in Myerson, 2002, and is included here for completeness.)

Proposition 3. Consider a scoring rule parameterized by (A,B) as above. Consider a three-candidate election where $K_1=\{1\}$ and $K_2=\{2,3\}$, but all three candidates are clean ($f(1)=f(2)=f(3)=0$). In this election, there exists an equilibrium where the voters treat candidates 2 and 3 symmetrically. But if this (A,B)-scoring rule is not approval voting, in that A>0 or B<1, then, for any finite n, this symmetric equilibrium yields a positive probability that the winner will be a candidate who is not preferred by a majority.

Proof. In the symmetric equilibrium, the leftist voters will randomize between voting (1,A,0) and (1,0,A) with equal probability, because they like candidate 1 best and are indifferent between dumping the smaller required middle value A on either of their less-preferred candidates. In this symmetric equilibrium, the rightist voters will randomize between voting (0,B,1) and (0,1,B) with equal probability, because they consider candidate 1 worst and are indifferent between giving the giving the larger middle value B to either of their more-preferred candidates. Now if $A > 0$ then it can happen that the leftist voters have a slight majority, but the leftists voters all vote (1,A,0) and the rightist voters all vote (0,1,B), making candidate 2 the winner. On the other hand, if $A = 0$ and $B < 1$, then it can happen that the rightist voters have a slight majority, but the leftists voters all vote (1,0,0) and the rightist voters split equally among (0,1,B) and (0,B,1), making candidate 1 the winner. QED

The failures of majoritarianism that are described in Proposition 3 can actually have probability 1 in the limit as $n\to\infty$ if $A+B \neq 1$. The key is to consider this quantity

$$R^* = (1+B)/(3+B-A),$$

which is Cox's threshold of diversity for these (A,B)-scoring rules with 3 candidates (see Cox 1987, 1990, and Myerson, 1993b). Let $\lambda = r(\{t|t<0\})$ denote the expected fraction of leftist voters. In the symmetric equilibrium, the expected per-capita score (points per voter) for each of the rightist candidates is $(1-\lambda)(B+1)/2 + \lambda(A+0)/2$, and the expected per-capita score for the leftist candidate is λ. In the limit as $n\to\infty$, the standard deviation in these per-capita scores goes to zero, and so the leftist is almost sure to win if

$$\lambda > (1-\lambda)(B+1)/2 + \lambda(A+0)/2,$$

but the leftist is almost sure to lose if

$$\lambda < (1-\lambda)(B+1)/2 + \lambda(A+0)/2.$$

These two inequalities are equivalent to $\lambda > R^*$ and $\lambda < R^*$ respectively. If $A+B > 1$ then $R^* > 0.5$, and so with $0.5 < \lambda < R^*$ we can get an example where the probability of the leftist voters being a majority but a rightist candidate winning goes to 1 as $n\to\infty$. On the other hand, if $A+B < 1$ then $R^* < 0.5$, and so with $0.5 > \lambda > R^*$ we can get an example where the probability of the leftist voters being a minority but a leftist candidate winning goes to 1 as $n\rightarrow\infty$.

Intuitively, in plurality voting and other top-rewarding rules where $A+B<1$, the existence of two candidates who appeal to the same bloc of voters can be weaken the bloc, if they divide their support symmetrically among these candidates. On the other hand, in negative voting and other bottom-punishing rules where $A+B>1$, the existence of two candidates who appeal to the same bloc of voters can strengthen the bloc, because opposing blocs will have to divide their bottom-rank punishments among these two candidates. Either way, we can get nonmajoritarian outcomes when a bloc of voters is strengthened or weakened by having multiple candidates.

Notice that Proposition 3 does not apply to approval voting. In the symmetric equilibrium under approval voting, with $A=0$ and $B=1$, the leftists all vote (1,0,0), and the rightists all vote (0,1,1), and so each candidate gets as many points as there are voters on his side of the policy question, and so the set of voters who prefer the winner cannot be a strict minority. In the next section, we prove a much stronger result, that approval voting always satisfies effectiveness against corruption and majoritarianism in our bipolar models with corruption with any number of candidates.

IV. Effectiveness and majoritarianism of approval voting

Proposition 4. Consider the general bipolar model with corruption as defined in Section 2, with any number of candidates. Suppose that the voting rule is approval voting. In a large equilibrium under approval voting, no corrupt candidates can be strong or serious, and there is probability 1 that the winner will be a candidate who is considered best by at least half of the voters.

Proof. Under approval voting, a voter can approve as many candidates as he wishes, and the winner is the candidate who is approved by the most voters. So a voter can never be hurt by adding an approval vote for a candidate whom he considers best among all candidates. So by a dominant-strategy argument, all leftist voters with types in $(-\infty,0]$ will approve any clean candidates in K_1. Similarly, all rightist voters with types in $[0,+\infty)$ will approve any clean candidates in K_2. A neutral voter of type 0 only cares about corruption and so will approve all clean candidates.

Thus, for a corrupt candidate to beat the clean candidates under approval voting, he would have to get approval votes from both leftist and rightist voters. But intuitively, the most corrupt among serious candidates would not get approval votes from any voters on the other side of the left-right divide. We now prove the theorem by formalizing this argument.

Consider an equilibrium σ_n for any finite expected population size n. If some type t approves some candidate i in K_1 and s < t then type s also approves i, because the net gains from making candidate i win instead of some other candidate k are at least as large for type s as for type t. (That is, $s<t$ and $i \in K_1$ implies $u(i|s) - u(k|s) \geq u(i|t) - u(k|t)$ for all k in K, with equality if $k \in K_1$ and strict inequality if $k \in K_2$.) So for each leftist candidate i in K_1, there exists some $\theta_n(i)$ such that voters of any type t approve i if $t < \theta_n(i)$ but do not approve i if $t > \theta_n(i)$. Similarly, for each rightist candidate j in K_2, there exists some $\theta_n(j)$ such that voters of any type t approve j if $t > \theta_n(j)$ but do not approve j if $t < \theta_n(j)$.

Taking the large-population limit, let $\theta(k) = \lim_{n\to\infty} \theta_n(k)$ for each candidate. Let H_1 and H_2 denote the candidates with the highest expected per-capita scores in the $n\to\infty$ limit among the leftists and rightist candidates respectively. That is

$$H_1 = \operatorname{argmax}_{i\in K_1}\theta(i), \;\; H_2 = \operatorname{argmin}_{j\in K_2}\theta(j).$$

Let h_1 be a leftist candidate in H_1, and let h_2 be a rightist candidate in H_2. Because voters of type 0 approve all clean candidates, we know that any clean candidate in K_1 has $\theta \geq 0$, and any clean candidate in K_2 has $\theta \leq 0$. So $\theta(h_2) \leq 0 \leq \theta(h_1)$. Let

$$r_1 = r([-\infty, \theta(h_2)]),$$

$$r_2 = r([\theta(h_1), +\infty]),$$

$$r_3 = r([\theta(h_2), \theta(h_1)]).$$

By the Lemma, the event of a close $\{h_1,h_2\}$-race has magnitude $2\sqrt{r_1 r_2} + r_3 - 1$, which is strictly greater than -1.

Now let i and j be any other candidates in K_1 and K_2 respectively. Let

$$s_0 = r([\theta(h_2),\theta(j)] \cup [\theta(i),\theta(h_1)])$$

which is the expected fraction of voters who approve h_2 but not j, or approve h_1 but not i. Let

$$s_1 = r([-\infty, \min\{\theta(i),\theta(h_2)\}]),$$

which is the expected fraction of voters who approve i but not h_2. Let

$$s_2 = r([\max\{\theta(j),\theta(h_1)\}, +\infty]),$$

which is the expected fraction of voters who approve j but not h_1. Let

$$s_3 = r([\theta(j), \theta(i)]),$$

which is the expected fraction of voters who approve both i and j. Here $s_3 = 0$ if $\theta(j) \geq \theta(i)$. Then by the Lemma, the event of a close {i,j}-race has magnitude $2\sqrt{s_1 s_2} + s_3 - 1$. But if $\theta(i)<\theta(h_1)$ or $\theta(h_2)<\theta(j)$ then $s_1 \leq r_1$, $s_2 \leq r_2$, $s_3 < r_3$, and so a close {i,j}-race has strictly lower magnitude than a close $\{h_1,h_2\}$-race. Thus, a serious race between a leftist and rightist candidate can only involve candidates in H_1 and H_2, the candidates with highest expected per-capita scores on each side of the binary policy question as $n\rightarrow\infty$.

Now suppose, contrary to the theorem, that some positively corrupt candidate is serious. Let i denote the most corrupt serious candidate. To be specific, we may suppose that $i \in K_1$. (A symmetric argument can cover the case of $i \in K_2$.) There must exist some j in H_2 such that the {i,j} race is serious, because nobody would vote for i if i's serious races were all with other less-corrupt candidates in K_1. Candidate i is the worst serious candidate for all voters in $[0,+\infty)$, and so $\theta_n(i) < 0$ for all n.

Let g be a clean candidate in K_1, who is approved by all voters in $(-\infty,0]$, and so $\theta_n(g) \geq 0$ for all. So the set of voters approving i is always a subset of those approving g. So candidate i can win only when all voters for-g-but-not-for-i vanish, leaving g in a tie with i. So whenever an additional vote for i could make i win, there is a positive limiting conditional probability that the winner would be g otherwise. But for type-0 voters, g is strictly better than i, and no serious candidate is worse than i. So in the limit, there are strictly negative conditional expected gains for type-0 voters from approving i, given the event that some serious race is close. So there must be some neighborhood of types around 0 that would have strictly negative conditional expected gains from approving i, given that some serious race is close. So

$$\theta(i) < 0 \leq \theta(g).$$

Thus, i is not in H_1. But then a close {i,j}-race must have lower magnitude than some other close race involving a higher-expected-scoring candidate in H_1. So the {i,j} race cannot be serious.

This contradiction shows that no positively corrupt candidate i can be serious. Thus, all serious candidates must be clean.

A similar argument shows that no positively corrupt candidate can be strong. Suppose to the contrary that some positively corrupt candidate i had a positive limiting probability of winning, and let g be a clean serious candidate in on the same side of the binary policy question as i. Then there would be a positive limiting probability of the event that no voters exist in the interval between $\theta_n(g)$ to $\theta_n(i)$. But then in the event that g is in a close race for first place, there would be a positive conditional probability of i also being in a close race for first place, and so i would also be serious, which is not possible.

A pair of clean candidates who are both in K_1 (or both in K_2) would not be distinct, and so every serious race involves a clean candidate in K_1 and a clean candidate in K_2. So the limiting cutoff θ for every clean candidate is 0. So in the large-equilibrium limit, the leftist voters in $(-\infty, 0)$ will all approve the clean candidates in K_1 but not in K_2, while rightist voters in $(0, +\infty)$ will all approve the clean candidates in K_2 but not in K_1. So with probability 1, the winner will be a clean candidate from the side of the political spectrum that has a majority (or at least half) of the electorate, and so the winner will be an optimal candidate for at least half of the voters. QED

Our results have shown that approval voting is unique in a wide class of voting rules for creating competitive pressure against political corruption. Such results naturally raise the question of why approval voting has not been used in real political systems. Although this paper has considered only one very simplified model of politics, this author does not know of any other models where equilibrium outcomes under other common voting rules might be considered distinctly better for the voters than approval voting. But a competitive electoral system that is better for the voters could also be worse for politicians, when our criterion is the amount of corrupt profit-taking that elected officials get to enjoy in equilibrium. Thus, our analysis also suggests that a reform to approval voting would not be in the interest of political leaders. If the voters do not understand how different voting rules would affect the quality of political competition, then political leaders are likely to get the less competitive voting systems that they prefer. This need for better public understanding of how voting rules affect political competition is a fundamental motivation for this research.

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