

Growth and the Cycle: Creative Destruction versus Entrenchment

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Newly established firms often try to secure their market position by building up a base of loyal customers. While recessions may not destroy technological leadership, they may be harmful for such firm–customer relationships. Without such customer bases, these firms find themselves more vulnerable to attacks by competitors. We formulate this idea within an Aghion–Howitt-type model of creative destruction and discuss its implications for growth. In the context of this model, recessions might be good for growth since they weaken the incumbent firm’s position and, thereby, stimulate research by outside firms. The model allows for the extreme case where the leading firm can be so entrenched that growth ceases, unless a recession shakes up its customer base. We find a one-to-one relationship between the average growth rate and the cyclical variability, a U-shaped relationship between the average speed of building up good customer relationships and the average growth rate, and a positive relationship between the arrival rate of recessions and average growth. It is finally shown that an appropriate stochastic tax program can implement the social planner’s solution. In some cases, general-equilibrium effects may generate interesting results, conflicting with intuition from a partial-equilibrium approach: we show that, in some cases, a social planner might want to subsidize research in order to discourage it.

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JEL classification: E32, H21, L16.

1 Introduction

Technological breakthroughs are often not enough to strongly establish a firm in the market. It also needs further marginal product improvements according to customer needs, or to build up consumer recognition to secure its position. Building up such a position takes time. And while recessions may not destroy technological breakthroughs, they may be seen as disrupt-

ing such firm–customer relationships. Thus, firms which have not yet built such relationships or whose relationships have been destroyed in a recession, find themselves more vulnerable to attacks by competitors than those that did. We formulate this idea within an Aghion–Howitt-type model of creative destruction (see Aghion and Howitt, 1992) and study its implications for growth. In particular, if the lead firm lacks an established position, the competitors’ incentives for attacks are increased, leading to higher research-and-development efforts on their part in the hope of leapfrogging the leader. Thus, in this model, recessions are actually good for growth, since they encourage new creative destruction. Booms and established market leads, on the other hand, can in extreme cases completely eliminate all desire for R&D, leading to complete entrenchment of the leader and to a standstill in growth, until the next recession destroys the secure market lead. We find a one-to-one relationship between the average growth rate and the cyclical variability, a U-shaped relationship between the average speed of building up good customer relationships and the average growth rate, and a positive relationship between the arrival rate of recessions and average growth. We do not view these claims as immediately testable empirical predictions of our model, however. Rather we like to think of our model as just analyzing one of many facets of economic fluctuations in isolation. For the same reason, we have abstained from attempting a serious calibration exercise. In our view, such a calibration exercise can only be done on the basis of a more complete, but thus also more complicated analysis of all the facets involved.

This paper fits within the literature on creative destruction, initiated by Schumpeter (1942), and more recently by Segerstrom et al. (1990) and Aghion and Howitt (1992).¹ Economic growth is driven by the process of creative destruction, i.e., the introduction of new products by innovating firms and the replacement of incumbent market leaders. The contribution of this paper is that we distinguish fundamental innovations leading to creative destruction from marginal innovations that slow down the process of creative destruction. The marginal innovations in our model capture the buildup of a well-functioning firm–customer relationship. Strong market leaders with a loyal customer base can, at least partially, insulate themselves from the threat of being leapfrogged by potential entrants. Secondly, along the lines of Caballero and Hammour (1994), we analyze the cleansing ef-

1 An overview of Schumpeterian growth theory can be found in Dinopoulos (1996).

fect of recessions by assuming that established firm–customer relationships will be destroyed in a recession.

Related ideas have received some attention in the recent literature. Cheng and Dinopoulos (1993) construct a model of Schumpeterian growth driven by asymmetric technological opportunities in the form of high-cost high-quality breakthroughs and low-cost low-quality improvements. They assume that each product generation starts with a quality breakthrough, followed by a single improvement. The pattern of growth and fluctuations can then be described as a stationary market equilibrium in which R&D races alternate between a breakthrough and an improvement. Our approach is different in the sense that we allow incumbent firms to gain from experience: firms that are longer in the market are more likely to carry out marginal improvements, or to establish a loyal customer base. Stein (1997) develops a model of creative destruction in which firms compete on product quality and on distribution costs. A firm's innovation in product quality ultimately spills over to new firms, whereas distribution costs are taken to be firm-specific: incumbent firms have an advantage over their potential competitors when they can reduce distribution costs through loyalty of their customers. In line with our results, Stein finds that firm-specific learning-by-doing may discourage research activity and thereby reduce long-term economic growth. In contrast with Stein's analysis, we take account of the possibility that such firm-specific advantages may suddenly be disrupted by the event of a recession. Caballero and Hammour (1996) analyze the timing, pace, and efficiency of job creation and destruction resulting from product and process innovation. While an efficient economy concentrates such job reallocation processes during recessions (because of the opportunity-cost effect), incomplete contracting between labor and capital as well as transactional difficulties may decouple the synchronized pattern of creation and destruction, leading to technological "sclerosis." Economic efficiency can be restored through an appropriate mix of government policies. We also aim to design optimal tax policies that restore economic efficiency, but in our story such a taxation scheme is shown to be state-contingent. Finally, there is an empirical literature on the countercyclicality of human-capital formation in connection with the opportunity-cost effect (cf. Bean, 1990; Davis and Haltiwanger, 1992; Saint-Paul, 1993, 1997). This opportunity-cost theory is not undisputed, however. In particular, some authors claim that innovation and human-capital accumulation tend to be procyclical. In this context one may think of learning-by-doing, inducing positive feedback effects from economic activity to productivity growth.

In addition, demand spillovers (Shleifer, 1986) or capital-market imperfections (Stiglitz, 1993) could make firms more willing to implement new technologies in expansions rather than in recessions. It is difficult to disentangle these effects empirically; nonetheless, Saint-Paul (1993, pp. 880) concludes that “there is more empirical support for the opportunity cost approach than one might have expected.”

We proceed along the following lines. Section 2 introduces the model, derives optimality conditions, and describes the equilibrium solution. Some interesting numerical examples are discussed in Sect. 3 to describe some special features of the model. In particular, we study the possibility of entrenchment: strong market leaders can in the extreme completely eliminate incentives to carry out R&D by potential competitors, leading to complete entrenchment in the market and to a standstill in economic growth. The model's implications for growth and the business cycle are more elaborately discussed in Sect. 4. Since the equilibrium solution of the model is not an efficient solution, we will investigate the policy selected by a benevolent social planner in Sect. 5. In Sect. 6, it is shown that an appropriate stochastic tax program can implement the social planner's solution in a decentralized economy. In some cases, general-equilibrium effects may generate interesting results, conflicting with intuition from a partial-equilibrium approach: we show that, in some cases, a government might want to subsidize research in order to discourage it. Briefly, the intuition is that the market leader has to pay taxes to finance these research subsidies. This may lower its value by a substantial amount, so that firms in the research sector expect substantially lower gains from innovative activity, and actually decide to undertake less research activity. If this does not yet sound convincing, we hope that it entices one to read Sect. 6 for a more extensive discussion and analysis. Finally, Sect. 7 concludes.

2 The Model

2.1 Environment

Consider an economy with three classes of tradeable objects: labor, a consumption good, and an intermediate good. Time is continuous. All markets are perfectly competitive, except for the intermediate-goods market. The economy is populated with a continuum of infinitely-lived, representative agents. These agents choose contingency plans for lifetime consumption,

evaluated at a constant rate of time preference $r > 0$ and linear instantaneous utility. Thus, r is also the rate of interest.

The agents also supply labor. Labor supply is constant, inelastic, and normalized to unity. Two categories of labor are distinguished: unskilled labor, which can only be used in the production of the final good which is used for consumption, and skilled labor, which can be employed in research and in the intermediate sector.

Production takes place in two sectors: a competitive final-goods sector and a monopolistic intermediate-goods sector. Furthermore, there is a sector undertaking research. A firm in the competitive final-goods sector hires unskilled labor m and purchases the amount x of the intermediate good to produce output y according to (omitting the time subscript)

$$y = A_f F(x/m)m, \quad (1)$$

where F is strictly increasing, strictly concave, and differentiable. The factor A_f is the current productivity of final-goods production “embodied” in the intermediate good: more advanced intermediate goods allow final-goods production firms to produce with higher total factor productivity. Normalizing the aggregate quantity of unskilled labor to unity, aggregate production is $y = A_f F(x)$.

The productivity A_f is thus intimately tied to the particular intermediate input x which is used, and which is sold by a monopoly. Fundamental innovations increase this productivity by a fixed factor γ . Therefore, the time profile of the productivity parameter is given by

$$A_f = A_0 \gamma^f, \quad \gamma > 1, \quad (2)$$

where $f = 0, 1, 2, 3, \dots$ denotes the fundamental innovation. A fundamental innovation brings about a new intermediate input allowing firms in the final-goods sector to produce more efficiently. The new intermediate product renders existing ones obsolete. Thus, fundamental innovations replace the existing intermediate firm with a new monopoly in the now leading technology: economic growth is driven by creative destruction. It will be assumed that fundamental innovations occur randomly with Poisson arrival rate λn , where n is the flow of skilled labor used in research. The research sector itself is competitive, but a successful innovator can protect his fundamental innovation by a patent which he can use to monopolize the intermediate sector. According to Eq. (2), the knowledge incorporated in a new intermediate input ultimately spills over to new firms: innovators stand on the shoulders of giants.

The production function of the intermediate good x is linear,

$$x = BL, \quad (3)$$

where L is the flow of skilled labor used in the intermediate sector. To capture the idea that an intermediate firm needs further marginal improvements of the product according to customer needs, or to build up consumer recognition to secure its position, we introduce the parameter B . For simplicity, we assume that B can only take two values: $B \in \{\delta, 1\}$. If $B = \delta > 1$, the intermediate firm is a strong market leader, but if $B = 1$ the monopolist is a weak market leader.

According to Eq. (3), the strength of the monopolist is reflected in parameter B . It should be noted, however, that there is no formal difference between technical improvements in the final-goods sector or in the intermediate-goods sector. Total factor productivity of the final-goods sector is determined by the current technology of the intermediate monopolist. Thus, the establishment of strong market leadership by the intermediate monopolist also translates into increased productivity in final-goods production.

We hasten to add that it is an exogenous lightswitching process that is responsible for changing B between 1 and δ . The interpretation of the mechanism behind this lightswitching process is therefore somewhat arbitrary. We do not explicitly model “marginal product improvements,” “loyal-customer relationships,” or “learning-by-doing.” The main idea here is that we distinguish public knowledge A that spills over to other firms (after patent expiration) from firm-specific knowledge B that does not spill over across the economy. Such firm-specific knowledge may create an additional advantage for the incumbent monopolist, but – as we will see – may at the same time discourage efforts to increase the stock of public knowledge through research activity.

Newly established firms can secure their market position by building up a base of loyal customers. Experience from being in the market can turn a weak market leader into a strong one. We specify this learning-by-doing as an exogenous stochastic Markov process, where μ is the Poisson arrival rate for a weak monopolist to become strong. In other words, we simply assume that older firms are more likely to have a loyal-customer base than young ones (*ceteris paribus*).

While recessions may not destroy technological breakthroughs, they may be seen as disrupting such firm–customer relationships. The event of a recession will consequently turn a strong monopolist into a weak one.

Specifically, let us assume that the Poisson arrival rate of a recession is given by ν .

To rule out the possibility of strategic behavior, we henceforth assume that $\gamma > \delta$: the size of drastic innovations in the productivity of final output is larger than the size of marginal innovations in the intermediate-goods sector from learning-by-doing.

Consider a firm which has made the f -th innovation. During its lifetime, the intermediate firm can find itself in two different states. In the first state, the incumbent firm is a weak market leader. In the second state, the incumbent firm is strong. After some random time span, the incumbent monopolist will be superseded by a new intermediate firm through the event of the $f + 1$ -th fundamental innovation.

The various transitions across states initiated by fundamental and marginal innovations can be tabulated as follows. Denoting the state of the f -th intermediate firm by i and the state of the new firm by j ($i/j = 1$ denotes weak market leadership; $i/j = 2$ denotes strong market leadership), we have the following transition structure during a small time interval dt :

f	$f + 1$	transition probability
$i = 1 \rightarrow j = 1$	$\lambda n_f^{(1)} dt$	
$i = 2 \rightarrow j = 1$	$\lambda n_f^{(2)} dt$	

Implicitly, we have assumed the labor input into research to depend only on the index f of the innovation, i.e., to be constant in the time interval during which the f -th but not the $f + 1$ -th innovation has been undertaken. In the further analysis we will see that this is justifiable in equilibrium.

Equivalently, denoting the state of the intermediate firm at t ($t + dt$) by i (j), we have the following transition structure in case of incremental leaps:

t	$t + dt$	transition probability
$i = 1 \rightarrow j = 2$	μdt	
$i = 2 \rightarrow j = 1$	νdt	

An equilibrium are lists of firm values ($V_f^{(i)}$), research labor ($n_f^{(i)}$), intermediate-goods production ($x_f^{(i)}$), intermediate-goods labor ($L_f^{(i)}$), wages for skilled labor ($w_f^{(i)}$), and profits ($\pi_f^{(i)}$) for $f = 0, 1, 2, \dots$ and $i = 1, 2$ so that at each level f and each state i ,

- i. the current intermediate-goods monopolist maximizes instantaneous profits, given wages $w_f^{(i)}$,

$$\pi_f^{(i)} = \max_{\{x_f^{(i)}, L_f^{(i)} \leq N\}} A_f F'(x_f^{(i)}) x_f^{(i)} - L_f^{(i)} w_f^{(i)} \quad \text{s. t. } x_f^{(i)} = B^{(i)} L_f^{(i)}$$

(note that we have substituted in the demand function for the intermediate-good sector, resulting from the final-goods-production sector);

- ii. the firm value is given by $V_f^{(i)} = \sum_{j=1}^2 \int_{t=0}^{\infty} e^{-rt} \pi_f^{(j)} P_f^{(i)}(t, j) dt$, where $P_f^{(i)}(t, j)$ is the probability that the current intermediate-good monopolist is still the market leader t time units from now, and is in state j then;
- iii. given the wage $w_f^{(i)}$, the competitive R&D firms maximize the instantaneous profits from R&D, calculated as the instantaneous value of a successful innovation times its instantaneous probability, minus the instantaneous wage costs,

$$\max_{\{n_f^{(i)} \geq 0\}} V_{f+1}^{(1)} \lambda n_f^{(i)} - w_f^{(i)} n_f^{(i)} ;$$

- iv. the market for skilled labor clears, $N = L_f^{(i)} + n_f^{(i)}$, where N denotes the mass of skilled individuals.

2.2 The Maximization Problems

Having finished the description of the economy, we now turn to the optimality conditions. At each instant in time, the monopolist can be in two different states, as described above. Therefore, two Bellman equations need to be constructed. For instance, when the intermediate firm is currently in the weak state, it makes the instantaneous profit, $\pi_f^{(1)}$. The probability of still being a monopolist after a small time interval dt has elapsed is equal to $1 - \lambda n_f^{(1)} dt$. Within this interval, the (unconditional) probability of a marginal innovation is μdt . By the event of a marginal innovation the monopolist switches to the second state, and the firm's value is given by $V_f^{(2)}$. With probability $1 - \mu dt - \lambda n_f^{(1)} dt$ the firm does not make the transition to state 2 during the time interval but is still the market leader, so that its value is still given by $V_f^{(1)}$. Proceeding along these lines, the Bellman equations

can be written in the form (details can be found in Appendix 1)

$$\begin{bmatrix} r + \lambda n_f^{(1)} + \mu & -\mu \\ -\nu & r + \lambda n_f^{(2)} + \nu \end{bmatrix} \begin{bmatrix} V_f^{(1)} \\ V_f^{(2)} \end{bmatrix} = \begin{bmatrix} \pi_f^{(1)} \\ \pi_f^{(2)} \end{bmatrix}; \quad (4)$$

or, abbreviated,

$$XV = \pi. \quad (5)$$

X is the 2×2 matrix from Eq. (4), $V = [V^{(1)} V^{(2)}]'$, and $\pi = [\pi^{(1)} \pi^{(2)}]'$.

Consider, next, the research sector. A potential entrant successfully doing research will start in state 1. The instantaneous expected gain for the f -th innovator when the current market leader is in state i is thus equal to $V_{f+1}^{(1)} \lambda n_f^{(i)} dt e^{-r dt}$. The instantaneous cost of doing research is $w_f^{(i)} n_f^{(i)} dt$, where $w_f^{(i)}$ denotes the wage of skilled labor. An optimizing R&D firm chooses $n_f^{(i)}$ so as to equalize both terms, taking V and w as given. It follows that

$$w_f^{(i)} \geq V_{f+1}^{(1)} \lambda, \quad n_f^{(i)} \geq 0 \quad (6)$$

with at least one equality.

Firms in the final-goods sector choose $x_f^{(i)}$ to maximize profits $A_f F(x_f^{(i)}) - p_f^{(i)} x_f^{(i)}$, taking the relative price of the intermediate good $p_f^{(i)}$ as given. The first-order condition for firms in the final-goods sector is thus given by

$$A_f F'(x_f^{(i)}) = p_f^{(i)}. \quad (7)$$

Consequently, the intermediate firm chooses $x_f^{(i)}$ to maximize $[A_f F'(x_f^{(i)}) - w_f^{(i)} / B^{(i)}] x_f^{(i)}$. The optimality condition is given by

$$\omega_f^{(i)} = B^{(i)} \{F''(x_f^{(i)}) x_f^{(i)} + F'(x_f^{(i)})\}, \quad (8)$$

where $\omega_f^{(i)} = w_f^{(i)} / A_f$ is the productivity-adjusted wage.

2.3 Stationary Equilibrium

In a stationary equilibrium, variables do not depend on the state f . Unless otherwise indicated, we concentrate in the sequel on interior equilibria,

while the situation where no research is undertaken by outside firms and the incumbent monopolist is entrenched and completely insulated from creative destruction will be discussed as a special case in Sect. 3 (Example 4). At each instant in time, the economy only needs to decide upon the allocation of skilled labor between manufacturing and research.

To determine $\omega^{(i)}$, we define $\tilde{V}^{(i)} = V_f^{(i)}/A_f$ and make use of the following proposition.

Proposition 1: In stationary interior equilibrium it must hold that $\omega^{(1)} = \omega^{(2)} = \omega$.

Proof: From Eq. (6) and the transition structure for fundamental innovations it follows that $w_f^{(i)} = w_f$, or $\omega^{(i)} = \omega$. \square

Notice that this proposition only holds for interior equilibria. Proposition 1 says that the productivity-adjusted wage of skilled labor is constant across both states. That is, skilled workers do not benefit from marginal innovations within the intermediate firm.

Using a Cobb–Douglas production function, $F(x) = x^\alpha$, we can readily express the solution in terms of ω :

$$x^{(i)} = \left[\frac{\alpha^2 B^{(i)}}{\omega^{(i)}} \right]^{1/(1-\alpha)}; \quad \tilde{\pi}^{(i)} = \frac{1-\alpha}{\alpha} \frac{\omega^{(i)} x^{(i)}}{B^{(i)}}; \quad \tilde{p}^{(i)} = \frac{\omega^{(i)}}{\alpha B^{(i)}},$$

where $\tilde{\pi}^{(i)} = \pi_f^{(i)}/A_f$ and $\tilde{p}^{(i)} = p_f^{(i)}/A_f$. For the Cobb–Douglas case, we obtain the stationary equilibria of the model from the following proposition.

Proposition 2: There are in general two solutions for ω , given by

$$\bar{\omega}_{1,2} = \left(\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a} \right)^2 - \frac{c}{a}} \right)^{1-\alpha},$$

where a , b , and c are as stated in Appendix 2.

A proof is given in Appendix 2.

3 Results

Using the Cobb–Douglas specification, we turn to some numerical examples to illustrate the stationary equilibrium (or equilibria). As a baseline, we more or less arbitrarily pick the following values: $r = 0.1$; $A_0 = B^{(1)} = N = 1$; $B^{(2)} = 1.2$; $\alpha = 0.5$; $\gamma = 1.4$; $\delta = 1.2$.

Example 1 (Aghion and Howitt): We discuss the model's equilibrium solution in the absence of learning-by-doing and recessions. By setting μ and ν equal to zero, we effectively are back in the Aghion–Howitt world. λ is set at 0.15. Although the fundamental quadratic from Proposition 2 delivers two equilibrium values for ω , only the “positive” root is economically meaningful (more precisely, only the “positive” root gives a nonnegative research intensity). In this example, 31% of the skilled-labor force is engaged in research activity.

Example 2 (Learning-by-doing): We allow intermediate firms to strengthen their market position by building up a base of loyal customers. We pick $\mu = 0.5$ (leaving other parameters equal). That is, we allow for the possibility of marginal innovations and assume that marginal leaps are more likely to take place than fundamental breakthroughs. We find $n^{(1)} = 0.40$ and $n^{(2)} = 0.28$. Since $n^{(1)} > n^{(2)}$ and n is positively related to the arrival rate of fundamental innovations, we can refer to state 1 (no marginal innovation) as the “high-growth equilibrium” and state 2 (with marginal innovation) as the “low-growth equilibrium.” To put it differently, the creation of a loyal-customer base by the intermediate firm discourages research activity by potential entrants, and thereby tends to lower economic growth.

Example 3 (Learning-by-doing and recessions): We consider the possibility that strong firm–customer relationships are destroyed in a recession. The flow probability of recessions, ν , is set at 0.2. With these parameter values, we have assumed that agents expect recessions to take place less often than marginal innovations, but more frequently than fundamental innovations. In equilibrium we have $n^{(1)} = 0.38$ and $n^{(2)} = 0.26$.

Example 4 (Entrenchment): Strong market leaders might completely eliminate all desire for R&D, leading to complete insulation of the incumbent monopolist from the process of creative destruction and to a standstill in growth. Such a scenario will emerge from our model when (for instance) fundamental innovations occur less frequently. For $\lambda = 0.08$ (and $\mu = 0.5$, $\nu = 0.2$) we have the knife-edge case and find $n^{(1)} = 0.17$ and $n^{(2)} = 0$.

Marginal innovations lead to a stop of research activity and economic growth. Only the extricating event of a recession can bring the economy out of such a “no-growth trap” back to a process where firms try to leapfrog each other.

4 Economic Growth and the Cycle

We will now discuss the model’s implications for economic growth and the cycle. In order to calculate the average growth rate in the economy, we need to find the expected fraction of time that the economy spends in each state. From a society’s perspective, we have the following transition scheme between both states during a small time interval dt :

t	$t + dt$	transition probability
$i = 1 \rightarrow j = 1$		$1 - \mu dt$,
$i = 1 \rightarrow j = 2$		μdt ,
$i = 2 \rightarrow j = 1$		$(v + \lambda n^{(2)}) dt$,
$i = 2 \rightarrow j = 2$		$1 - (v + \lambda n^{(2)}) dt$.

To see this, consider first an intermediate monopolist which is currently in state 1. With probability $\lambda n^{(1)} dt$ this monopolist will be superseded by a new intermediate firm through the event of a fundamental innovation during that time interval. This new intermediate firm will start operation in state 1. A monopolist currently in state 1 will make the transition to state 2 during dt with probability μdt . Thus, an incumbent firm which is in state 1 will maintain its current position during a small time span dt with probability $1 - \lambda n^{(1)} dt - \mu dt$. From a society’s perspective, we thus find that the probability of the economy still being in the first state after dt has elapsed is equal to $1 - \mu dt$, whereas with probability μdt the economy has moved to the second state. Likewise, consider an intermediate monopolist which is currently in state 2. With probability $\lambda n^{(2)} dt$ this monopolist will be replaced by a new intermediate firm through the event of a fundamental innovation during that time span. Again, this new intermediate firm will start operation in state 1. A monopolist currently in state 2 will face a recession and fall back to state 1 during dt with probability νdt . Consequently, an incumbent firm which is in state 2 will maintain its current position during a small time span dt with probability $1 - \lambda n^{(2)} dt - \nu dt$. From a society’s perspective, we thus find that the probability of the economy still being in the

second state after dt has elapsed is equal to $1 - \lambda n^{(2)} dt - \nu dt$ whereas with probability $\nu dt + \lambda n^{(2)} dt$ the economy has moved back to the first state.

Denoting the stationary probability that the firm is in state i by $q^{(i)}$ and using that $q^{(2)} = 1 - q^{(1)}$, we have in stationary flow $q^{(1)} = q^{(1)}(1 - \mu dt) + (1 - q^{(1)})(\nu + \lambda n^{(2)}) dt$, or

$$q^{(1)} = \frac{\nu + \lambda n^{(2)}}{\mu + \nu + \lambda n^{(2)}}; \quad q^{(2)} = \frac{\mu}{\mu + \nu + \lambda n^{(2)}}. \quad (9)$$

Naturally, the firm will never become a strong market leader when $\mu = 0$. In Aghion and Howitt (1992) the average growth rate in the economy equals $\lambda n \ln(\gamma)$, where λn is the arrival rate of fundamental innovations. A similar expression can easily be derived by weighting the research intensity in each state, $n^{(1)}$ and $n^{(2)}$, by the expected fraction of time that the economy spends in each state, determined by Eq. (9). The average growth rate (*AGR*) is thus found to be given by

$$AGR = \frac{\nu n^{(1)} + \lambda n^{(1)} n^{(2)} + \mu n^{(2)}}{\mu + \nu + \lambda n^{(2)}} \lambda \ln \gamma. \quad (10)$$

Following a similar methodology, the variance of the rate of economic growth (*VGR*) can be expressed as

$$VGR = \frac{\nu n^{(1)} + \lambda n^{(1)} n^{(2)} + \mu n^{(2)}}{\mu + \nu + \lambda n^{(2)}} \lambda (\ln \gamma)^2. \quad (11)$$

The ratio of *AGR* over *VGR* is constant and equal to $1 / \ln \gamma$: *AGR* and *VGR* are thus related in a linear fashion. This result is in line with Aghion and Howitt (1992); it does not emerge from particular features introduced in our model.

Having determined the economy's average growth rate and cyclical variability, we next turn to an evaluation of the effect of learning-by-doing and recessions on this growth rate. First we vary the speed μ of a marginal innovation, i.e., the effect of learning-by-doing, within the closed unit interval and study its implications for growth and research in Fig. 1. Figure 1a shows a kind of U-shaped relation between the economy's average growth rate and the flow probability of marginal innovations: an increase in μ will tend to lower economic growth when firms need a relatively long time to learn about their customers' needs, whereas an opposite relation is found when firms learn fast. Figure 1b explains the intuition behind this result. A strong market leader discourages R&D activity by potential entrants by increasing its expected lifetime. As μ is increased, firms tend to spend

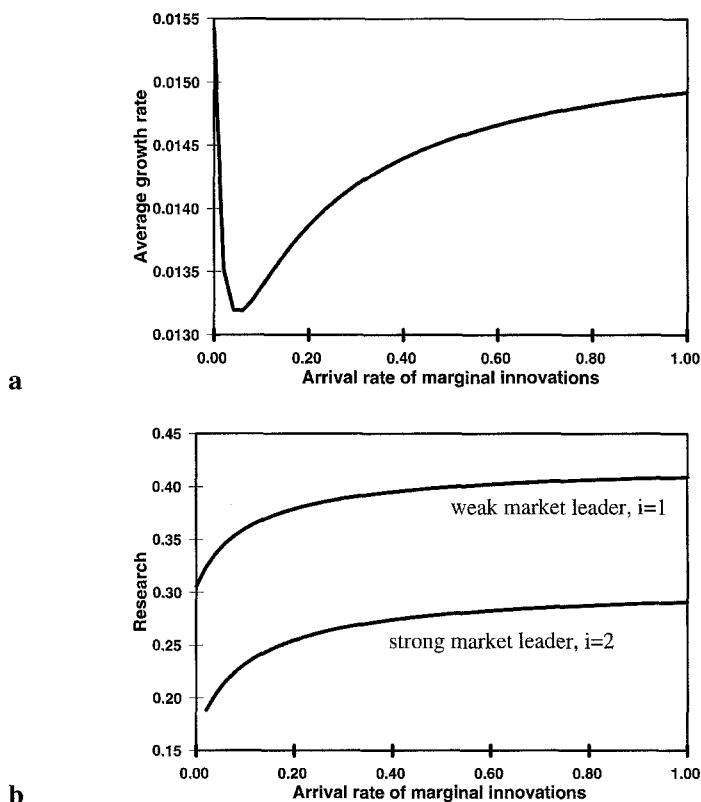


Fig. 1. Effect of learning-by-doing on economic growth (a) and research (b)

more time in the strong state. This discourages R&D. Call this the “discouragement effect.” On the other hand, research intensity $n^{(i)}$ is a positive function of the flow probability μ of marginal innovations: the prospect of being a strong market leader during a larger fraction of its lifetime increases the expected gains from fundamental innovations, and thereby stimulates research activity. Let us refer to this as the “reward effect.” Overall, the discouragement effect dominates the reward effect when firms need a long learning period, whereas the opposite holds when firms learn fast, leading to the observed U-shaped relation between AGR and μ .

We secondly vary the arrival rate of recessions, ν , within the closed unit interval (setting $\mu = 0.5$) and study its implications for growth and research in Fig. 2. Figure 2a shows a positive relation between the economy’s average growth rate and this arrival rate: an increase in ν will stim-

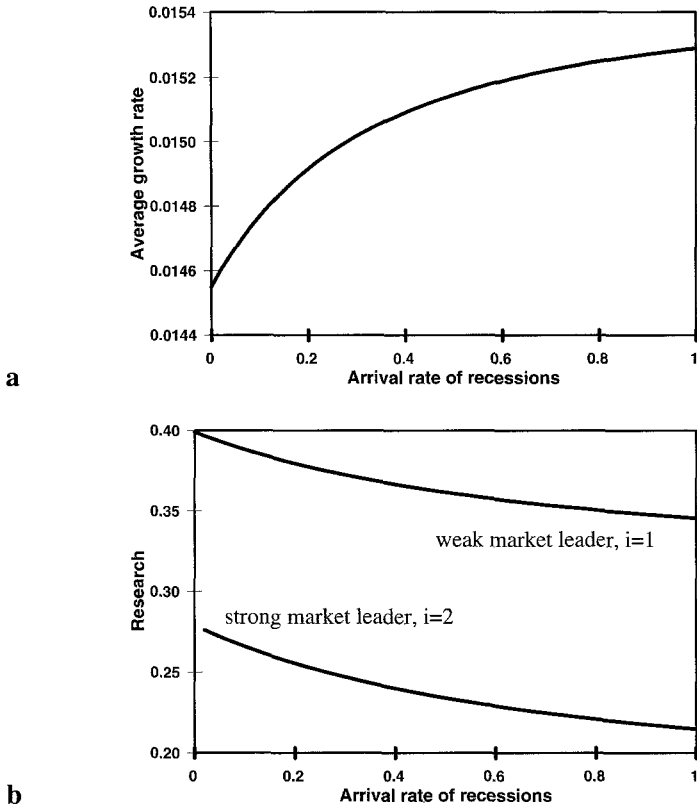


Fig. 2. Effect of recessions on economic growth (a) and research (b)

ulate economic growth. Figure 2b again shows that a strong market leader discourages R&D activity by potential entrants; $n^{(2)}$ is smaller than $n^{(1)}$ over the whole relevant domain. An increase in the arrival rate of recessions makes it more likely that the market leader is weak, encouraging R&D: this is the discouragement effect “in reverse.” This should increase growth. On the other hand, research activity is a negative function of the arrival rate of recessions; the intuition being that the prospect of losing strong market leadership earlier decreases the expected gains from fundamental innovations. This reward effect “in reverse” decreases growth. Overall, the discouragement effect “in reverse” dominates.

5 Social Planner

The balanced-growth rate in a market economy may not be optimal from a society's point of view because of two external effects and an additional distortion. Firstly, intermediate firms cannot fully appropriate the rents generated by their fundamental innovations: the new technology ultimately spills over to other firms. Because of this intertemporal spillover effect, research activity and economic growth tend to be too low under *laissez faire*. On the other hand, innovating firms do not internalize the destruction of rents from leapfrogging the incumbent monopolist. This business-stealing effect will tend to overemphasize the benefits from research. Finally, the intermediate-goods producer chooses its quantity monopolistically, possibly distorting the first-best solution. In addition to these market imperfections, we want to raise the question whether strong firm–customer relationships are socially desirable or not. As we have seen, the discouragement effect that strong market leaders exert on potential entrants will lead to less R&D activity in the economy. However, the fact that strong market leaders can appropriate a larger share of the social value of their innovation since they can partly shelter from the threat of being leapfrogged by a new entrant will encourage research activity.

The objective of a social planner is to choose R&D labor and thus quantities of the intermediate good in order to maximize the expected present value of consumption, subject to the constraints of feasibility. Following a similar methodology as in Sect. 2.2, the social planner's problem can be written as a dynamic program, which can be rewritten as

$$\begin{aligned} & \begin{bmatrix} r + \lambda n^{(1)}(1 - \gamma) + \mu & -\mu \\ -v - \lambda n^{(2)}\gamma & r + \lambda n^{(2)} + v \end{bmatrix} \begin{bmatrix} U^{(1)} \\ U^{(2)} \end{bmatrix} \\ &= \begin{bmatrix} F(B^{(1)}(N - n^{(1)})) \\ F(B^{(2)}(N - n^{(2)})) \end{bmatrix}, \end{aligned} \quad (12)$$

where $U^{(i)}$ is the utility level when the economy is in state i . Compare Eq. (12) to Eq. (4) for the firm values in equilibrium. The upper left-hand element of the matrix in Eq. (12) contains the additional term $-\lambda n^{(1)}\gamma$ compared to Eq. (4), reflecting the fact that R&D is valuable to the social planner, but not to the existing firm. The same holds true for the term $-\lambda n^{(2)}\gamma$ in the lower left-hand element of that matrix.

Inverting the 2×2 matrix in Eq. (12), and weighting lifetime utility in each state by the average fraction of time that the economy will spend in

each state [cf. Eq. (9)], we finally express lifetime utility as

$$U = \sum_{i=1}^2 q^{(i)} U^{(i)} = \Xi(n^{(1)}, n^{(2)}), \quad (13)$$

where Ξ is a complicated function.

An optimizing social planner would select $n^{(i)}$ such that

$$\frac{\partial \Xi}{\partial n^{(1)}} = \frac{\partial \Xi}{\partial n^{(2)}} = 0. \quad (14)$$

Since we were not able to obtain an equilibrium solution in closed form, we resort to numerical simulations to discuss the effect of learning-by-doing and recessions on the economy's growth rate. The implications for growth and research from variation in μ within the closed unit interval are illustrated in Fig. 3. Figure 3a shows a positive relation between the economy's average growth rate and the arrival rate μ of marginal innovations for values of μ exceeding (say) 0.1. The intuition behind this result, and why and how it differs from Fig. 1, can most easily be developed with the help of Fig. 3b. In Fig. 3b we show the optimal research program that a social planner would implement. It shows that the social planner allocates more workers to research if the market leader is weak. Intertemporal reallocations of skilled labor between production and research activities are intensified compared to the decentralized equilibrium situation. What is at work here, is that the gains related to a particular state of the economy are optimally used. An economy can better reallocate skilled workers from production towards research activity when the incumbent monopolist is weak, in order to fully exploit the temporary lower opportunity costs in terms of production foregone. Likewise, during a boom when the market leader is particularly good at producing intermediate inputs, one can better concentrate efforts in this direction, by relieving employees from research activity and allocating these workers to the monopolistic firm. Figure 3b also shows that research activity is intensified when μ is increased: the prospect of being a strong market leader during a larger fraction of its lifetime increases the expected gains from fundamental innovations, so that it is optimal to allocate more labor to research activity. Two additional comments are in order. Firstly, research activity during periods of strong market leadership is strongly reduced when it takes a long time to build up such a leading position. When μ is in the interval between 0 and (say) 0.1, all research activity is stopped and there is a standstill in economic growth

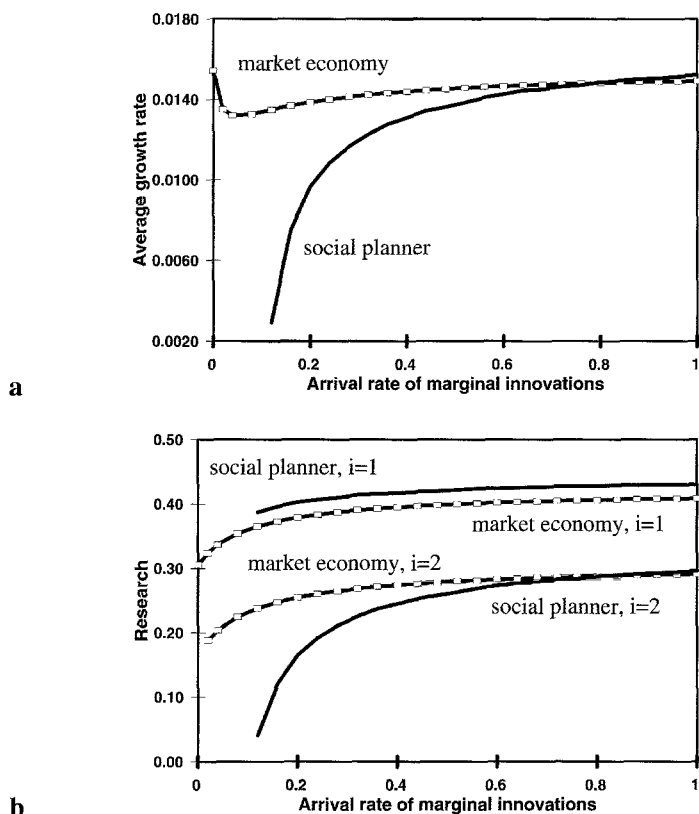


Fig. 3. Social planner: effect of learning-by-doing on economic growth (a) and research (b). $i = 1$, weak market leadership; $i = 2$, strong market leadership.

when the market leader is strong. By doing so, a social planner thus chooses to completely entrench the incumbent monopolist in the market. In the absence of the threat of a recession, this means that the economy will settle down in a no-growth equilibrium and enjoy permanently well-established market relationships. Secondly, whereas a social planner would choose to do more research compared to the decentralized equilibrium when the leading firm is weak, the reverse does not necessarily hold true in case of strong market leadership. A social planner indeed opts for less research compared to a market economy in state 2 for a wide range of μ , but may increase research activity relative to the decentralized equilibrium at higher values for μ . The reward effect is relatively strong in a social planner's economy with strong market leadership.

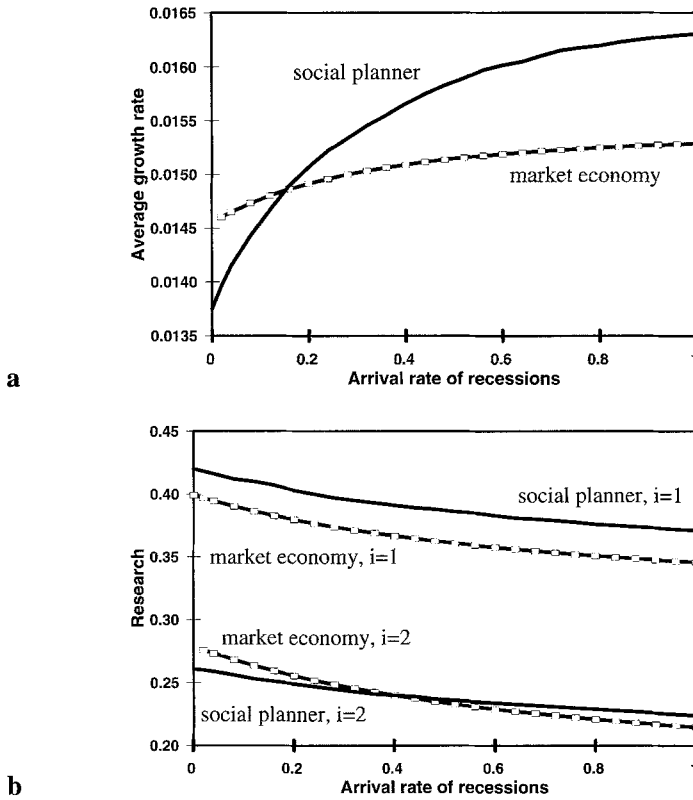


Fig. 4. Social planner: effect of recessions on economic growth (a) and research (b). $i = 1$, weak market leadership; $i = 2$, strong market leadership.

We secondly vary the arrival rate of a recession ν within the closed unit interval (setting $\mu = 0.5$) and study its implications for growth and research in Fig. 4. Figure 4a shows a positive relationship between the economy's average growth rate and ν . This result is explained by the fact that research activity is much higher when the monopolistic firm is a weak market leader (see Fig. 4b), and the economy will on average spend more time in the recessionary state as ν increases. However, Fig. 4b also shows that research efforts in both states of the economy decline when recessions become more likely, but this turns out not to alter the positive effect from recessions on economic growth in this example.

6 Implications for Policy

In the previous section we derived the optimal research program that a benevolent social planner would implement under the extreme assumption that this social planner can decide upon the optimal allocation of skilled labor across production and research activity. Here we relax this assumption and investigate the possibility of “finetuning” by the government through implementation of an optimal tax program.

A government can implement the social planner’s solution by using the appropriate tax instruments as follows. We concentrate the analysis on the steady-state interior solution. Let a social planner’s solution $(n_S^{(1)}, n_S^{(2)})$ be given. Denoting the tax rate on production workers in the monopolistic firm in state i by $\tau_P^{(i)}$, the optimality condition [Eq. (8)] now rewrites to (suppressing the subscript for the fundamental innovation, f)

$$(1 + \tau_P^{(i)})\omega^{(i)} = B^{(i)}\{F''(x^{(i)})x^{(i)} + F'(x^{(i)})\}. \quad (8')$$

Similarly, and denoting the tax rate on research workers in the research sector when the leading monopolist is in state i by $\tau_R^{(i)}$, the optimality condition [Eq. (6)] now reads as

$$(1 + \tau_R^{(i)})\omega^{(i)} = \gamma \tilde{V}^{(1)}\lambda. \quad (6')$$

Tax rates are not restricted to be positive, and effectively turn into subsidies when they are negative. The government is supposed to stick to a balanced-budget rule

$$0 = \tau_P^{(i)}(N - n^{(i)}) + \tau_R^{(i)}n^{(i)}. \quad (15)$$

Notice that this equation makes use of the fact that net wage payments to workers are equal in the two sectors for a given state of the economy i . It can be shown that the optimal tax rates are given by (see Canton and Uhlig, 1997, technical appendix, for details)

$$\tau_P^{(i)} = \frac{n^{(i)}}{N} \left(1 - \frac{\hat{\omega}_R^{(i)}}{\hat{\omega}_P^{(i)}}\right) \left[1 - \frac{n^{(i)}}{N} \left(1 - \frac{\hat{\omega}_R^{(i)}}{\hat{\omega}_P^{(i)}}\right)\right]^{-1}, \quad (16a)$$

$$\tau_R^{(i)} = \tau_P^{(i)}(1 - N/n^{(i)}), \quad (16b)$$

where $\hat{\omega}$ denotes gross wages. In words, Eq. (16b) (which directly follows from the government’s balanced-budget assumption) says that both tax rates are opposite in sign (since $n^{(i)}$ is strictly less than N for interior solu-

tions), and their mutual proportion is determined by the sectoral allocation of skilled workers: for a given tax (subsidy) on production labor, subsidies (taxes) on research workers increase when less people are engaged in research activity.

Let us return to some numerical examples from Sect. 3 to illustrate these policy implications. In the first example we assumed the absence of marginal leaps and recessions, so that we are back in the Aghion–Howitt world. Research intensity in decentralized equilibrium equals 0.31, whereas $n_S = 0.33$ is the socially desirable research effort. A tax on production labor of 2.24% in combination with a subsidy on research activity of 4.49% leads to an optimal solution in the market economy. In our second example (learning-by-doing), we allowed intermediate firms to strengthen their market position by building up a base of loyal customers. The arrival rate of marginal leaps (μ) was set at 0.5. In that example we found $n^{(1)} = 0.40$ and $n^{(2)} = 0.28$: the creation of a loyal-customer base by the intermediate firm discourages research activity by potential entrants. A benevolent social planner would choose $n_S^{(1)} = 0.42$ together with $n_S^{(2)} = 0.26$. A social optimum can be implemented in the decentralized economy by the following tax program: $\tau_P^{(1)} = 0.24\%$, $\tau_P^{(2)} = -0.69\%$, $\tau_R^{(1)} = -0.33\%$, $\tau_R^{(2)} = 1.93\%$. Thus, it is optimal to introduce a stochastic tax system in which the use of production labor is taxed when the market leader is weak and subsidized in case of strong market leadership, whereas research activity is subsidized when the leading monopolist is weak and taxed under a strong intermediate monopolist. At first glance, this may seem counterintuitive: production activity should be encouraged during good times, and discouraged during recessions and when the leading firm is weak. What is at work here is that the gains related to a particular state of the economy are optimally used. One can better tax production labor when the incumbent monopolist is weak, and subsidize research activity in order to fully exploit the temporary lower opportunity costs in terms of production foregone. Likewise, during a boom when the leading monopolist is particularly good at producing intermediate inputs, one can better give an additional incentive for production labor and discourage research activity.

The possibility of recessions was introduced in the third example (learning-by-doing and recessions) by setting the flow probability of recessions, ν , at 0.2. An equilibrium solution was found for $n^{(1)} = 0.38$ and $n^{(2)} = 0.26$. By setting $n_S^{(1)} = 0.40$ and $n_S^{(2)} = 0.25$, a social planner again increases research activity when the monopolist is weak and reduces R&D

when the market leader is strong (compared to the decentralized equilibrium without taxation). The optimal tax program is now given by $\tau_P^{(1)} = 1.16\%$, $\tau_P^{(2)} = 0.13\%$, $\tau_R^{(1)} = -1.72\%$, $\tau_R^{(2)} = -0.39\%$. For the case of weak market leadership, this result has a straightforward interpretation: too much production activity and too little research is going on, so that the former activity should be discouraged via taxation and the latter encouraged via subsidies. But when the market leader is strong one should actually tax production labor and subsidize research activity in order to reduce research intensity! What is at work here, is a general-equilibrium effect. The introduction of recessions implies that boom states become less likely, and the economy will more often be in a recessionary state. Compared to the decentralized equilibrium, the social planner needs to subsidize R&D when the leading monopolist is weak. This weak market leader has to pay taxes to finance the research subsidies. Since the market leader spends a larger fraction of its lifetime in a weak state (compared to the previous example), this may lower its value by a substantial amount. Since firms in the research sector expect substantially lower gains from innovative activity, they may actually decide to undertake less research activity than in a competitive equilibrium without taxation. This R&D fall might already be more than the social planner wants, so that research activity should be subsidized in a boom. This again lowers the value of the monopolistic firm, so that the social planner needs to stimulate R&D even more, and so on.

As a final example, we look at the case in which marginal leaps become more likely compared to the previous example by increasing the arrival rate of marginal innovations to 1 (holding the other parameters constant). A market equilibrium solution is given by $n^{(1)} = 0.40$ and $n^{(2)} = 0.275$. A social optimum is attained when $n_S^{(1)} = 0.42$ and $n_S^{(2)} = 0.284$. As before, a social planner increases research activity (in comparison with the decentralized equilibrium) when the intermediate firm is weak. But now the optimal research intensity when the leading monopolist is strong is higher than in the market economy without taxation. The optimal tax program that implements the social optimum in a decentralized economy is now given by $\tau_P^{(1)} = 1.83\%$, $\tau_P^{(2)} = 0.83\%$, $\tau_R^{(1)} = -2.52\%$, $\tau_R^{(2)} = -2.10\%$. Production activity is too high in the market economy for both states, and taxing production labor gives the appropriate incentives to establish the social optimum. Likewise, too little research is going on in both states without government intervention. Subsidizing research labor can restore the social optimum.

Table 1 summarizes the main findings from these examples.

Table 1. Optimal taxation

Model	$\tau_P^{(1)}$ (%)	$\tau_P^{(2)}$ (%)	$\tau_R^{(1)}$ (%)	$\tau_R^{(2)}$ (%)
Aghion and Howitt $\mu = 0, \nu = 0$	2.24		-4.49	
Learning-by-doing $\mu = 0.5, \nu = 0$	0.24	-0.69	-0.33	1.93
Learning-by-doing and recessions $\mu = 0.5, \nu = 0.2$	1.16	0.13	-1.72	0.39
$\mu = 1, \nu = 0.2$	1.83	0.83	-2.52	-2.10

7 Conclusion

Newly established firms often try to secure their market position by building up a base of loyal customers. Learning about customer needs or building up consumer recognition is a time-consuming process, but without such customer bases, these firms find themselves more vulnerable to attacks by competitors. While recessions may not destroy technological leadership, they may be harmful for such firm–customer relationships.

These ideas have been introduced within an Aghion–Howitt-type model of creative destruction. In the context of this model, recessions might be good for growth since they weaken the incumbent firm’s position, and thereby stimulate research by outside firms. The model allows for the extreme case where the leading firm can be so entrenched that growth ceases, unless a recession shakes up its customer base. We find a one-to-one relationship between the average growth rate and the cyclical variability, a U-shaped relationship between the average speed of building up good customer relationships and the average growth rate, and a positive relationship between the arrival rate of recessions and average growth. The optimal use of skilled labor by a benevolent social planner has been shown to exhibit larger reallocations between the intermediate monopolist and the research sector when the leading firm moves from one state to the other. It is finally shown that an appropriate stochastic tax program can restore the social planner’s solution. In some cases, general-equilibrium effects may generate interesting results, conflicting with intuition from a partial-equilibrium approach.

The analysis can be extended in several ways. Firstly, unemployment could be introduced into the model by allowing for search on the labor

market (cf. Aghion and Howitt, 1994). This would give a more plausible interpretation of recessions in our story. Secondly, it would be more realistic to have a richer sector structure than the simple structure of a single intermediate firm that was used here. Thirdly, our assumption that learning-by-doing is an exogenous stochastic event rules out the possibility of strategic behavior on the part of the incumbent monopolist. It would be interesting to introduce endogenous factors that affect the probability to become a strong market leader. These issues are left for future research.

Technical Appendix

1 The Maximization Problems

The f -th intermediate monopolist wants to maximize the value V_f of the firm. Let π_f denote the monopolist's profit. At any instant in time, the monopolist can be in two different states. Therefore, we find the following Bellman expressions:

$$V_f^{(1)} = \pi_f^{(1)} dt + e^{-r dt} \{ [1 - \lambda n_f^{(1)} dt] [\mu dt V_f^{(2)} + (1 - \mu dt) V_f^{(1)}] \}, \quad (\text{A1a})$$

$$V_f^{(2)} = \pi_f^{(2)} dt + e^{-r dt} \{ [1 - \lambda n_f^{(2)} dt] [\nu dt V_f^{(1)} + (1 - \nu dt) V_f^{(2)}] \}. \quad (\text{A1b})$$

In words, Eq. (A1a) says that when the intermediate firm is currently in the first state, it makes a profit $\pi^{(1)}$. The probability of still being a monopolist after a small time interval dt has elapsed is equal to $1 - \lambda n_f^{(1)} dt$. Within this interval, the (unconditional) probability of a marginal innovation is μdt . By the event of a marginal innovation the monopolist switches to the second state, and the firm's value is given by $V^{(2)}$. With probability $1 - \mu dt$ the firm does not make the transition to state 2 during the time interval, so that its value is still given by $V^{(1)}$. In Eq. (A1b) the monopolist is in the second state at time t , earning a profit $\pi^{(2)}$. Now, the probability of still being alive after a small time interval dt has elapsed is equal to $1 - \lambda n_f^{(2)} dt$. During this interval, the (unconditional) probability of a recession is νdt . A recession destroys firm-customer relationships, so that the monopolist switches back to the first state, and the firm's value is given by $V^{(1)}$. With probability $1 - \nu dt$ the firm does not suffer from a recession after the time interval has elapsed, so that its value is still given by $V^{(2)}$.

Exploiting $e^{-r dt} \approx 1 - r dt$ and leaving out higher-order terms, we

rewrite the Bellman equations to

$$(r + \lambda n_f^{(1)} + \mu) V_f^{(1)} = \pi_f^{(1)} + \mu V_f^{(2)} , \quad (\text{A2a})$$

$$(r + \lambda n_f^{(2)} + \nu) V_f^{(2)} = \pi_f^{(2)} + \nu V_f^{(1)} . \quad (\text{A2b})$$

It will be convenient to use the following matrix notation

$$\begin{bmatrix} r + \lambda n_f^{(1)} + \mu & -\mu \\ -\nu & r + \lambda n_f^{(2)} + \nu \end{bmatrix} \begin{bmatrix} V_f^{(1)} \\ V_f^{(2)} \end{bmatrix} = \begin{bmatrix} \pi_f^{(1)} \\ \pi_f^{(2)} \end{bmatrix} ; \quad (\text{A3})$$

or, abbreviated,

$$XV = \pi . \quad (\text{A4})$$

X is the 2×2 matrix from Eq. (A3), $V = [V^{(1)} V^{(2)}]'$, and $\pi = [\pi^{(1)} \pi^{(2)}]'$. (A3) and (A4) correspond to Eqs. (4) and (5) in the text.

2 Equilibrium

Step 1. Suppose $\tilde{V}^{(1)}$ is given. From Proposition 1, the transition scheme for fundamental innovations, Eqs. (2) and (6), and the definition of ω and \tilde{V} , in stationary equilibrium it holds that

$$\omega^{(i)} = \omega = \gamma \lambda \tilde{V}^{(1)} . \quad (\text{A5})$$

Step 2. Given ω , we find $x^{(i)}$ and $\tilde{\pi}^{(i)}$ from

$$x^{(i)} = [\alpha^2 B^{(i)} / \omega^{(i)}]^{1/(1-\alpha)} , \quad (\text{A6})$$

$$\tilde{\pi}^{(i)} = \frac{1 - \alpha}{\alpha} \frac{\omega^{(i)} x^{(i)}}{B^{(i)}} . \quad (\text{A7})$$

(A6) and (A7) follow from Eqs. (7) and (8), the Cobb–Douglas production function, and the expression for the monopolist's profits.

Step 3. Given $x^{(i)}$, the number of researchers follows from the condition for labor-market equilibrium

$$n^{(i)} = N - x^{(i)} / B^{(i)} . \quad (\text{A8})$$

Step 4. From Proposition 1, Steps 2 and 3, and Eqs. (4) and (5) we have

$$X = \begin{bmatrix} r + \mu + \lambda \left(N - \left(\frac{\alpha^2}{\omega} \right)^{\frac{1}{1-\alpha}} \right) & -\mu \\ -\nu & r + \nu + \lambda \left(N - \left(\frac{\alpha^2 \delta^\alpha}{\omega} \right)^{\frac{1}{1-\alpha}} \right) \end{bmatrix}, \quad (\text{A9})$$

and

$$\pi = \left[\frac{1}{\delta^{\alpha/(1-\alpha)}} \right] \frac{1-\alpha}{\alpha} \alpha^{2/(1-\alpha)} \left(\frac{1}{\omega} \right)^{\alpha/(1-\alpha)}. \quad (\text{A10})$$

Using these four steps, we proceed with our proof of Proposition 2 by defining

$$\tilde{V}^{(2)} = \tilde{V}^{(1)} + \Lambda. \quad (\text{A11})$$

After some substitutions we end up with two equations in two unknowns

$$\left[r + \lambda \left(N - \left(\frac{\alpha^2}{\omega} \right)^{1/(1-\alpha)} \right) \right] \frac{\omega}{\gamma \lambda} = \frac{1-\alpha}{\alpha} \alpha^{2/(1-\alpha)} \left(\frac{1}{\omega} \right)^{\alpha/(1-\alpha)} + \mu \Lambda, \quad (\text{A12})$$

$$\begin{aligned} & \left[r + \lambda \left(N - \left(\frac{\alpha^2 \delta^\alpha}{\omega} \right)^{1/(1-\alpha)} \right) \right] \frac{\omega}{\gamma \lambda} \\ &= \frac{1-\alpha}{\alpha} \alpha^{2/(1-\alpha)} \left(\frac{\delta}{\omega} \right)^{\alpha/(1-\alpha)} - \left[r + \nu + \lambda \left(N - \left(\frac{\alpha^2 \delta^\alpha}{\omega} \right)^{1/(1-\alpha)} \right) \right] \Lambda. \end{aligned} \quad (\text{A13})$$

Subtracting (A13) from (A12) gives after some manipulation

$$\frac{\omega}{\Lambda} = \gamma \frac{(r + \nu + \lambda N + \mu) \omega^{1/(1-\alpha)} - \lambda (\alpha^2 \delta^\alpha)^{1/(1-\alpha)}}{(1 + \gamma(1-\alpha)/\alpha)(\delta^{\alpha/(1-\alpha)} - 1) \alpha^{2/(1-\alpha)}}. \quad (\text{A14})$$

Multiplying (A14) with $\omega^{1/(1-\alpha)}/\Lambda$ finally completes the proof of Proposition 2: There are in general two solutions for ω , given by

$$\bar{\omega}_{1,2} = \left(\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a} \right)^2 - \frac{c}{a}} \right)^{1-\alpha},$$

where

$$\begin{aligned}
 a &= (r + \lambda N)\kappa(r + v + \lambda N + \mu)/(\gamma\lambda), \\
 b &= (r + \lambda N)\kappa\lambda(\alpha^2\delta^\alpha)^{1/(1-\alpha)}/(\gamma\lambda) \\
 &\quad + \alpha^{2/(1-\alpha)}\kappa(r + v + \lambda N + \mu)\left[\frac{1}{\gamma} + \frac{1-\alpha}{\alpha}\right] + \mu, \\
 c &= \alpha^{2/(1-\alpha)}\kappa\lambda(\alpha^2\delta^\alpha)^{1/(1-\alpha)}\left[\frac{1}{\gamma} + \frac{1-\alpha}{\alpha}\right], \\
 \kappa &= \frac{\gamma}{(1 + \gamma(1-\alpha)/\alpha)(\delta^{\alpha/(1-\alpha)} - 1)\alpha^{2/(1-\alpha)}}.
 \end{aligned}$$

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