## Midterm Exam: New Employee Tax Credits

## You have 1:30 to complete the exam

Consider a long-lived firm/employer. In year $t$, the firm hires labor in the amount $n_{t}$, at wage rate $w$, thereby producing goods in the amount $f\left(n_{t}\right)$. The firm's customers pay a price one for each unit of those goods, and the wage rate $w$ does not vary over time. The firm's cash flows for year $t$ are therefore $f\left(n_{t}\right)$ - wn $n_{t}-P_{t}$, where $P_{t}$ denotes payroll taxes owed in year $t$, and $w n_{t}$ is total payroll spending for that year. You may assume that employees pay zero payroll taxes.

The firm chooses the amount of labor to hire at each date with the objective of maximizing the present value of its cash flows, using a constant interest rate $r$. [Hint: the present value of cash flows is the current cash flow, plus next year's cash flow divided by ( $1+r$ ), plus the following year's cash flow divided by $(1+r)^{2}$, etc.]
(1) Calculate the demand for labor, assuming that the year t payroll tax is simply a constant fraction $\tau$ of payroll spending for that year. What is the effect of the tax rate on the amount of labor that is hired?

The first step is to write down a formula for payroll taxes: $P_{t}=\tau w n_{t}$. The employer recognizes that this is how payroll taxes are determine, so we substitute the tax formula into his cash flow formula:

$$
f\left(n_{t}\right)-w n_{t}-P_{t}=f\left(n_{t}\right)-(1+\tau) w n_{t}
$$

$f^{\prime}\left(n_{t}\right)$ is the benefit from hiring a bit more labor, and $(1+\tau) w$ is the cost. See also page 2 or equation (6) in "The Behavioral Implications of Tax Distortions." Inverting the marginal product of labor schedule, we have:

$$
n_{t}=g((1+\tau) w)
$$

The function $g$ has a negative derivative because the marginal product of labor slopes down (try drawing the marginal product of labor schedule $f^{\prime}(n)$ in the $[n, w]$ plane). The greater is the payroll tax rate, the less labor is hired.
(2) President Obama has proposed that employers be given a payroll tax credit for increases in their payroll spending (hereafter, "new-hires tax credit"). Let $\sigma$ denote the credit rate. For a year that the credit is in place, write down a formula for total payroll taxes $P_{t}$.

If the employer spends $w n_{t}$ on payroll in year $t$, and $w n_{t-1}$ in year $t-1$, then his increase in payroll spending for year $t$ is:

$$
w n_{t}-w n_{t-1}
$$

Payroll taxes are $P_{t}=\tau w n_{t}-\left(w n_{t}-w n_{t-1}\right) \sigma$, where the second term is the credit for payroll increases.
(3) Continuing from part (2): At the moment, employers get no credit for increasing payroll
spending. Assume that the credit is expected to take effect in 2012, and last only for that year. Calculate the demand for labor in 2011.

As usual, the second step is to substitute the tax formula into the firm's objective. Making the substitution, the present value of cash flows is:

$$
\begin{align*}
& f\left(n_{2011}\right)-w n_{2011}-\tau w n_{2011}+\frac{f\left(n_{2012}\right)-w n_{2012}-\tau w n_{2012}+\left(w n_{2012}-w n_{2011}\right) \sigma}{(1+r)} \\
& +\sum_{t=2012}^{\infty} \frac{f\left(n_{t}\right)-w n_{t}-\tau w n_{t}}{(1+r)^{t-2011}} \tag{1}
\end{align*}
$$

The first term is the cash flows for 2011, the second term the cash flows from 2012, and the last term is the cash flows for all years thereafter. Notice that 2011 labor appears in four places: one of them a benefit and the other three costs. The marginal cost of hiring labor in 2011 is therefore:

$$
\begin{equation*}
w+\tau w+\frac{w \sigma}{(1+r)}=\left[1+\tau+\frac{\sigma}{(1+r)}\right] w \tag{2}
\end{equation*}
$$

Firms demand labor up to the point where marginal product equals marginal cost:

$$
\begin{equation*}
n_{2011}=g\left(\left[1+\tau+\frac{\sigma}{(1+r)}\right] w\right)<g((1+\tau) w) \tag{3}
\end{equation*}
$$

(4) Continuing from part (3): What is the employment impact of announcing, in 2011, a new-hires tax credit that takes effect in 2012? What is the impact of that news on plans for 2012 employment?

Before employers knew that a new-hires tax credit was coming, their 2011 labor demand was $g((1+\tau) w)$ (see part (1) above). But knowing that the credit is coming, their 2011 labor demand is given by equation (3) above, which is less than $g((1+\tau) w)$. Thus, the announcement of the credit causes them to demand less labor in 2011 than they would have if they had not expected a new-hires tax credit to be coming.

To find the marginal cost of 2012 labor, inspect the cash flow (1) for 2012 terms. The marginal cost is $(1+\tau-\sigma) w$. It follows that 2012 demand for labor is $g((1+\tau-\sigma) w)$ rather than the $g((1+\tau) w)$ that it would be without the credit. The credit thereby increases 2012 labor demand.
(5) Suppose that the new-hires tax credit, to begin in 2012, is expected to be in place forever. What is the impact of the credit on 2012 employment?

We need to repeat the second step of substituting the tax formula into the firm's objective, recognizing that the credit lasts forever:

$$
\begin{align*}
& f\left(n_{2011}\right)-w n_{2011}-\tau w n_{2011}+\frac{f\left(n_{2012}\right)-w n_{2012}-\tau w n_{2012}+\left(w n_{2012}-w n_{2011}\right) \sigma}{(1+r)} \\
& +\sum_{t=2012}^{\infty} \frac{f\left(n_{t}\right)-w n_{t}-\tau w n_{t}+\left(w n_{t}-w n_{t-1}\right) \sigma}{(1+r)^{t-2011}} \tag{4}
\end{align*}
$$

Now 2012 labor appears in five places: two benefits in 2012, two costs in 2012, and one cost in 2013. The 2012 demand for labor is:

$$
\begin{equation*}
n_{2012}=g\left(\left[1+\tau-\frac{\sigma r}{1+r}\right] w\right) \tag{5}
\end{equation*}
$$

where the term in parentheses is the marginal cost of hiring in 2012. This demand is greater than it would be without the permanent credit, but not nearly as much as it is with a temporary credit because $r /(1+r)$ is pretty close to zero.
(6) Return to your answer from part (4). How many jobs (in millions) would be created or lost in 2011 by the announcement of the new hires tax credit. [Hint: first make a list of model parameters you need to know. Then go through your lecture notes to find estimates of those parameters]

We need to take the difference between the left- and right-hand sides of the inequality (3): the left has 2011 employment with credit announced, and the right has 2011 employment without it announced. The most important things we need to know are: (a) the slope of the labor demand function $g$, (b) the payroll tax rate $\tau$, and (c) the proposed credit rate $\sigma$. In order to give an answer in millions of jobs, rather than proportional terms, we need to know how many people were employed in 2011.

If the aggregate production function were Cobb-Douglas in employment with exponent 0.7 , then the elasticity of the labor demand curve would be $-1 / 0.3$ (just look at the marginal product schedule and invert it). Now take the ratio of the two sides of the inequality (3):

$$
\begin{equation*}
\frac{g\left(\left[1+\tau+\frac{\sigma}{(1+r)}\right] w\right)}{g((1+\tau) w)}=\left(1+\frac{\sigma}{(1+\tau)(1+r)}\right)^{-1 / 0.3} \tag{6}
\end{equation*}
$$

Using $\tau$ of 0.09 (roughly the employer payroll tax rate) and a credit rate of 0.09 (part of President Obama's proposal was to let payroll increases go tax free), and an annual interest rate of 0.05, I get

$$
\begin{equation*}
\frac{g\left(\left[1+\tau+\frac{\sigma}{(1+r)}\right] w\right)}{g((1+\tau) w)}=\left(1+\frac{0.09}{(1.09)(1.05)}\right)^{-1 / 0.3} \approx 0.777 \tag{7}
\end{equation*}
$$

It's OK if you used different numbers, as long as you explained. Actual employment in 2011 was about 130 million - if you take that as the numerator of (7), then employment (in millions) would have been $130 / 0.777$ = 167 million. In other words, employment was 37 million less than it would have been without the announcement!

37 million is quite a large number, and you might speculate that this calculation misses something. For example, people are probably unsure as to whether the tax credit will ever be in place, and to the amount of the credit if it were in place. Our calculation holds the wage constant, but a change in the demand for labor by 37 million would certainly have an effect on wages (recall the "incidence parameter" from our analysis of the seasons and of tax incidence - likely 75 percent or more of the tax credit's effect would be to raise wages rather than employment). By incorporating some of these factors, one might expect the impact to be 1 or 2 million jobs rather than 37 million, but that's still a lot!

True, False, or Uncertain. Answer true, false or uncertain to each of the statements below. Your grade will be based on your justification of your answer.
(7) It would be better if employers paid all the payroll taxes, rather than requiring employees to pay half.

False. See our "equivalence formulas" - it doesn’t really matter who is liable for the payroll tax as long as wages can adjust (the federal minimum wage is one case where wages cannot adjust).
(8) If President Obama were to reduce the labor income tax, that would increase the "Compensation of Employees" shown in the National Accounts.
"Compensation of employees" is the employee income before taxes are deducted. Employee income tends to rise from the substitution effect. Employee income tends to fall from the wealth effect (of which there would be one only to the extent that Obama paid for the tax cut by reducing government purchases of goods and services rather than reducing transfers).

