ECON 202: Macroeconomics I
Lecture 18 - Review

John Grigsby

March 8, 2017
Section 1

Measurement
Measuring GDP

- Nominal GDP calculated as sum of prices times quantities

\[ GDP_{t}^{NOM} = \sum_{i} p_{it} q_{it} \]

- Only include goods sold to end users to avoid double counting

- Real GDP fixes prices at a particular point in time \( b \)

\[ GDP_{t}^{REAL} = \sum_{i} p_{ib} q_{it} \]

- Chain-weighting (averaging subsequent time periods’) reduces substitution bias and better measures new products
Section 2

Finding Equilibrium
Finding Equilibrium

The Cookbook to find equilibrium

1. Write down household maximization problem
2. Write down household Lagrangean, take FOCs, solve for Marshallian Demand
   \[ MRS = MRT \]
3. (If applicable) write down firm’s maximization problem. Unless explicitly stated, firm’s problem does not have constraint.
4. Write down FOCs, solve for input (a.k.a. factor) demands
   \[ \text{Marginal Revenue Product} = \text{Marginal Cost} \]
5. Impose market clearing by equating supply and demand
Permanent Income Hypothesis

- In the absence of frictions (e.g. consumers can borrow), fluctuations in consumption will be driven by fluctuations in permanent income, not transitory income.
- Thus temporary shocks should not affect consumption greatly in the absence of large propagation mechanisms.
- Came from a problem like

\[
\max_{c_0, c_1} \ln c_0 + \beta \ln c_1 \quad s.t. \quad c_0 + b_1 = y_0 \\
\quad c_1 = y_1 + (1 + r)b_1
\]

to yield Euler equation

\[
\frac{u'(c_0)}{\beta u'(c_1)} = 1 + r
\]
Section 3

Growth
Empirics

1. Growth in per capita output of about 2% per year in the U.S.
2. Poorer countries converge to richer countries on average
3. Savings rate highly correlated with wealth
4. Innovation positively correlated with growth
Solow Growth - Set up

- Output produced with capital and labor according to
  \[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]
  for \( Y_t \) aggregate output (GDP), \( A_t \) total factor productivity (TFP), \( K_t \) capital, \( L_t \) labor, and \( \alpha \) the capital share in production

- Capital evolves according to law of motion:
  \[ K_{t+1} = (1 - \delta) K_t + I_t \]
  for \( \delta \) the (constant) depreciation rate of capital and \( I_t \) investment

- Agents save a fixed share \( s \) of their income so that
  \[ I_t = s Y_t \quad \text{and} \quad C_t = (1 - s) Y_t \]
Solow Growth - Dynamics

- Over time, converge to a steady state value of capital, output, and consumption.

\[ \bar{K} = \left( \frac{sAL^{1-\alpha}}{\delta} \right) \frac{1}{1-\alpha} \quad \bar{Y} = A\bar{K}^\alpha L^{1-\alpha} \quad \bar{C} = (1-s)\bar{Y} \]

- If productivity or labor increasing, converge to steady state value of capital etc. *per effective labor unit*

\[ \bar{k} = \left( \frac{s}{\delta + g_A + g_L + g_A g_L} \right) \frac{1}{1-\alpha} \quad \bar{Y} = \bar{k}^\alpha \quad \bar{c} = (1-s)\bar{y} \]

for \( g_A \) growth rate in productivity, \( g_L \) growth rate of labor.

- Increases in savings rate will not lead to higher consumption unambiguously: have more stuff but eat less of it. Optimal \( s^* = \alpha \)

- Only source of long run growth in per capita output: productivity
Neoclassical Growth

- Same premise as Solow, but people choose savings:
- Consumers solve

\[
\max \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.} \quad Y_t = A_t K_t^\alpha L_t^{1-\alpha}
\]

\[
K_{t+1} = (1 - \delta)K_t + I_t
\]

\[
l_t = Y_t - c_t
\]

Substitute constraints into each other to get

\[
K_{t+1} = (1 - \delta)K_t + A_t K_t^\alpha L_t^{1-\alpha} - c_t
\]

- Get Euler Equation a before.
- Savings rate is

\[
s = \frac{\delta \alpha}{\rho + \delta} \Rightarrow s = s^* = \alpha \iff \rho = 0
\]
Modern Growth

- Growth only comes from increases in productivity
- This requires innovation/input improvements
- Knowledge is a *public good*
- Thus have strong *intellectual property rights* (patent law)
- Public goods tend to be underprovided relative to social optimum
Section 4

Business Cycles
Economic Fluctuations

- Cycles happen recurrently but not periodically
- Multiple indicators fall at the same time
  - Output, investment, consumption, wages, etc.
- Investment and other highly income elastic goods fluctuate more over the cycle
- People predict with *leading indicators*, including the yield curve.
- Need propagation mechanism to make small shock large
  - People get poorer so invest less, have less capital, thus less output next period
  - Granularity: shocks to large firms can have aggregate effects
  - Network: shocks to highly connected sectors can have aggregate effects
Real Business Cycles - Firms

Firms solve

$$\max_{L_t, K_t} A_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t$$

so that

$$w_t = \underbrace{A_t (1 - \alpha) L_t^{-\alpha} K_t^\alpha}_{MP_L}$$

$$r_t = \underbrace{A_t \alpha L_t^{1-\alpha} K_t^{\alpha-1}}_{MP_K}$$
Real Business Cycles - Households

- Two generations: old and young
- Households solve

\[
\max_{c_t^t, c_{t+1}^t, k_{t+1}} \ln c_t^t + \ln c_{t+1}^t \quad \text{s.t.} \quad c_t^t + k_{t+1} = w_t
\]

\[
c_{t+1}^t = (1 - \delta)k_{t+1} + r_{t+1}k_{t+1}
\]

- Writing Lagrangean and FOC eventually yields

\[
k_{t+1} = \frac{w_t}{2}
\]

- Higher \(w_t\) ⇒ higher \(k_{t+1}\) ⇒ higher \(Y_{t+1}\)
Equilibrium

- Labor market clearing implies $L_t = 1$
- Plug firms’ FOC into household maximization problem to get
  \[ K_{t+1} = \left[ \frac{A_t(1 - \alpha)K_t^\alpha}{2} \right] \]
- If $A_t \uparrow$, $w_t \uparrow$, $K_{t+1} \uparrow$
- Define $I_t = K_{t+1} - (1 - \delta)K_t$, $C_t = Y_t - I_t$.
- Plugging in for $Y_t = A_tK_t^\alpha$, $K_{t+1}$, take derivative with respect to $A_t$ to get elasticity:
  \[ \epsilon_{IA} > 1 > \epsilon_{CA} \]
  so investment moves more than consumption through the cycle.
Dynamics

1. $A_t \downarrow$ so labor and capital demand falls
2. $w_t \downarrow$ and $r_t \downarrow$
3. $K_{t+1} \downarrow$
4. $A_{t+1}$ rebounds, so labor and capital demand rebound
5. Lower $K_{t+1}$ implies lower marginal product of labor, so wage doesn’t rebound all the way
6. Lower $K_{t+1}$ and old labor supply means $r_{t+1}$ jumps above old steady state
Section 5

Labor Markets
Labor Supply

- Labor supply comes from people trading off leisure and consumption.
- Two forces when wages rise:
  1. Substitution effect: higher wage makes labor more expensive ⇒ work more.
  2. Income effect: higher wage means higher income; leisure normal good ⇒ work less.
- If leisure and consumption substitutes, strong substitution effect ⇒ upward-sloping labor supply.
- If leisure and consumption complements, weak substitution effect ⇒ downward-sloping labor supply (possibly).
- Short run changes in wages do not affect permanent income ⇒ small income effect.
## Size of effects

<table>
<thead>
<tr>
<th>Permanent $w \uparrow$</th>
<th>Income Effect</th>
<th>Substitution Effect</th>
<th>Slope of Uncompensated Labor Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary $w \uparrow$</td>
<td>Large</td>
<td>?</td>
<td>Less positive</td>
</tr>
<tr>
<td>$c, l$ substitutes</td>
<td>Small</td>
<td>?</td>
<td>More positive</td>
</tr>
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</table>
Labor Supply Shifters

- Non-labor income ↑ shifts labor supply in
- Taxes: unclear as makes people poorer but leisure cheaper
- Population growth: add more supply curves together
- Increased value of leisure (e.g. improved leisure technology)

Laffer Curve: revenue maximizing income tax does not equal 0 or 1.
Unemployment comes from frictions

- Wage stickiness
- Search and matching frictions
  - Individuals need time to find a job
  - Suppose a fraction $\lambda$ of unemployed $U_t$ find a job each period
  - A fraction $\delta$ of employed lose their job each period
  - Yields law of motion

$$U_{t+1} = (1 - \lambda)U_t + \delta E_t$$

- Divide by $L$, get

$$u_{t+1} = (1 - \lambda)u_t + \delta(1 - u_t)$$

- At steady state, natural unemployment rate is

$$u_n = \frac{\delta}{\delta + \lambda} > 0$$
Job Search

- Individuals draw a wage $w$ from a distribution $F(w)$
- Can either accept it and earn that wage for two periods, or reject and search again
- If reject, get unemployment benefit $b$
- Choose:

$$\max \left\{ w + \beta w, b + \beta \mathbb{E}[w'] \right\}$$

- Leads to reservation rate strategy: accept all wages above some $w^*$
- $w^* \uparrow$ and so too does unemployment duration if:
  1. Unemployment benefit $b \uparrow$
  2. Expected next period wage offer $\mathbb{E}[w'] \uparrow$
  3. Discount factor $\beta \uparrow$
Section 6

Inflation and Money
Use of money

- Money used to trade for goods and services
- But by holding money, give up on holding high yield bonds
- Thus holding money has carrying cost
  - Forgone interest $r$
  - Possibility of theft
- But costly to take money out
  - “Shoe-leather cost” $\gamma$
  - It’s costly to go to ATM or bank.

Quantity Theory of Money

$$M_t V_t = P_t Y_t$$
Baumol-Tobin model of Cash Management

- Need money to make $pc$ purchases in a period
- Go to bank every $T$ periods
- Holding money has carrying cost $r$
- Have to pay cost $\gamma$ to go to bank
- Hold $pcT/2$ dollars on average
- Thus choose frequency of going to bank $T$ to minimize total cost:

$$\min_T \frac{1}{2} pcTr + \frac{\gamma}{T}$$

yields

$$T^* = \sqrt{\frac{2\gamma}{pcr}} \quad \Rightarrow \quad m^D = pcT^*/2 = p\sqrt{\frac{c\gamma^{REAL}}{2r}}$$
Inflation and Money

CIA & Friedman Rule

Cash-in-Advance

- Prices flexible, money supply grows constantly from Fed
- Choose money, bonds, consumption, and labor to maximize utility
- Two constraints:
  1. Cash in Advance (CIA): \( p_t c_t \leq m_t \)
  2. Budget Constraint (BC):
     \[ p_t c_t + b_{t+1} + m_{t+1} = m_t + (1 + R_t)b_t + p_t l_t + \tau_t \]
- To solve:
  1. Set up Lagrangean
  2. Take first order conditions
  3. Use market clearing \((b_t + 1 = 0, m_t^D = m_t^S)\)
Lessons from CIA

1. Growth rate of prices = growth rate of money
2. \((1 + R) = (1 + \pi)(1 + r)\) for \(R\) nominal interest rate, \(\pi\) inflation rate, \(r\) real interest rate
3. Friedman rule: should set money growth and inflation negative, to have nominal interest rate \(= 0\)
4. Output negatively related to inflation
5. If prices fully flexible, money market does not affect goods market.
6. However, reverse could be true by changing the real interest rate
If prices fixed (possibly true in short run), monetary expansions increase output.

- Draw curves of output ($Y$) vs real interest rate ($r$).
- Investment-Savings (IS) curve downward sloping because if $r$ low, cheaper to borrow and invest, so investment rises and so does output as:

$$Y = C + I + G + NX$$

- Liquidity Management (LM) curve upward sloping because as $Y$ increases, demand for money increases for every value of $r$.
- Where they intersect is economywide general equilibrium.
- Increases in money supply shift out LM curve $\Rightarrow$ higher output, lower real interest rate.
Section 7

Uncertainty and Asset Pricing
Expected utility

- Assume people maximize expected utility.
- Define:
  1. Risk averse: \( u(E[y]) > E[u(y)] \)
  2. Risk neutral: \( u(E[y]) = E[u(y)] \)
  3. Risk loving: \( u(E[y]) < E[u(y)] \)
- Risk averse if and only if \( u(c) \) concave.
Asset pricing

- If know price of states $q_s$, can price any asset as weighted combination of $q_s$
  - An asset that pays $x_s$ in state $s$ for each state $s$ will have price

\[ p = \sum_s q_s x_s \]

- Price of burger + fries = Price burger + price fries
- Risk aversion implies people want to buy insurance
  \[ \Rightarrow \] Price of bad states higher than price of good state
- Rate of return (Expected payout/price) higher for risky assets – those that pay out when output already high
- Higher rate of return generally inversely related to price