

# COLLECTIVE GOALS

# THE PROJECT

Many normative frameworks are compelling

Reasonable people can disagree

Perhaps, even though we can't agree on normative principles, we can agree on procedural principles

Find an appealing *aggregation rule* that takes individual preferences and returns collective preferences

# HOW TO FIND AN APPEALING PROCEDURE?

Identify normatively appealing conditions we want the aggregation rule to satisfy

Rule out aggregation rules that fail these conditions

Choose a procedure from the set of remaining aggregation rules

# UNIVERSAL DOMAIN

You don't know what issues will face your society in the future

Want the aggregation rule to perform sensibly regardless of what issue comes up

Aggregation rule must satisfy normative criteria for *all* possible collections of alternatives and rational individual preferences

# RATIONAL PREFERENCES

Let  $A$  be the set of alternatives

**Completeness:** For any  $x, y \in A$  one of the following is true

- ▶  $x \succ y$
- ▶  $y \succ x$
- ▶  $x \sim y$

**Transitivity:** For any  $x, y, z \in A$

- ▶ If  $x \succ y$  and  $y \succ z$ , then  $x \succ z$
- ▶ If  $x \sim y$  and  $y \sim z$ , then  $x \sim z$

# MAJORITY RULE AGGREGATION: AN EXAMPLE

3 alternatives: Low, Medium, or High Taxes

3 individuals

$$H \succ_1 M \succ_1 L$$

$$M \succ_2 H \succ_2 L$$

$$L \succ_3 M \succ_3 H$$

Social preferences under majority rule aggregation

$$M \succ_{S,maj} H \succ_{S,maj} L$$

# BORDA COUNT AGGREGATION: AN EXAMPLE

Same policies and individuals

$$\mathbf{H} : 2 + 1 + 0 = 3$$

$$\mathbf{M} : 1 + 2 + 1 = 4$$

$$\mathbf{L} : 0 + 0 + 2 = 2$$

Social preferences under Borda count aggregation

$$M \succ_{S, bor} H \succ_{S, bor} L$$

# TRANSITIVITY

If  $x \succ_S y$  and  $y \succ_S z$ , then  $x \succ_S z$

This is a basic standard of coherence for social preferences

# MAJORITY RULE NEED NOT RETURN TRANSITIVE SOCIAL PREFERENCES

3 alternatives ( $x, y, z$ ) and 3 people (1, 2, 3)

Individual preferences

$$x \succ_1 y \succ_1 z$$

$$y \succ_2 z \succ_2 x$$

$$z \succ_3 x \succ_3 y$$

Social preference

$$x \succ_{S,maj} y \quad y \succ_{S,maj} z \quad z \succ_{S,maj} x$$

# UNANIMITY

If every individual in society prefers  $x$  to  $y$ , then the social preference should prefer  $x$  to  $y$

This is a basic standard of the aggregation rule actually representing the views of the individuals in society

# AMENDMENT PROCEDURE AND UNANIMITY

5 alternatives ( $x^1, \dots, x^5$ ) and 3 people (1, 2, 3)

$$x^2 \succ_1 x^1 \succ_1 x^4 \succ_1 x^3 \succ_1 x^5$$

$$x^3 \succ_2 x^2 \succ_2 x^1 \succ_2 x^4 \succ_2 x^5$$

$$x^5 \succ_3 x^1 \succ_3 x^4 \succ_3 x^3 \succ_3 x^2$$

$x^4$  wins, but  $x^1$  is unanimously preferred

# INDEPENDENCE OF IRRELEVANT ALTERNATIVES

Social preference with respect to  $x$  and  $y$  should depend only on individual preferences over  $x$  and  $y$

Shouldn't be able to manipulate social preference by introducing new alternatives

Also a kind of basic coherence or consistency

# BORDA COUNT AND IIA

$$x \succ_{1,2} y \succ_{1,2} z$$

$$y \succ_3 z \succ_3 x$$

$$y \succ_4 x \succ_4 z$$

$y$  is Borda winner with 6 points

Add  $w$  while holding all other preferences fixed

$$x \succ_{1,2} w \succ_{1,2} y \succ_{1,2} z$$

$$y \succ_3 z \succ_3 x \succ_3 w$$

$$w \succ_4 y \succ_4 x \succ_4 z$$

$x$  is the Borda winner with 8 points

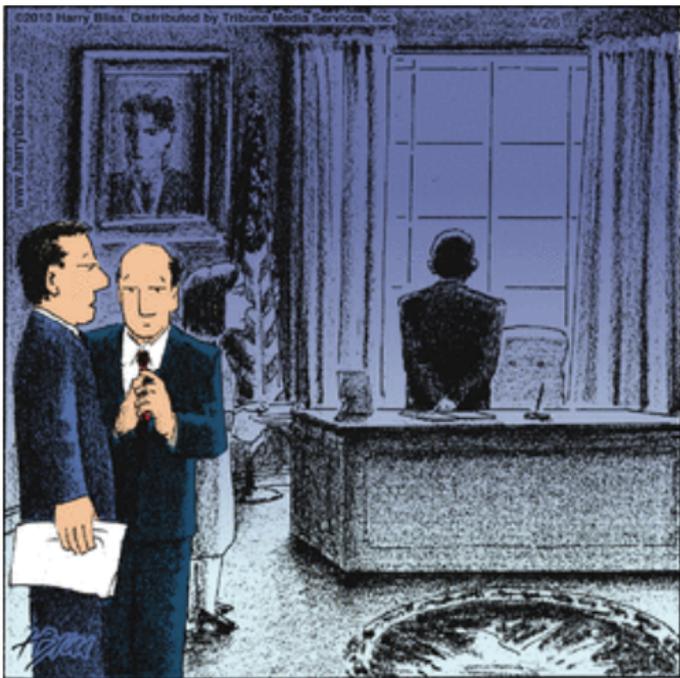
# ARROW'S THEOREM

Suppose there are at least 3 alternatives and at least 2 people

Only one aggregation rule satisfies

- ▶ Universal Domain
- ▶ Transitivity
- ▶ Unanimity
- ▶ Independence of Irrelevant Alternatives

That rule is dictatorship



"Mr. President, the people have spoken ...  
and they're not making any sense."

# WHAT TO DO IN LIGHT OF ARROW'S THEOREM

Abandon the idea of the public interest?

Weaken our ambitions a bit more

- ▶ Let's consider a few ways

# THE CASE FOR MAJORITY RULE

Although we've seen majority rule can be problematic, there are two settings in which we can make a case for majority rule as a principled aggregation procedure

Let's consider them in turn

- ▶ Only two alternatives
- ▶ Restricted domain

## ONLY TWO ALTERNATIVES

Arrow's theorem applies when there are at least three alternatives under consideration

*Anonymity*: When the preferences of two people are switched, social preference doesn't change

*Neutrality*: When everyone's preference between two alternatives flips, rank ordering of those alternatives flips

*Positively responsiveness*: Suppose society is indifferent between  $x$  and  $y$ . All else equal, if one person who preferred  $y$  to  $x$  moves  $x$  up in her preference order, then society strictly prefers  $x$  to  $y$ .

# MAY'S THEOREM

Suppose society has at least two people considering exactly two alternatives. The only aggregation procedure that satisfies universal domain, anonymity, neutrality, and positive responsiveness is **simple majority rule**—if a plurality of individuals strictly prefer one alternative to the other, then so does society and otherwise society is indifferent between the two alternative.

# RESTRICTING THE DOMAIN

Arrow's theorem applies when all preference profiles are possible (universal domain)

Suppose we think certain profiles are unrealistic?

Let's focus on a particular, substantively motivated, domain restriction

# AN IDEOLOGY

For any collection of alternatives, line them up according to an *ideological order*

Write  $a_1 < a_2 < a_3$  for  $a_1$  is to the left of  $a_2$  which is to the left of  $a_3$

For any set of alternatives there are many possible ideological orders

Say that  $i$  is more conservative than  $j$  under a given ideological order if for any  $a_1 < a_2$

- ▶ If  $j$  likes  $a_2$  at least as much as  $a_1$ , then so does  $i$

Define *strictly more conservative* in the natural way

## DESCRIBED BY AN IDEOLOGY

An issue is *described by an ideology* if we can find some ideological order such that individuals can be rank ordered from liberal to conservative

This isn't always true

- ▶ For example, the Condorcet Triple is not described by an ideology

Substantively, an issue is described by an ideology if we agree on a single dimension that describes the issue

- ▶ If the richer you are, the less you like redistribution, then tax policy is described by an ideology

# THE MEDIAN VOTER

For any issue described by an ideology, majority rule will yield coherent social preferences

Line people up from liberal to conservative. There is a median person (called the *median voter*)

Consider  $x > y$

- ▶ If the median voter prefers  $x$  to  $y$ , so does everyone who is more conservative than the median (a majority)
- ▶ If the median voter prefers  $y$  to  $x$ , so does everyone who is more liberal than the median (a majority)

# THE MEDIAN VOTER THEOREM

Suppose society has an odd number of people and is considering an issue that is described by an ideology. Then social preferences under majority rule are identical to the preferences of the median voter. That is, society prefers an alternative  $x$  to an alternative  $y$  if and only if the median voter prefers  $x$  to  $y$ .

- ▶ If we restrict the domain to issues described by an ideology, majority rule is a coherent aggregation rule

# INTENSITY OF PREFERENCE

Arrow's theorem only makes use of ordinal information

- ▶ I like  $x$  better than  $y$  better than  $z$

But sometimes we might also have information about

- ▶ *Intensity of preferences*: My preference for  $x$  over  $y$  is much stronger than my preference for  $y$  over  $z$
- ▶ *Interpersonal comparisons*: The choice between  $x$  and  $y$  matters a lot more to me than it does to you

With this kind of information, we can get back to arguments for things like utilitarianism

# AGREEMENT

Look for situations in which it is likely that any sensible rule will yield the same social choice

Suppose we all agree on certain policies

This is only a useful thought if there are plausible situations in which agreement might arise

Thinking about how we might construct such situations is the topic of the next lecture