

### Mortgage Modification Arithmetic

Submit your answers by email to [psets@caseybmuligan.com](mailto:psets@caseybmuligan.com)

Consider a homeowner/worker/consumer whose mortgage obligates him to pay  $x$  per year for  $T > 5$  years in order to occupy his house. Denote time  $t = 0, 1, 2, \dots$ , in years, and assume that this homeowner has incomes  $\{y_0, y_1, \dots\}$ . Let  $c$  denote the present value of what the homeowner can spend on things other than his mortgage.

Using a constant annual real interest rate  $r$  to future discount cash flows, the present value budget constraint for the consumer equates the combination of non-mortgage spending  $c$  and mortgage spending  $x$  to the present value of income:

$$c + x + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots + \frac{x}{(1+r)^T} = y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + \dots$$

- (1) Draw the above budget constraint in the  $[y_0, c]$  plane, holding future incomes  $\{y_1, y_2, \dots\}$  constant.
- (2) The U.S. government has a “mortgage modification” program that reduces homeowners’ mortgage payments during years 1 – 5 to be 31 percent of their income  $y_0$ . If the original mortgage payment  $x$  were already less than  $0.31y_0$ , then mortgage payments are left at  $x$ . Write down a formula for the modified mortgage payment  $m$  in years 1 – 5.
- (3) Write down the present value budget constraint for a homeowner that recognizes the possibility of such modifications. [Hint: replace some of the  $x$ ’s in the consumer budget constraint with the formula for  $m$  you found above]
- (4) Draw the budget constraint in the  $[y_0, c]$  plane, holding future incomes constant. Is it possible for a household to reduce its income and increase its spending?
- (5) Suppose that homeowners work only for the purpose of having money to spend  $c$ . What do you think the mortgage modification program does to their income in period zero?
- (6) Without government programs like this, would mortgage lenders modify loans (that is, reduce the borrowers payment below the originally contracted amount  $m$ )? If so, would the lenders modify differently than your derived in part (2)?