Territorial Conflict over Endogenous Rents*

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Abstract

I study a model of conflict over territories from which rents are endogenously generated. Territorial conquest affects market power and, thus, rents. As such, there is an endogenous relationship between conflict behavior and economic behavior. Consistent with standard intuitions, changes to economic conditions that increase market power or market size at all territories lead to a positive association between rents and conflict. However, contrary to these same intuitions, changes in local economic conditions at a territory under dispute can lead to a negative, positive, or non-monotone association between rents and conflict. The local comparative statics show that shocks at one territory have effects on violence at other territories and that the sign of these effects is heterogeneous. These local comparative statics also facilitate a theoretical exploration of the sign and magnitude of the bias associated with standard empirical strategies for estimating the effect of economic shocks on conflict outcomes.

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In many settings, armed groups compete for control of territory that can be used to extract economic rents.[1]

Some of the most significant such violence over the last decade in Afghanistan occurred in Helmand Province, a Taliban stronghold and Afghanistan's leading poppy producer. The United Nations Office of Drugs and Crime reports that local leaders use control over these territories and transshipment routes to extract economic rents by levying taxes on drug traffickers, poppy farmers, and owners of heroin laboratories.[2]

Over 500 people were murdered in Chicago in 2012. "Most of Chicago's violent crime," according to he head of the DEA for the five-state region that includes Illinois, "comes from gangs trying to maintain control of drug-selling territories."[3] Investigative journalist John Lippert reports that the gangs are motivated precisely by access to territorial rents: "[i]f you want to expand your sales, you have to expand your street corners. You know, you have to physically take street corners, which is a violent act."[4] A similar story of territorial conquest for economic rent extraction is told about violent conflict among Brazil's drug gangs for control of the *favelas* (Lessing, 2013).

From March through June of 2009 violence in the Mexican state of Michoacán more than tripled, reaching an average 67 drug-related homicides per month. The proximate cause was a war over territory. Los Zetas, in the midst of splitting from the Gulf Cartel, sought to wrest control over Michoacán and its valuable transshipment routes from La Familia. Such territorial conflicts reached their apex in 2011 and 2012 when over 10,000 people per year lost their lives as a result of the fight between two of the largest Mexican drug trafficking organizations—Los Zetas and the Sinaloa Cartel—for control of transshipment routes ranging from Veracruz, to Guadalajara, to Nuevo Laredo (Rios, 2013).

[1] For theoretical models of conflict over economic rents, see, for example, Hirshleifer (1991); Grossman (1999); Hafer (2006); Fearon (2008*a*,*b*); Chassang and Padro i Miguel (2009); Besley and Persson (2011); Dal Bó and Dal Bó (2011). Carter (2010) and Goemans and Schultz (2017) discuss the extensive scholarship territorial conflict with non-economic motivations (e.g., irredentism, security).

[2] United Nations Office on Drugs and Crime. "The Global Afghan Opium Trade: A Threat Assessment." July, 2011. `http://www.unodc.org/documents/data-and-analysis/Studies/Global_Afghan_Opium_Trade_2011-web.pdf`

[3] John Lippert. "Heroin Pushed on Chicago by Cartel Fueling Gang Murders." *Bloomberg Markets Magazine*. September 16, 2013. http://www.bloomberg.com/news/2013-09-17/heroin-pushed-on-chicago-by-cartel-fueling-gang-murders.html

[4] "Probing Ties Between Mexican Cartel And Chicago's Violence." *National Public Radio*. http://www.npr.org/2013/09/17/223309103/probing-ties-between-mexican-drug-cartel-and-chicagos-violence

Motivated by such conflicts, a recent empirical literature is increasingly interested in the relationship between territorial control, economic rents, and violence. (See, for example, Angrist and Kugler (2008); Castillo, Mejia and Restrepo (2013); Mejia and Restrepo (2013); Dube, García-Ponce and Thom (2016); Dell (2015); Caselli, Morelli and Rohner (2015).)

I propose a model to investigate such relationships. The model makes three types of contributions. First, it yields testable hypotheses about the effects of market power and market size on conflict outcomes. Second, it facilitates a theoretical exploration of standard empirical identification strategies. Third, it highlights the conceptual value of endogenizing the economic returns to territorial conquest.

Market Size and Market Power I study two types of comparative statics regarding the effects of market power and market size. The global comparative statics show that when market power or size increase at all territories, expected violence increases. The comparative statics regarding the effect of variation in local economic conditions on conflict are more interesting. Unlike the global comparative statics, local economic shocks at a disputed territory can create a negative, positive, or non-monotone association between rents and violence. To get a sense of why there could be a negative association, consider a shock to market size surrounding some disputed territory. An increase in local market size increases the marginal costs to raising prices (in terms of foregone demand) for the groups that control surrounding territories. Hence, as local market size at some territory increases, prices at the surrounding territories decrease, which spills over into lower prices at all territories. While this price decline tends to reduce all groups' rents, the rents decrease more slowly for whichever group ends up with control over the shocked territory, since increased market size also has a direct positive effect on demand at that territory. As a consequence, the returns to territorial conquest may be increasing in local market size. Hence, even though the shock decreases all groups' rents, it increases expected violence. More generally, economic shocks at one territory have differing effects on conflict at different territories, creating scope for a variety of relationships between rents and conflict.

Relationship to Empirical Literature The model has at least three implications relevant for empirical scholarship on conflict. First, empirical work often assumes that territorial conflict is expected to increase with rents. But that intuition comes from thinking about changes akin to my global comparative statics. The model here suggests that the relationship is more varied when we study local shocks. As the empirical literature becomes increasingly concerned with identification, this is precisely the kind of variation being studied.

Second, because the model has economic spillovers, local shocks at one territory affect violence at other territories in subtle ways. The fact that these spillovers can be negative, positive, or even non-monotone suggests that estimates of the average effects of local economic shocks could mask interesting heterogeneity.

Third, these results on heterogeneous spillovers are relevant for thinking about the difference-in-differences identification strategy used in many studies estimating the effect of economic shocks on conflict.[5] While it is well known that spillovers bias difference-in-differences, the model goes one step further—allowing empirical and theoretical research to constructively engage by using theory to explore the sign and magnitude of the resulting bias. In the case of a positive shock to market size, the model predicts that difference-in-differences yields overestimates—increased market size increases violence at the shocked territory and decreases violence at other territories. In the case of shocks to local market power, matters are more complicated. Difference-in-difference may over- or under-estimate the effect, and which it does depends both on which territories are used as a baseline of control and on the magnitude of the shock. Hence, the sign and magnitude of the bias are probably unknowable by the empirical researcher.

The model also suggests that the standard practice of excluding neighboring territories in response to concerns over spillovers may, in some circumstances, increase rather than decrease the bias from difference-in-differences.

Relationship to the Theoretical Literature The theoretical conflict literature is vast and I do not attempt to summarize it. But it is worth noting that all of the predicted relationships in the model are driven by the fact that the value of territorial conquest is determined endogenously by future economic behavior, which, in turn, depends on market power and market size. Hence, the model highlights, in one setting, the value of endogenizing the economic returns to conflict for understanding how conflict plays out.[6]

[5]See, among others, Deininger (2003); Angrist and Kugler (2008); Brückner and Ciccone (2010); Hidalgo et al. (2010); Besley and Persson (2011); Dube and Vargas (2013); Bazzi and Blattman (2014); Dube, García-Ponce and Thom (2016); Maystadt and Ecker (2014); Mitra and Ray (2014); Castillo, Mejía and Restrepo (2018). A related literature looks at the effect of local development aid on local conflict (e.g., Berman, Shapiro and Felter, 2011; Crost, Felter and Johnston, 2014).

[6]For models that consider other aspects of the two-way relationship between economic and conflict outcomes, see, Fearon (2008*b*); Besley and Persson (2010); Rohner, Thoenig and Zilibotti (2013).

1 The Model

There are four fixed territories, labeled $A-D$, located at equal intervals on the perimeter of a circle. The territories are arrayed in alphabetical order (so territory D is contiguous with territories A and C). Each territory is controlled by one of two groups, 1 and 2. A population of mass N is located uniformly on the perimeter of the circle.

The game is played as follows.

1. Nature chooses one territory to become *vulnerable*.

2. Each group, i, chooses an amount, $a_i \in \mathbb{R}_+$, to invest in fighting for control of the vulnerable territory.

3. At the end of the conflict either the territory is still controlled by its original owner or has changed hands. Groups then set prices for the single good traded in the economy. A group can set a different price at each territory it controls. The price at territory j is $p_j \in [0,1]$.

4. Each population member decides whether and from which territory to buy the good.

Conflict is modeled as an all-pay auction (Krishna and Morgan, 1997; Epstein and Gang, 2007). Call the initial holder of a vulnerable territory the *defender* and the other group the *attacker*. If one of the groups involved in fighting invests strictly more than the other, it wins the territory.[7]

Each population member gets a benefit of 1 from consuming the good. Population members bear linear transportation costs, $t \in (0,1]$. If a population member buys the good for price p from a territory at distance x from her location, her payoff is $1-p-tx$. If she doesn't buy the good, her payoff is zero.

If a group makes revenues r and invests a in conflict, its payoff is $r-a$.

I will primarily be interested in the amount of *observed violence*. Say that violence is observed if both groups make a positive investment. If violence is observed, it is the sum of the investments:

$$v = \begin{cases} a_1 + a_2 & \text{if} \quad \min\{a_1, a_2\} > 0 \\ 0 & \text{else.} \end{cases}$$

The solution concept is subgame perfect Nash equilibrium.

[7]Since ties never occur in equilibrium, the tie breaking rule is irrelevant.

1.1 Comments on the Model

I briefly discuss some assumptions and matters of interpretation.

Most important is the interpretation of transportation costs and market size. Transportation costs, because they create imperfect competition, are the source of market power in the model. In the case of drug gangs that control street corners in the United States or *favelas* in Brazil, transportation costs can be understood as a model of consumers' search and travel costs for finding alternative suppliers. Afghan groups often tax travel on roads they control and charge for protection services.[8] The associated market power depends on the availability of alternative routes, which are reasonably modeled as transportation costs. For Mexican drug transshipment, and in some other potential applications, market power derives from sources that map less cleanly onto transportation costs. Even so, the model may provide some insight if we think of transportation costs as a metaphor for market power more generally.

To relate transportation costs more directly to the empirical literature, consider Dube, García-Ponce and Thom's (2016) study of the effect on violence of local economic shocks in Mexico that derive from weather related changes in US maize production. In this setting, criminal organizations control territory from which they extract rents from local farmers producing illegal crops. A decrease in maize prices reduces farmers' outside options (especially in locations particularly suitable for maize production) and, thus, increases the rents criminal groups can extract from them. This is similar to the way transportation costs work in the model. An increase in transportation costs reduces the viability of a consumers' outside option (i.e., purchasing from a different territory) and, thus, allows more rents to be extracted.

Market size in the model corresponds to factors in the world that scale the rents available to be extracted. For instance, Angrist and Kugler (2008) argue that the disruption in the "air bridge" from Peru and Bolivia to Colombia was a shock to demand for Colombian coca. They use this shock to study the effect of a demand increase on drug related violence. An increase in market size in the model is precisely such an increase in demand. Castillo, Mejía and Restrepo (2018) use a similar enforcement-based demand shock to study the relationship between economic factors and violent conflict in Mexico.

Of course, one could also ask a variety of interesting questions about the spillovers of violence or about groups' strategic use of violence to manipulate market power or market

[8]United Nations Office on Drugs and Crime. "The Global Afghan Opium Trade: A Threat Assessment." July, 2011. http://www.unodc.org/documents/data-and-analysis/Studies/Global_Afghan_Opium_Trade_2011-web.pdf

size. Those questions are left for future research, as are potentially important dynamic considerations.[9]

It is also worth highlighting a few assumptions. First, because total rents are increasing in market concentration, the groups would benefit from forming a cartel. Hence, the model implicitly assumes a commitment problem preventing such agreements (Fearon, 1995; Powell, 2004).

Second, while the model assumes a symmetric conflict technology, the literature on conventional warfare suggests that defenders may have technological advantages over attackers. For instance, one important analysis argues that in some settings an attacker's force must be two- to three-times the size of a defender's in order to succeed (Mearsheimer, 1988). Moreover, there might be specific territories where this defensive advantage is particularly large. It is straightforward to capture the idea of a defensive advantage in the model by assuming the defender has lower costs for investing in conflict (and those lower costs could be different at different territories). Formally, this is equivalent to inflating the defender's incremental returns from winning. Qualitative results are unaffected by such an extension (though the point at which the defender has a higher implied incremental return shifts), so I abstract away from them.

Finally, a group bears the costs of investment in conflict even if the other group does not invest (ceding the territory). Since preparing for conflict involves converting resources into training and weapons, groups surely bear some costs in such circumstances. A somewhat more satisfying assumption might be that these costs are lower when no actual fighting occurs. However, the benefits of such an assumption, in terms of verisimilitude, come at a significant cost in tractability.

2 Conflict for General Incremental Returns

A group deciding how much to invest in fighting for the vulnerable territory compares its expected economic rents should it win versus lose the fight. Call the difference in group i's expected equilibrium rents should it win versus lose its *incremental return to winning*, IR_i. If $\mathrm{IR}_i > \mathrm{IR}_j$, say that group i is *more motivated* than group j.

The following result from the literature on all-pay auctions are key:

[9]One particularly interesting issue raised by dynamic considerations is the possibility that groups should worry about ceding themselves out of existence and, so, may be willing to fight even when the short-run returns are negative, in order to realize the option value of staying "in the game" (Fearon, 1996; Bueno de Mesquita, 2013).

Theorem 2.1 *(Hillman and Riley, 1989) In an all-pay auction with linear costs and two bidders, let $\overline{\mathrm{IR}} \geq \underline{\mathrm{IR}}$ be the two players' incremental returns from winning the auction. In the unique equilibrium, the player with the larger incremental return bids the realization of a random variable drawn from the uniform distribution on $[0, \underline{\mathrm{IR}}]$ and the player with the smaller incremental return bids 0 with probability $\frac{\overline{\mathrm{IR}}-\underline{\mathrm{IR}}}{\overline{\mathrm{IR}}}$ and with complementary probability bids the realization of an independent random variable drawn from the uniform distribution on $[0, \underline{\mathrm{IR}}]$.*

Given this, from an ex ante perspective, the amount of observed violence is a random variable. With probability $\frac{\overline{\mathrm{IR}}-\underline{\mathrm{IR}}}{\overline{\mathrm{IR}}}$, the less motivated group cedes and v takes the value 0. With complementary probability, v is the sum of two uniform random variables on $[0, \underline{\mathrm{IR}}]$ and, so, has a symmetric triangular distribution on $[0, 2\underline{\mathrm{IR}}]$. Hence, v has a CDF given by

$$\Phi(v) = \begin{cases} 1 - \frac{\underline{\mathrm{IR}}}{\overline{\mathrm{IR}}} + \frac{\underline{\mathrm{IR}}}{\overline{\mathrm{IR}}}\left(\frac{v^2}{2\underline{\mathrm{IR}}^2}\right) & \text{if } v \in [0, \underline{\mathrm{IR}}] \\ \\ 1 - \frac{\underline{\mathrm{IR}}}{\overline{\mathrm{IR}}} + \frac{\underline{\mathrm{IR}}}{\overline{\mathrm{IR}}}\left(1 - \frac{(2\underline{\mathrm{IR}}-v)^2}{2\underline{\mathrm{IR}}^2}\right) & \text{if } v \in [\underline{\mathrm{IR}}, 2\underline{\mathrm{IR}}]. \end{cases} \tag{1}$$

With this, it is straightforward to calculate expected observed violence:

$$E[v] = \int_0^{2\underline{\mathrm{IR}}} v \, d\Phi(v) = \frac{\underline{\mathrm{IR}}^2}{\overline{\mathrm{IR}}}. \tag{2}$$

Let's unpack the intuition. As the more motivated group's incremental return increases, it becomes more willing to invest in conflict. Were it to do so, this would make the less motivated group unwilling to fight at all, since it would be so likely to lose. But if the more motivated group is certain the less motivated group will not fight, then it has no incentive to invest. To maintain equilibrium, as $\overline{\mathrm{IR}}$ increases, the more motivated group's increased willingness to invest leads the less motivated group to cede more often, which establishes equilibrium by decreasing the more motivated group's incentive to invest. Thus, this *scare-off* effect of an increase in $\overline{\mathrm{IR}}$ tends to reduce the expected amount of observed violence by increasing the probability that the territory is ceded.

An increase in $\underline{\mathrm{IR}}$ has two effects. First, as the less motivated group's incremental return to winning increases, it becomes less willing to cede the territory. This *anti-scare-off* effect increases expected observed violence. Second, as the less motivated group's incremental return increases, it becomes willing to invest more. This *stakes* effect increases both groups' expected investment and, thus, also increases expected observed violence.

Often some factor will simultaneously increase both $\overline{\mathrm{IR}}$ and $\underline{\mathrm{IR}}$. Such a change can increase or decrease expected observed violence, depending on the relative effects on the two incremental returns. Note, however, that $\underline{\text{IR}}$ increases expected observed violence through two mechanisms—anti-scare-off and stakes—while $\overline{\text{IR}}$ decreases expected observed violence through only one mechanism—scare-off. Hence, if some factor were to change both incremental returns by similar amounts, the effect on $\underline{\text{IR}}$ would dominate. Indeed, in order for the effect on $\overline{\text{IR}}$ to dominate, it must be more than twice as large. To see this, suppose that both incremental returns are strictly increasing, differentiable functions of some parameter θ. Then expected observed violence is decreasing in θ if and only if:

$$\frac{\partial \underline{\text{IR}}(\theta)/\partial\theta}{\partial \overline{\text{IR}}(\theta)/\partial\theta} < \frac{\underline{\text{IR}}(\theta)}{2\overline{\text{IR}}(\theta)}. \tag{3}$$

3 Economic Equilibrium

To calculate groups' incremental returns to winning, we need to know each groups' rents in the economic equilibrium that follows conflict. Each group chooses prices at each territory it controls to maximize its profits across its territories. The basic trade-off is simple. The marginal cost of a price increase at a territory is decreased demand at that territory. The marginal benefit of a price increase at a territory is higher revenue per customer. Several factors affect pricing, and thus rents, by changing these marginal benefits and costs.

First, as a group adds territories, it increases its price at every territory. Suppose group i controls only one territory, say A. If it increases prices, it loses customers to both D and B. Now suppose that group i controls territories A and B. If it increases prices at A, the same number of consumers depart for D and B. But group i's costs from those departures are now lower because the consumers that depart for territory B remain group i's customers, just at a different location. Hence, the marginal cost of increasing prices goes down the more territories a group controls.

A second point follows from similar logic. Groups charge higher prices at interior territories than at territories that border competitors. If a group controls A, B, and C, it charges the highest prices at B, since all the lost demand from a price increase at B flows to the group's other territories. By contrast, a price increase at, say, A results in some consumers moving to B (remaining that group's customers) and some consumers moving to D (becoming the other group's customers).

Third, the price charged at a given territory is increasing in the price charged at neighboring territories. When prices at D or B are higher, a price increase at territory A results in a smaller loss of demand at A. This is because the high prices at D and B make consumers' less inclined to abandon territory A. Hence, when prices at neighboring territories are high, the marginal cost of price increases is low.

In Appendix A, I formally characterize equilibrium in the economic game and compute the rents for a configuration where each group controls two territories $(2,2)$ and a configuration where one group controls three territories and the other controls one $(3,1)$. Below I report the relevant rents. For the asymmetric configuration, the relevant group's number of territories is in bold, so $u^{\mathbf{3},1}$ is the rents of the group that controls three territories in the $(3,1)$ configuration. Recall that N corresponds to the size of the market (i.e., the mass of consumers) and t corresponds to market power (i.e., transportation costs).

$$u^{2,2} = \frac{Nt}{4} \qquad u^{\mathbf{3},1} = \frac{205Nt}{576} \qquad u^{3,\mathbf{1}} = \frac{25Nt}{144}.$$

4 Equilibrium and Global Comparative Statics

It is straightforward to characterize equilibrium and to calculate the amount of expected observed violence. Regardless of which territory is vulnerable, the defender's incremental return from winning is

$$\mathrm{IR}_{\mathrm{def}} = u^{2,2} - u^{3,\mathbf{1}} = \frac{11Nt}{144}$$

and the attacker's incremental return is

$$\mathrm{IR}_{\mathrm{att}} = u^{\mathbf{3},1} - u^{2,2} = \frac{61Nt}{576}.$$

Given this, Theorem 2.1 and Equation 1 immediately imply the following:

Proposition 4.1 *Equilibrium play at the conflict stage is as follows:*

- *With probability $\frac{44}{61}$ the defender's investment is drawn from a uniform distribution on $\left[0, \frac{11Nt}{144}\right]$ and with complementary probability the defender invests zero.*
- *The attacker's investment is drawn independently from a uniform distribution on $\left[0, \frac{11Nt}{144}\right]$.*

Expected observed violence is:

$$\frac{\mathrm{IR}_{def}^2}{\mathrm{IR}_{att}} = \frac{121Nt}{2196}.$$

The global comparative statics—how violent outcomes change as market size or market power change at all territories—are now straightforward. Both groups' incremental returns are linearly increasing in both transportation costs and market size. Hence, a change to either of those parameters has no net effect on scare-off, which is determined by the ratio of the incremental returns. Such a change affects observed violence only through the stakes effect. The value of a territory is increasing in both N and t, resulting in increased expected observed violence.

Corollary 4.1 *Expected observed violence is increasing in both N and t.*

5 Local Comparative Statics

Now I turn to local comparative statics—what happens to expected observed violence when there are changes to the market size surrounding or the transportation costs associated with getting to one particular territory? For concreteness, suppose one group controls AB, the other group controls CD, and the *shocked territory* is D. I analyze the effects of such shocks when each territory is vulnerable.

5.1 Local Market Size

Consider a situation in which the population in the quarter of the circle surrounding territory D increases to $\frac{\eta N}{4}$, for some $\eta \in [1, \overline{\eta}]$ (with $1 < \overline{\eta} \leq 2$), while the population elsewhere stays as it was. The parameter η represents the magnitude of the shock. The larger is η, the larger the market size around territory D compared to other territories.

An increase in local market size at D has two channels through which it may affect rents.

First, there is a direct effect. For a fixed vector of prices, demand at D increases in η. This increase in demand tends to increase rents for the group that ends up with control of D. Moreover, it raises the marginal benefit to increased prices at D, which tends to push prices up at all territories because the economy has complementarities.

Second, there can be an indirect effect. When local market size around D increases, if both groups are competing for that population, then the marginal cost (in terms of foregone demand) associated with a price increase at A and C increases. This tends to decrease prices at all territories and decrease rents for both groups.

In the case of the configuration where each group controls two territories—AB and CD—both effects are at work, since the group that controls AB is competing to attract

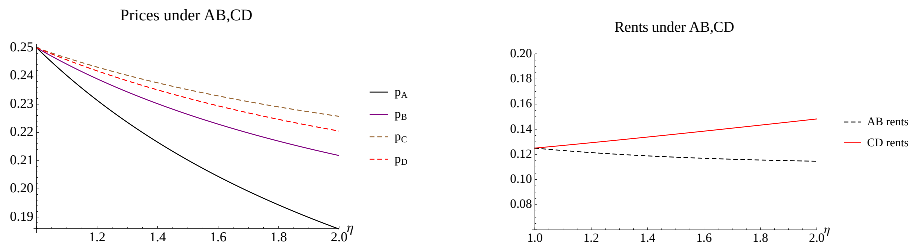

Figure 5.1: Prices and rents as a function of market size surrounding D for the configuration AB, CD.

some of the population surrounding D to buy at A instead. As illustrated in Figure 5.1, as a result of the indirect effect, prices are decreasing in η at all territories. (For the remainder of the paper, all figures are drawn for the case of $t = \frac{1}{2}$.) For the group that controls D, the direct effect on demand dominates in terms of rents, so its rents are increasing in η. Of course, for the group that does not control D, the only effect on rents is through the decrease in prices, so its rents are decreasing in η.

How expected observed violence changes as a function of η depends on how these rents compare to the rents under the configuration that results if the attacker wins the vulnerable territory (since this will tell us about the incremental returns as a function of η). This, in turn, depends on how the direct and indirect effects operate in those alternative configurations, which is what we turn to now. In all cases, the economic equilibria are characterized in Appendix B.1.

5.1.1 Shocked Territory (D) is Vulnerable

When territory D itself is vulnerable, the group that initially controls AB is the attacker. To compute the incremental returns, we compare equilibrium rents (as a function of η) in two scenarios: AB, CD and ABD, C.

As in the case of AB, CD, both the direct and the indirect effects are at work in the ABD, C configuration, since the group that controls C is competing to attract demand away from D. The indirect effect dominates with respect to prices, so as illustrated in Figure 5.2, prices are decreasing in both configurations.

The left-hand panel of Figure 5.3 illustrates the net effect on rents. Since the group that does not control territory D at the end of the conflict experiences only the indirect effect, its rents are decreasing in η. (This is represented by the dashed curves in the figure.) However, for the group that ends up in control of territory D the direct effect dominates

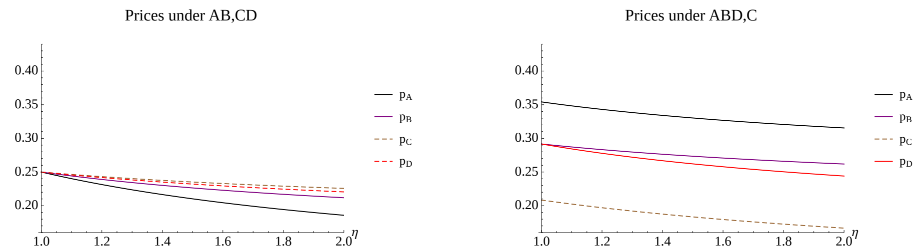

Figure 5.2: Prices as a function of local market size at D for both the AB, CD and ABD, C configurations.

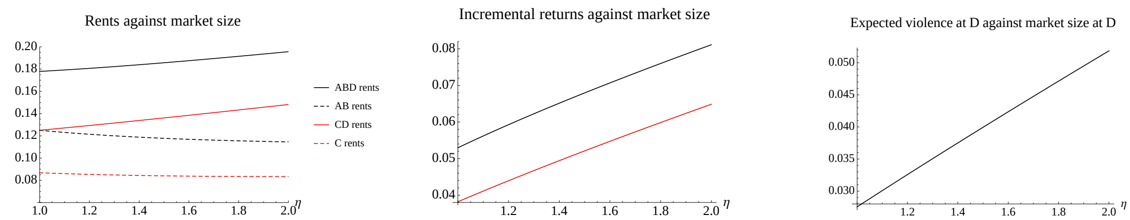

Figure 5.3: Rents, incremental returns, and expected observed violence at D as a function of market size at D.

and rents are increasing in η. (Represented by the solid curves.)

What does this imply about incremental returns? A group's rents increase in local market size at D if it controls D but decrease in η if it does not control D. Hence, both groups' incremental returns to winning D are increasing in η. This fact is illustrated in the middle panel of Figure 5.3 (and formalized in Proposition 5.1).

Since both groups' incremental returns are increasing in η, there are competing effects on expected observed violence. As shown in Condition 3, the effect on the smaller incremental return (here the defender's) dominates unless the larger incremental return changes a lot more, which is not the case here. Hence, an increase in local market size at the vulnerable territory increases incremental returns and expected observed violence. These facts are illustrated in the right-hand panel of Figure 5.3 and formalized below.

Proposition 5.1 *Suppose the population on the quarter of the circle with the vulnerable territory at its center is of mass* $\frac{\eta N}{4}$ *for some* $\eta \in [1, \overline{\eta}]$*, while population elsewhere on the circle remains fixed:*

1. *Both groups' incremental returns to winning the conflict are increasing in* η*.*

2. *Expected observed violence is increasing in η.*

Proof. See Appendix C.1. ■

5.1.2 Territory A is Vulnerable

Continue to consider a shock to market size at D, but suppose territory A is vulnerable. Now the group that controls CD is the attacker and we compute the incremental returns by comparing rents under AB, CD and B, ACD.

If the attacker wins, so that one group controls ACD, the indirect effect becomes unimportant. This is because, whichever territory the consumers surrounding D buy from, they are customers of the same group. Hence, the incentive to lower prices to capture the new demand at D is much diminished. As a consequence, the direct effect dominates in this scenario. As illustrated in the right-hand side of Figure 5.4, this means that in the B, ACD configuration, prices are increasing at all territories. Moreover, as illustrated in the left-hand panel of Figure 5.5, this means that rents are increasing for both groups, though they are increasing faster for the group that controls D, since it benefits from both the increase in prices and the direct increase in demand.

What does this imply for incremental returns? The attacker's rents are increasing in η, whether it wins or loses. But if it wins, its rents are increasing faster, since, when it wins, it benefits from both increased prices and increased demand. Hence, the attacker's incremental return is increasing in η. By contrast, as we've just seen, the defender's rents are increasing in η if it loses (due to increasing prices) but are decreasing in η if it wins (because of decreasing prices). Hence, the defender's incremental return is decreasing in η. Moreover, if η is large enough, the defender's incremental return is negative—it would prefer to cede territory A because doing so leads to increased rents at its remaining territory that compensate for the loss of territory A. These points are illustrated in the middle panel of Figure 5.5.

Putting all of this together, as illustrated in the right-hand panel of Figure 5.5 and stated formally in the next result, expected observed violence is decreasing in η and, if η is sufficiently large, it drops to zero.

Proposition 5.2 *Suppose the population on the quarter of the circle with territory j at its center is of mass $\frac{\eta N}{4}$ for some $\eta \in [1, \overline{\eta}]$, while the population elsewhere on the circle remains fixed. If the territory contiguous with j and controlled by the other group is vulnerable:*

1. *The incremental returns to winning are increasing in η for the attacker and decreasing in η for the defender.*

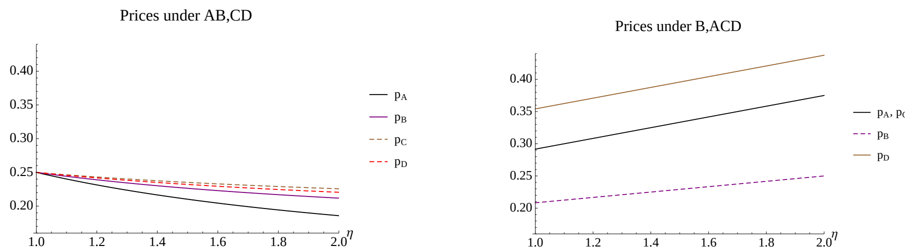

Figure 5.4: Prices as a function of local market size at D for both the AB, CD and B, ACD configurations.

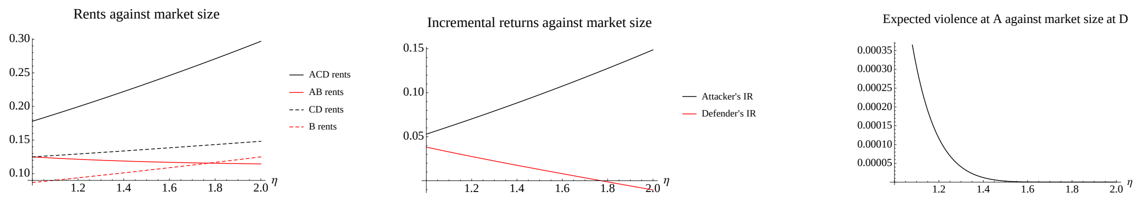

Figure 5.5: Rents, incremental returns, and expected observed violence at territory A as a function of local market size at D.

2. *There is a critical threshold $\hat{\eta} \in (1, 2)$ such that the defender's incremental return is positive for all $\eta \in [1, \hat{\eta})$ and negative for all $\eta \in (\hat{\eta}, 2)$.*

3. *Expected observed violence is strictly decreasing in η for $\eta < \hat{\eta}$ and is zero for $\eta \geq \hat{\eta}$.*

Proof. See Appendix C.1. ■

5.1.3 Territory B is Vulnerable

Continue to consider a shock to market size at D, but suppose territory B is vulnerable. Now the group that controls CD is the attacker and we compute incremental returns by comparing rents under AB, CD and A, BCD.

As in the case where D was vulnerable, even if the attacker wins, both effects are still operative because the group that controls A competes for some of the population surrounding D. As a consequence of the indirect effect, prices are decreasing at all territories (see the right-hand panel of Figure 5.6). In the A, BCD configuration, rents are increasing for the group that controls BCD because the direct effect dominates, whereas rents are decreasing for the group that controls A since it only experiences the indirect effect.

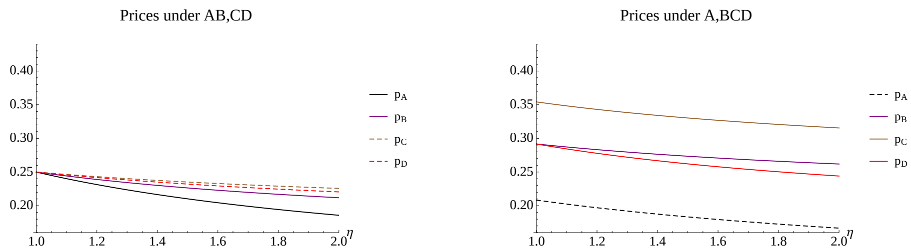

Figure 5.6: Prices as a function of local market size at D for both the AB, CD and A, BCD configurations.

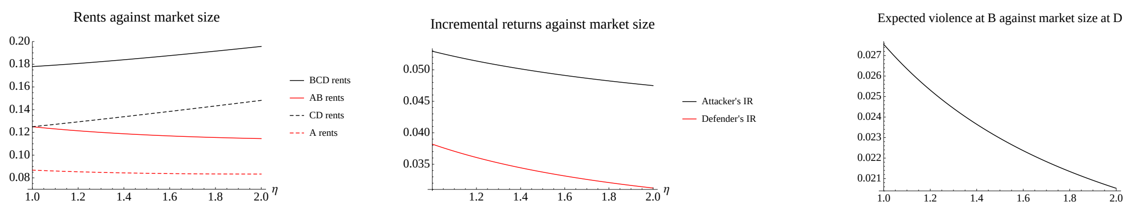

Figure 5.7: Rents, incremental returns, and expected observed violence at territory B as a function of local market size at D.

Interestingly, the effect on incremental returns is the opposite of the case where D was vulnerable. When the groups were fighting over D—a territory that, because of the direct effect, was increasing in value as η increased—incremental returns were increasing. When the groups are fighting over territory B—a territory that, because of the indirect effect, is decreasing in value as η increases—incremental returns are decreasing.

Since both the attacker's and the defender's incremental returns are decreasing in η (see the middle panel of Figure 5.7) there are offsetting effects on expected observed violence. As shown in Condition 3, the effect on the smaller incremental return (here the defender's) dominates unless the larger incremental return changes a lot more, which is not the case. Hence, when conflict is over B, an increase in local market size at D decreases incremental returns and expected observed violence. These facts are illustrated in the right-hand panel of Figure 5.7 and formalized in the next result.

Proposition 5.3 *Suppose the population on the quarter of the circle with territory j at its center is of mass $\frac{\eta N}{4}$ for some $\eta \in [1, \overline{\eta}]$, while the population elsewhere on the circle remains fixed. If the territory that is not contiguous with j is vulnerable:*

1. *The incremental returns to winning are decreasing in η for the attacker and the de-*

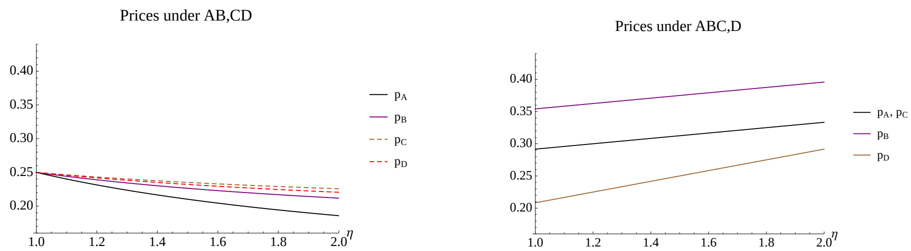

Figure 5.8: Prices as a function of local market size at D for both the AB, CD and B, ACD configurations.

fender.

2. *Expected observed violence is decreasing in η.*

Proof. See Appendix C.1. ■

5.1.4 Territory C is Vulnerable

Finally, continue to consider a shock to market size at D, but suppose territory C is vulnerable. Now the group that controls AB is the attacker and to compute the incremental returns we compare equilibrium rents (as a function of η) under AB, CD and ABC, D.

If the attacker wins, then the defender controls only territory D. In this case, as in the case when A was vulnerable, only the direct effect is important, but for different reasons. When territory A was vulnerable, if the attacker won, the same group controlled A, C, and D, so there was no cross-group competition over the population surrounding D. Hence, the indirect effect was inconsequential. When territory C is vulnerable and the attacker wins, the group the controls D does not control any other territories. This leads that group to set a low price at D. Consequently, it captures all of the consumers on its quarter of the circle, as well as some consumers who are closer to A or C. This means that the group that controls A and C is not competing for the population surrounding D, it cedes them to the group that controls D. As such, there is no indirect effect of an increase in η on prices. Like in the case where A was vulnerable, because only the direct effect is operating, if C is vulnerable and the attacker wins, prices are increasing in η. This is illustrated in the right-hand side of Figure 5.8.

Because both groups benefit from the price increases, both groups' rents are increasing in η if the attacker wins. The incremental returns of the defender (who only controls D) are increasing particularly quickly because the defender benefits not only from the price

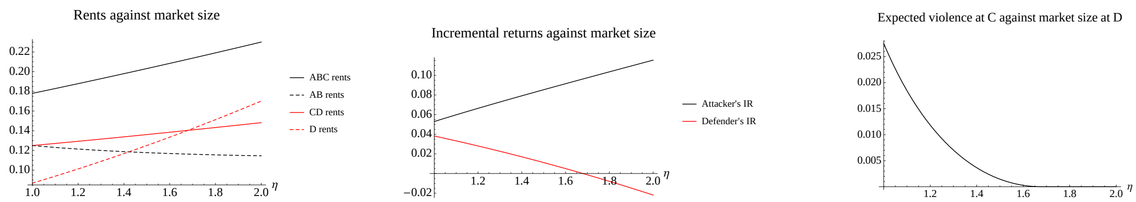

Figure 5.9: Rents, incremental returns, and expected observed violence at territory C as a function of local market size at D.

increase, but from the direct increase in demand at territory D. This fact is illustrated in the left-hand panel of Figure 5.9.

As we've just seen, the attacker's rents are increasing in η if she wins (so she controls ABC). And, as we saw earlier, because of the indirect effect, her rents are decreasing in η if she loses (so she controls AB). Hence, as illustrated in the middle panel of Figure 5.9, the attacker's incremental returns are increasing in η. The defender's rents are increasing in η whether she wins or loses. If she wins, the rents are increasingly relatively slowly because both the direct and indirect effects are at work. If she loses, her rents are increasing relatively quickly because only the direct effect is at work. Hence, as illustrated in the middle panel of Figure 5.9, the defender's incremental returns are decreasing in η. Moreover, if η is sufficiently large, the increment return becomes negative—the defender would rather cede territory C to the attacker. As shown in the right-hand panel of Figure 5.9, the consequence of the attacker's increasing incremental return and the defender's decreasing incremental return is that expected observed violence is decreasing in η and, for η sufficiently large, it drops to zero. These facts are recorded in the next result.

Proposition 5.4 *Suppose the population on the quarter of the circle with territory j at its center is of mass $\frac{\eta N}{4}$ for some $\eta \in [1, \overline{\eta}]$, while the population elsewhere on the circle remains fixed. If the territory contiguous with j and controlled by the same group is vulnerable:*

1. *The incremental returns to winning are increasing in η for the attacker and decreasing in η for the defender.*

2. *There is a critical threshold $\hat{\eta} \in (1, 2)$ such that the defender's incremental return is positive for all $\eta \in [1, \hat{\eta})$ and negative for all $\eta \in (\hat{\eta}, \overline{\eta})$.*

3. *Expected observed violence is strictly decreasing in η for $\eta < \hat{\eta}$ and is zero for $\eta \geq \hat{\eta}$.*

Proof. See Appendix C.1. ■

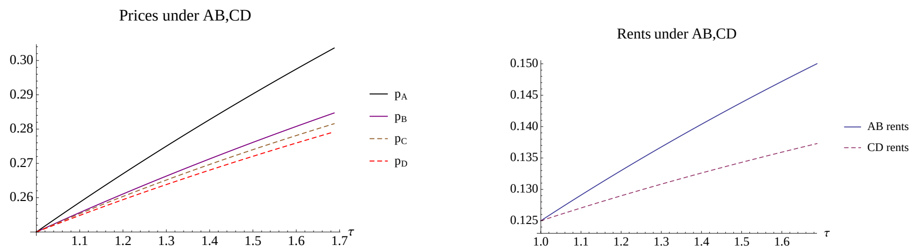

Figure 5.10: Prices and rents as a function of transportation costs at D for the configuration AB, CD.

5.2 Local Transportation Costs

Now consider a situation in which the transportation costs for getting to territory D increase from t to τt for some $\tau \in [1, \overline{\tau}]$, with $1 < \overline{\tau} \leq \frac{27}{16}$.[10] The parameter τ now represents the size of the shock—the larger is τ, the larger are the transportation costs at territory D compared to other territories.

As with market size, an increase in local transportation costs at D has two channels through which it may affect rents.

First, there is a direct effect. For a fixed vector of prices, demand at D decreases in τ, which tends to decrease rents for the group that ends up with control of D.

Second, there is an indirect effect. When local transportation costs at D increase, the marginal costs (in terms of foregone demand) associated with a price increase at A or C increase. This tends to increase prices at all territories and increase rents for both groups.

In the case of the configuration where each group controls two territories—AB, and CD—both effects are at work. As illustrated in Figure 5.10, because of the indirect effect, prices are increasing in τ at all territories. As a result, rents are increasing in τ for both groups. Because the group that controls AB experiences only the indirect effect, its rents are increasing faster than those of the group that controls CD, which is harmed by the direct effect.

How expected observed violence changes as a function of τ depends on how these rents compare to the rents under the configuration that results if the attacker wins the vulnerable territory (since this will tell us about the incremental returns as a function of τ). This, in turn, depends on how the direct and indirect effects operate in those alternative configurations, which is what we turn to now. In all cases, the economic equilibria are characterized

[10] I restrict $\tau \leq \frac{27}{16}$ to avoid the potential problem of population members who might not wish to purchase at all because transportation costs are too high.

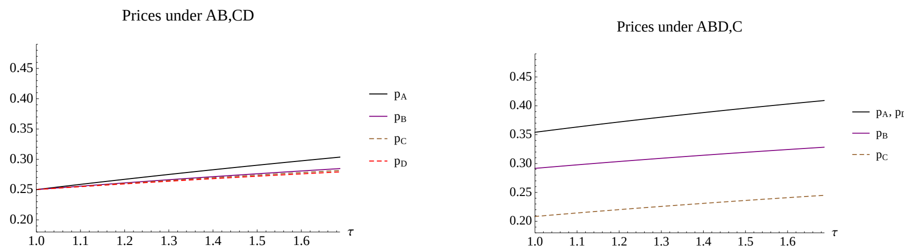

Figure 5.11: Prices as a function of local transportation costs at D.

in Appendix B.2.

5.2.1 Territory D is Vulnerable

When territory D itself is vulnerable, the group that initially controls AB is the attacker. To compute the incremental returns, we compare equilibrium rents (as a function of τ) in two scenarios: AB, CD and ABD, C.

As in the case of AB, CD, both the direct and the indirect effects are at work in the ABD, C configuration. Because of the indirect effect, prices are increasing at all territories in both configurations, as illustrated in Figure 5.11.

The left-hand panel of Figure 5.12 illustrates the net effect on rents. Because prices are increasing, both groups' rents are increasing in τ in both configurations. However, whichever group controls D also experiences the (negative) direct effect. Hence, rents are increasing faster in τ for the group that does not control D. As a consequence, as illustrated in the middle panel of Figure 5.12, both groups' incremental returns are decreasing in τ.

Since both groups' incremental returns are decreasing in τ, as shown in Condition 3, the effect on the smaller incremental return (the defender's) dominates unless the larger incremental return changes a lot more, which is not the case here. Hence, as illustrated in Figure 5.12, the effect of an increase in local transportation costs at the vulnerable territory is to increase rents, but decrease incremental returns and expected observed violence at that territory.

Proposition 5.5 *Suppose the transportation costs associated with the vulnerable territory are τt for some $\tau \in [1, \overline{\tau}]$:*

1. *Both group's incremental returns to winning the conflict are decreasing in τ.*

2. *Expected observed violence is decreasing in τ.*

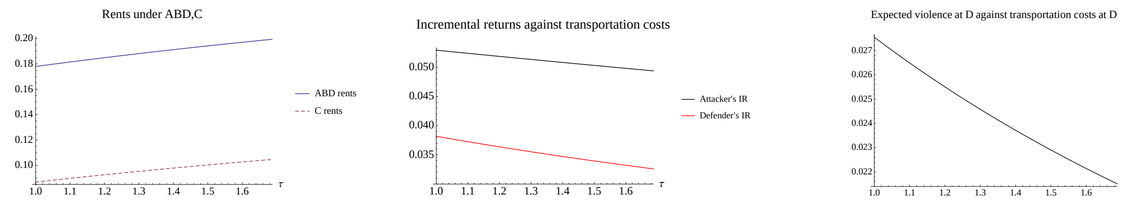

Figure 5.12: Rents, incremental returns, and expected observed violence at D as a function of transportation costs at D.

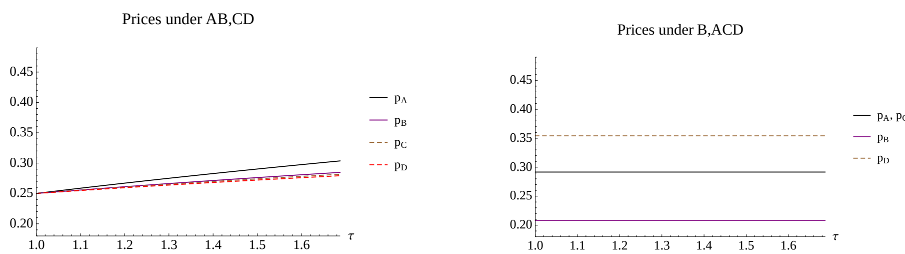

Figure 5.13: Prices as a function of local transportation costs at D.

Proof. See Appendix C.2. ■

5.2.2 Territory A is Vulnerable

Continue to consider a shock to transportation costs at D, but now suppose territory A is vulnerable. Now the group that controls CD is the attacker and we compute incremental returns by comparing compare rentsunder AB, CD and B, ACD.

If the attacker wins, so that one group controls ACD, the indirect effect becomes unimportant. This is because, whichever territory the consumers surrounding D buy from, they are customers of the same group. Hence, the desire to raise prices to increase profits from consumers who do not want to purchase from D is much diminished. As a consequence, the direct effect dominates in this scenario.

This is illustrated in the right-hand panel of Figure 5.13, where we see that prices are in fact unresponsive to transportation costs at D under the B, ACD configuration. Moreover, as illustrated in the left-hand panel of Figure 5.14, this means that rents are also barely responsive to τ in this configuration. (The group that controls ACD's rents are slightly responsive to τ because, while a change in τ doesn't change prices, it does change how many consumers buy from D.)

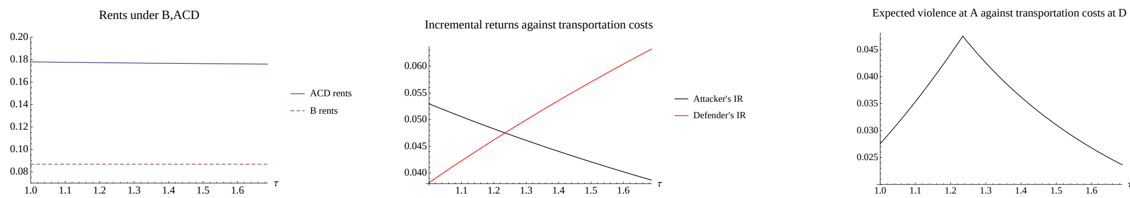

Figure 5.14: Rents, incremental returns, and expected observed violence at territory B as a function of local transportation costs at D.

As consequence, when A is vulnerable, the effect of transportation costs at D on incremental returns is driven by their effect in the AB, CD configuration. Since, as we've already seen, in the AB, CD configuration, an increase in τ leads to an increase in prices and an increase in rents, this means that incremental returns are increasing in τ for the defender, but decreasing for the attacker. And, indeed, for τ sufficiently large, the defender's incremental return becomes larger than the attacker's. All of this is illustrated in the middle panel of Figure 5.14.

What are the implications for expected observed violence? Recall, expected observed violence is increasing in the smaller of the two incremental returns and decreasing in the larger. The defender's incremental return is increasing in τ while the attacker's is decreasing. As such, when τ is small, so that the attacker's incremental return is larger than the defender's, expected observed violence is increasing in τ. However, when τ is sufficiently large, so that the defender's incremental return is larger than the attacker's, expected observed violence is decreasing in τ. Hence, as illustrated in the right-hand panel of Figure 5.14 and stated formally in the next result, expected observed violence at A is non-monotone in transportation costs at D.

Proposition 5.6 *Let the transportation costs associated with a territory j be τt for some $\tau \in [1, \overline{\tau}]$. If the territory that is contiguous with j and controlled by the other group is vulnerable:*

1. *The attacker's incremental returns are decreasing and the defender's incremental returns are increasing in τ.*

2. *Expected observed violence is non-monotone in τ. In particular, there is a critical threshold $\tilde{\tau} \in \left(1, \frac{27}{16}\right)$ such that expected observed violence is increasing in τ for $\tau < \tilde{\tau}$ and decreasing in τ for $\tau > \tilde{\tau}$.*

Proof. See Appendix C.2. ■

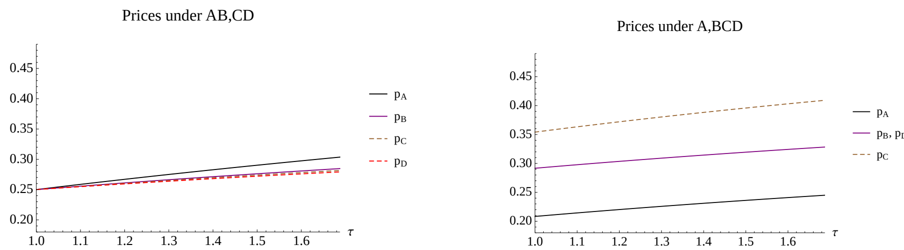

Figure 5.15: Prices as a function of local transportation costs at D.

5.2.3 Territory B is Vulnerable

Continue to consider a shock to transportation costs at D, but suppose territory B is vulnerable. Now the group that controls CD is the attacker and we compute incremental returns by comparing rents under AB, CD and A, BCD.

As in the case where D was vulnerable, even if the attacker wins, both effects are still operating because the group that controls A competes for some of the population surrounding D. As a consequence of the indirect effect, as illustrated in the right-hand panel of Figure 5.15, prices are increasing at all territories. And, as a result of the price increases, rents are increasing for both groups, as illustrated in the left-hand panel of Figure 5.16.

Since the territory under dispute experiences only the (positive) indirect effect, rents are increasing faster in τ for whichever group wins the conflict. Hence, as illustrated in the middle panel of Figure 5.16, both incremental returns are increasing in τ.

Because both groups' incremental returns are increasing in τ, as shown in Condition 3, the effect on the smaller incremental return (the defender's) dominates unless the larger incremental return changes a lot more, which is not the case here. Hence, as illustrated in the right-hand panel of Figure 5.16 and stated formally in the next result, an increase in local transportation costs at D increases expected observed violence at B.

Proposition 5.7 *Let the transportation costs associated with j be τt for some $\tau \in [1, \overline{\tau}]$. If the territory that is not contiguous with j is vulnerable:*

1. *Both groups' incremental returns to winning are increasing in τ.*

2. *Expected observed violence is increasing in τ.*

Proof. See Appendix C.2. ∎

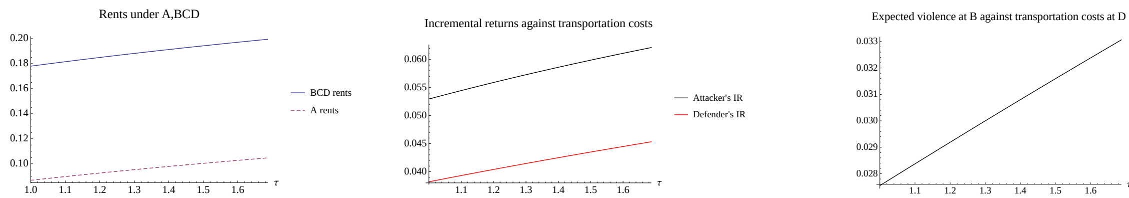

Figure 5.16: Rents, incremental returns, and expected observed violence at territory B as a function of local transportation costs at D.

5.2.4 Territory C is Vulnerable

Finally, continue to consider a shock to transportation costs at D, but suppose territory C is vulnerable. Now the group that controls AB is the attacker and to compute the incremental returns we compare equilibrium rents (as a function of η) under AB, CD and ABC, D.

Again, because of the indirect effect, prices are increasing in τ at all territories, as illustrated in Figure 5.17. The groups are fighting over territory C, which is of increasing value as τ increases because of the indirect effect on prices. For this reason, the attacker's incremental is increasing in τ, as illustrated in Figure 5.18. However, the defender's incremental return is more subtle. Because of the direct effect, territory D is of decreasing value as τ increases. If the defender loses, it controls only territory D. So we might expect its incremental returns to be increasing. However, there is a second effect. If the defender loses, the other group controls all three territories. This leads it to increase prices faster as a function of τ, which diminishes the loss of demand at D. As illustrated in Figure 5.18, this effect is sufficiently important that the defender's rents are increasing faster in τ if it loses than if it wins. Consequently, the defender's incremental return is decreasing. As shown in the right-hand panel of Figure 5.18 and formalized in the next result, because the attacker's incremental return is increasing and the defender's incremental return is decreasing, expected observed violence at C is decreasing in τ.

Proposition 5.8 *Let the transportation costs associated with a border territory j be τt for some $\tau \in [1, \overline{\tau}]$. If the territory contiguous with j and controlled by the same group is vulnerable:*

1. *The defender's incremental return to winning is decreasing in τ. The attacker's incremental return to winning is increasing in τ.*

2. *Expected observed violence is decreasing in τ.*

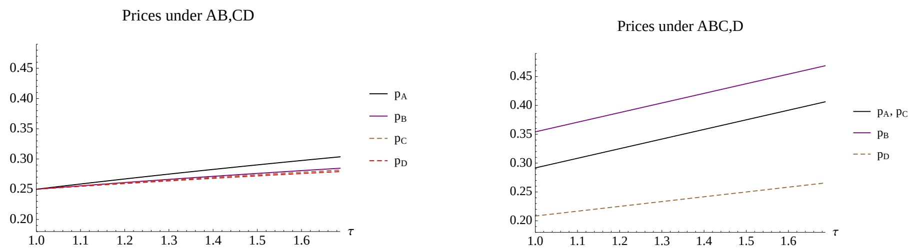

Figure 5.17: Prices as a function of local transportation costs at D.

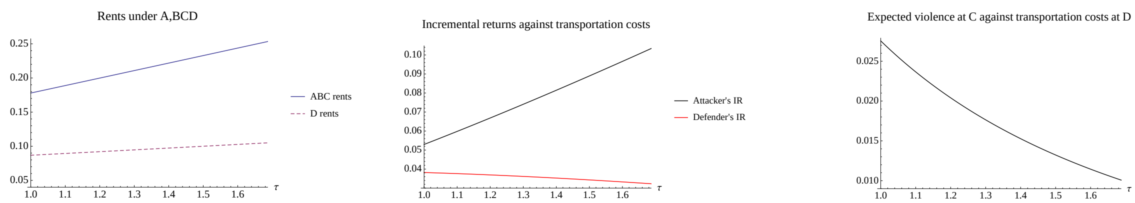

Figure 5.18: Rents, incremental returns, and expected observed violence at territory A as a function of local transportation costs at D.

Proof. See Appendix C.2. ■

6 Implications for Empirical Work

The results on the effects of local economic shocks on observed violence have a variety of implications for empirical scholarship. Of course, each comparative static is itself a testable hypothesis. But a few of more specific points are worth highlighting.

First, consider the contrast between the global and local comparative statics. In the global comparative statics there was a clear prediction of a positive correlation between economic rents and expected observed violence. There is no such prediction for the local comparative statics. As the results above show, the two different groups' rents can move in opposite directions as a result of the shock. This, for instance, is the case when there is a market size shock at D. And even when the two groups' rents move in the same direction, expected observed violence can move in the opposite direction. This, for instance, is the case for a transportation cost shock at D when D is vulnerable—rents for both groups are increasing in the shock (regardless of who wins), while expected observed violence at D is decreasing in the magnitude of the shock. Moreover, it need not even be the case that expected observed violence moves in the same direction even as the local rents extracted just from the territory under dispute. This is illustrated in the case of transportation cost shocks at D when A is vulnerable. There, a change in τ has a monotone effect on the value of territory A, but a non-monotone effect on the level of violence.

Second, these shocks do not have a uniform effect on violence at all territories. A shock to market size at D increases expected observed violence at D and decreases expected observed violence at A, B, and C. A shock to transportation costs at D decreases expected observed violence at C and D, increases expected observed violence at B, and has a non-monotone effect on expected observed violence at A. The fact that these spillovers can be negative, positive, or non-monotone suggests that empirical estimates of the average effects of local economic shocks could mask interesting heterogeneity.

Finally, these varied effects of economics shocks at one territory on violence at other territories has implications for empirical practice. A common empirical approach uses difference-in-differences to estimate the effect of a local economic shock on observed violence.[11] The effect of interest is the change in observed violence at territory j following an economic shock to territory j. To isolate the effect of the shock, the researcher studies the change in observed violence at territory j before and after the shock relative to the change in observed violence at nearby territories. Under a parallel trends assumption, difference-in-differences identifies the effect of the economic shock at territory j on observed violence at territory j.

It is obvious that, in a political economy with economic spillovers like the one modeled here, the parallel trends assumption does not hold. But we can go further, asking what the model says about the sign and magnitude of the bias. The answer depends on the type, magnitude, and location of the shock.

Write the expected observed violence at territory i given a shock of size σ (which equals τ for shocks to transportation costs or η for shocks to market size) at territory j as:

$$\mathbb{E}[v_i \mid (j, \sigma)].$$

The true effect on expected observed violence at i of a shock of size σ at j is

$$\delta_i(j, \sigma) = \mathbb{E}[v_i \mid (j, \sigma)] - \mathbb{E}[v_i \mid (j, 1)].$$

Difference-in-differences estimates the change in expected observed violence at j minus the

[11]See, among others, Deininger (2003); Angrist and Kugler (2008); Brückner and Ciccone (2010); Hidalgo et al. (2010); Besley and Persson (2011); Berman, Shapiro and Felter (2011); Dube and Vargas (2013); Bazzi and Blattman (2014); Dube, García-Ponce and Thom (2016); Maystadt and Ecker (2014); Mitra and Ray (2014).

change in expected observed violence at i following a shock of size σ at j:

$$\Delta_{j,i}(j,\sigma) = \delta_j(j,\sigma) - \delta_i(j,\sigma).$$

This difference-in-differences is biased to the extent that it does not equal the true effect of a shock at j on j. Thus, the bias from the difference-in-differences using i as a baseline for j is:

$$b_{j,i} = \Delta_{j,i}(j,\sigma) - \delta_j(j,\sigma).$$

6.1 Transportation Cost Shocks

As we've seen, a shock to transportation costs at D decreases expected observed violence at D, but has different effects on observed violence at other territories. Because of these spillovers, difference-in-differences typically will not recover the true effect of the shock at D on expected observed violence at D. More disturbingly, the model suggests that it may be difficult for a researcher even to know the sign of the bias. This is the case for two reasons. First, because the sign of the effect of a shock at D on violence at other territories differs, the sign of the bias from difference-in-differences depends on which territory is used as the baseline for comparison. Second, for territory A, shocks at D have a non-monotone effect on violence. So, if territory A is used as the baseline for comparison, the sign of the bias depends on the magnitude of the shock. For small shocks (so the effect of τ at A is positive), difference-in-differences overestimates the magnitude of the effect (i.e., says it is more negative than it is). For sufficiently large shocks (so the effect of τ at A is sufficiently negative), difference-in-differences actually gets the sign of the effect wrong (i.e., says the effect is positive). For some moderate shocks (so the effect of τ at A is close to zero), difference-in-differences comes close to the true effect. For both of these reasons, it seems likely that the empirical researcher will be unable to learn the sign of the bias, or even the sign of the true effect, from difference-in-differences.

These facts are illustrated in Figure 6.1. The left-hand panel shows the effect of a shock of size τ to transportation costs at D on expected observed violence at each territory i ($\delta_i(D,\tau)$) and on the difference-in-differences ($\Delta_{D,i}(D,\tau)$). The right-hand side shows that the sign of the bias ($\Delta_{D,i}(D,\tau) - \delta_D(D,\tau)$) can be positive or negative, depending on the territory used for comparison or the size of the shock. When the bias is positive and larger in magnitude than the true effect, difference-in-differences gets the sign of the effect itself wrong.

In empirical applications, a standard approach when there are concerns about spillovers

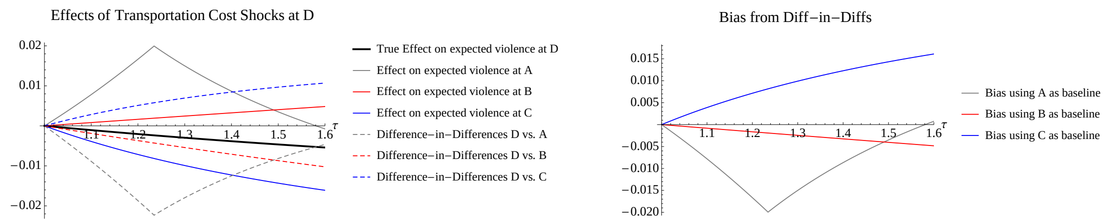

Figure 6.1: Difference-in-differences for the effect of a local transportation cost shock.

is to drop neighboring territories and focus empirical analysis on somewhat more distant neighbors. It is, thus, worth noting that the analysis here does not unequivocally confirm that this strategy improves matters. For many values of the shock, the bias is smallest in magnitude when the most distant territory (B) is used as the baseline for control. But this is not the case for every value of the shock.

6.2 Market Size

As we've seen, a shock to market size at D increases expected observed violence at D and decreases expected observed violence at other territories. These spillovers again mean that difference-in-differences does not recover the true effect of a shock at D on expected observed violence at D. But, unlike the case of transportation cost shocks, here we at least know the sign of the bias—difference-in-differences overestimates the effect.

This fact is illustrated in Figure 6.2. The left-hand panel shows the effect of a shock of size η to market size at D on expected observed violence at each territory i ($\delta_i(D, \eta)$) and on the difference-in-differences ($\Delta_{D,i}(D, \eta)$). The right-hand side shows that the bias associated with the difference-in-differences ($\Delta_{D,i}(D, \eta) - \delta_D(D, \eta)$) is positive and its magnitude is increasing in the size of the shock.

Here, the model is perhaps even less supportive of the standard practice of excluding nearest neighbors to reduce bias. In particular, in this case, the bias from difference-in-differences is minimized by using territory A as the baseline of comparison.

7 Conclusion

I study a model of groups fighting over control of territories from which they endogenously extract economic rents. The analysis yields several results worth reemphasizing.

First, local and global changes to market conditions have different effects on conflict.

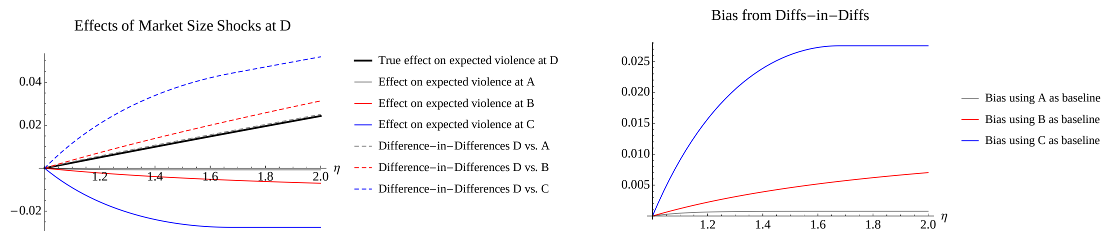

Figure 6.2: Difference-in-differences for market size shocks.

Most of the modern empirical literature exploits local variation. Yet, the model's predictions about the effects of local changes are different from conventional hypotheses (which are more similar to the model's predictions regarding global changes). In particular, the model predicts that changes to local economic conditions do not necessarily generate a positive association between (overall or local) rents and expected observed violence.

Second, local economic shocks affect conflict at other territories in subtle ways. The fact that these spillovers can be negative, positive, or even non-monotone suggests that estimates of the average effects of local economic shocks could mask interesting heterogeneity.

Third, in the presence of such spillovers, a difference-in-differences research design, of course, produces biased estimates. The model facilitates a theoretical exploration of the sign and magnitude of the bias. In the case of shocks to local market size, the bias is always positive, so that difference-in-differences leads to overestimates. In the case of shocks to local transportation costs (market power), the sign and magnitude of the bias depend on both the magnitude of the shock and the territory used as a baseline for comparison. Thus, the empirical researcher may not be able to learn the magnitude or the sign of the effect from a difference-in-differences exercise. In both cases, using territories more distant from the shock as the baseline for comparison need not reduce the bias.

Both the divergence between the local and the global comparative statics, and the usefulness of the local comparative statics for understanding what difference-in-differences estimates in such a setting, point to a complementarity between identification-oriented, micro-empirical scholarship on conflict and theoretical models within which we can think about the sources of variation used in such studies.

Finally, the model highlights a conceptual point. The results here arise because conflict outcomes feedback into economic behavior, which affects the returns to winning the conflict. Hence, the model demonstrates the importance of a political economy approach that takes seriously the two-way relationship between economic and conflict outcomes.

References

Angrist, Joshua D. and Adriana D. Kugler. 2008. "Rural Windfall or a New Resource Curse? Coca, Income, and Civil Conflict in Colombia." *Review of Economics and Statistics* 90(2):191–215.

Bazzi, Samuel and Christopher Blattman. 2014. "Economic Shocks and Conflict: The Evidence from Commodity Prices." *American Economic Journal: Macroeconomics* 6(4):1–38.

Berman, Eli, Jacob N. Shapiro and Joseph H. Felter. 2011. "Can Hearts and Minds be Bought? The Economics of Counterinsurgency in Iraq." *Journal of Political Economy* 119:766–819.

Besley, Timothy and Torsten Persson. 2010. "State Capacity, Conflict, and Development." *Econometrica* 78(1):1–34.

Besley, Timothy and Torsten Persson. 2011. "The Logic of Political Violence." *Quarterly Journal of Economics* 126(3):1411–1445.

Brückner, Markus and Antonio Ciccone. 2010. "International Commodity Prices, Growth and the Outbreak of Civil War in Sub-Saharan Africa." *The Economic Journal* 120(544):519–534.

Bueno de Mesquita, Ethan. 2013. "Rebel Tactics." *Journal of Political Economy* 121(2):323–357.

Carter, David B. 2010. "The Strategy of Territorial Conflict." *American Journal of Political Science* 54(4):969–987.

Caselli, Francesco, Massimo Morelli and Dominic Rohner. 2015. "The Geography of Interstate Resource Wars." *Quarterly Journal of Economics* 130(1):267–315.

Castillo, Juan Damilo, Daniel Mejia and Pascual Restrepo. 2013. "Illegal Drug Markets and Violence in Mexico: The Causes Beyond Calderon." Universidad de los Andes typescript. Available at http://fsi.stanford.edu/sites/default/files/143.illegaldrug.pdf.

Castillo, Juan, Daniel Mejía and Pascual Restrepo. 2018. "Scarcity without Leviathan: The Violent Effects of Cocaine Supply Shortages in the Mexican Drug War." Available at https://ssrn.com/abstract=2409268.

Chassang, Sylvain and Gerard Padro i Miguel. 2009. "Economic Shocks and Civil War." *Quarterly Journal of Political Science* 4(3):211–228.

Crost, Benjamin, Joseph Felter and Patrick Johnston. 2014. "Aid Under Fire: Development Projects and Civil Conflict." *American Economic Review* 104(6):1833–1856.

Dal Bó, Ernesto and Pedro Dal Bó. 2011. "Workers, Warriors, and Criminals: Social Conflict in General Equilibrium." *Journal of the European Economic Association* 9(4):646–677.

Deininger, Klaus. 2003. "Causes and Consequences of Civil Strife: Micro-Level Evidence from Uganda." *Oxford Economic Papers* 55(4):579–606.

Dell, Melissa. 2015. "Trafficking Networks and the Mexican Drug War." *American Economic Review* 105(6):1738–79.

Dube, Oeindrila and Juan Vargas. 2013. "Commonity Price Shocks and Civil Conflict: Evidence from Colombia." *Review of Economic Studies* 80(4):1384–1421.

Dube, Oeindrila, Omar García-Ponce and Kevin Thom. 2016. "From Maize to Haze: Trade Liberalization and the Growth of the Mexican Drug Sector." *Journal of the European Economic Association* 14(5):1181–1224.

Epstein, Gil S. and Ira N. Gang. 2007. "Who is the Enemy?" *Defence and Peace Economics* 18(6):469–484.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Fearon, James D. 1996. "Bargaining Over Objects that Influence Future Bargaining Power." Stanford University typescript. Available at https://web.stanford.edu/group/fearon-research/cgi-bin/wordpress/wp-content/uploads/2013/10/Bargining-Over-Objects-That-Influence-Future-Bargaining-Power.pdf.

Fearon, James D. 2008*a*. Economic Development, Insurgency, and Civil War. In *Institutions and Economic Performance*, ed. Elhanan Helpman. Cambridge, MA: Harvard University Press pp. 292–328.

Fearon, James D. 2008*b*. "A Simple Political Economy of Relations among Democracies and Autocracies." Stanford University typescript. Available at https://web.stanford.edu/group/fearon-research/cgi-bin/wordpress/wp-content/uploads/2013/10/ASimplePolitical-Economy-ofRelations-amongDemocraciesand-Autocracies.pdf.

Goemans, Hein E. and Kenneth A. Schultz. 2017. "The Politics of Territorial Claims: A Geospatial Approach Applied to Africa." *International Organization* 71(1):31–64.

Grossman, Herschel I. 1999. "Kleptocracy and Revolutions." *Oxford Economic Papers* 51(2):267–283.

Hafer, Catherine. 2006. "On the Origins of Property Rights: Conflict and Production in the State of Nature." *Review of Economic Studies* 73(1):119–143.

Hidalgo, F. Daniel, Suresh Naidu, Simeon Nichter and Neal Richardson. 2010. "Economic Determinants of Land Invasions." *Review of Economics and Statistics* 92(3):505–523.

Hillman, Arye L. and John G. Riley. 1989. "Politically Contestable Rents and Transfers." *Economics & Politics* 1(1):17–39.

Hirshleifer, Jack. 1991. "The Technology of Conflict as an Economic Activity." *American Economic Review (Papers & Proceedings)* pp. 130–134.

Krishna, Vijay and John Morgan. 1997. "An Analysis of the War of Attrition and the All-Pay Auction." *Journal of Economic Theory* 72(2):343–362.

Lessing, Benjamin. 2013. When Business Gets Bloody: State Policy and Drug Violence. In *Small Arms Survey 2012*. New York: Cambridge University Press.

Maystadt, Jean-François and Olivier Ecker. 2014. "Extreme Weather and Civil War: Does Drought Fuel Conflict in Somalia through Livestock Price Shocks?" *American Journal of Agricultural Economics* 96:1157–1181.

Mearsheimer, John J. 1988. "Numbers, Strategy, and the European Balance." *International Security* 12(4):174–185.

Mejia, Daniel and Pascual Restrepo. 2013. "Bushes and Bullets: Illegal Cocaine Markets and Violence in Colombia." Documento CEDE No. 2013-53. Available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2353979.

Mitra, Anirban and Debraj Ray. 2014. "Implications of an Economic Theory of Conflict: Hindu-Muslim Violence in India." *Journal of Political Economy* 122(4):719–765.

Powell, Robert. 2004. "The Inefficient Use of Power: Costly Conflict with Complete Information." *American Political Science Review* 98(2):231–241.

Rios, Viridiana. 2013. "Why Did Mexico Become So Violent? A Self-Reinforcing Violent Equilibrium Caused by Competition and Enforcement." *Trends in Organized Crime* 16(2):138–155.

Rohner, Dominic, Mathias Thoenig and Fabrizio Zilibotti. 2013. "War Signals: A Theory of Trade, Trust, and Conflict." *Review of Economic Studies* 80(3):1114–1147.

Appendices
"Territorial Conflict over Endogenous Rents"

A Economic Equilibrium

Consider two contiguous territories, i and j, charging prices p_i and p_j. A population member located between i and j at distance x from i will purchase from i rather than purchasing from j or staying home if:

$$p_i + tx \leq p_j + t\left(\frac{1}{4} - x\right) \qquad \text{and} \qquad 1 - p_i - tx \geq 0.$$

The population member who is indifferent between purchasing from i and j is located at distance $x^*_{i,j}$ from i, given by:

$$x^*_{i,j} = \frac{1}{8} + \frac{p_j - p_i}{2t}.$$

Plugging this in and rearranging, this population member will purchase if

$$p_i \leq 2 - p_j - \frac{t}{4}. \tag{4}$$

If Condition 4 holds, demand at territory i from population members located between i and j is:

$$\mathcal{D}_i(p_i, p_j) = \begin{cases} \frac{N}{4} & \text{if } p_i < p_j - \frac{t}{4} \\ N\left(\frac{1}{8} + \frac{p_j - p_i}{2t}\right) & \text{if } p_i \in \left[p_j - \frac{t}{4}, p_j + \frac{t}{4}\right] \\ 0 & \text{if } p_i > p_j + \frac{t}{4}. \end{cases} \tag{5}$$

A.1 Two Territories Each

Suppose each group controls two territories.

If demand is characterized by Equation 5 at some vector of prices, profits from territory i are:

$$p_i \left[\mathcal{D}_i(p_i, p_{i+1}) + \mathcal{D}_i(p_i, p_{i-1})\right].$$

Given the symmetry of the groups, equilibrium prices are characterized by the following condition:

$$N\left[\frac{2p^* - 2p^*}{2t} + \frac{1}{4}\right] - \frac{Np^*}{t} = 0.$$

This implies that in equilibrium the common price is

$$p^*_{2,2} = \frac{t}{2}$$

Note that for any $t \leq 1$, we have $p_i \leq 2 - p_j - \frac{t}{4}$ for all i, j, so demand is in fact characterized by Equation 5. Each group's equilibrium rents are

$$u^{2,2} = 2 \cdot \frac{t}{2} \cdot \frac{N}{4} = \frac{Nt}{4}.$$

A.2 Three Territories, One Territory

Suppose one group controls three territories and the other group controls two. Without loss of generality, suppose the large group controls territories A, B, and C. If demand is characterized by Equation 5 at some vector of prices, the large group's profits are:

$$N\left[p_A\left(\frac{1}{4} + \frac{p_B + p_D - 2p_A}{2t}\right) + p_B\left(\frac{1}{4} + \frac{p_A + p_C - 2p_B}{2t}\right) + p_C\left(\frac{1}{4} + \frac{p_B + p_D - 2p_C}{2t}\right)\right],$$

and the small group's profits are:

$$Np_D\left(\frac{1}{4} + \frac{p_A + p_C - 2p_D}{2t}\right).$$

An equilibrium is described by the following first-order conditions:

$$\frac{1}{4} + \frac{p^*_B + p^*_D - 2p^*_A}{2t} - \frac{p^*_A}{t} + \frac{p^*_B}{2t} = 0$$

$$\frac{p^*_A}{2t} + \frac{1}{4} + \frac{p^*_A + p^*_C - 2p^*_A}{2t} - \frac{p^*_B}{t} + \frac{p^*_C}{2t} = 0$$

$$\frac{p^*_B}{2t}\frac{1}{4} + \frac{p^*_B + p^*_D - 2p^*_C}{2t} - \frac{p^*_C}{t} = 0$$

$$\frac{1}{4} + \frac{p^*_A + p^*_C - 2p^*_D}{2t} - \frac{p^*_D}{t} = 0.$$

This implies that in equilibrium we have:

$$p^*_A = p^*_C = \frac{7t}{12} \qquad p^*_B = \frac{17t}{24} \qquad p^*_D = \frac{5t}{12}.$$

Note that for any $t \leq 1$, we have $p_i \leq 2 - p_j - \frac{t}{4}$ for all i, j, so demand is in fact characterized by Equation 5.

Rents for the large group, the border groups, and the interior groups, respectively, are

$$u^{\mathbf{3},1} = \frac{205Nt}{576} \qquad u^{3,\mathbf{1}} = \frac{25Nt}{144}.$$

B Economic Equilibria for Local Comparative Statics

B.1 Local Market Size

Without loss of generality, suppose the two factions start controlling A, B and C, D. To find the incremental returns, I start by characterizing equilibrium in the scenarios: AB, CD, ABD, C, B, ACD, A, BCD, and ABC, D.

For a given vector of prices, demand is the same as in Equation 5 at territory B but may be changed at A, C, and D. Assuming $p_D \leq 2 - p_j - \frac{t}{6}$, for $j \in \{A, C\}$, demand at territory D from the part of the population between D and j is:

$$\mathcal{D}_D(p_D, p_j) = \begin{cases} \frac{(1+\eta)N}{4} & \text{if } p_D \leq p_j - \frac{t}{4} \\ \frac{\eta N}{8} + N\left(\frac{p_j - p_D}{2t}\right) & \text{if } p_D \in \left(p_j - \frac{t}{4}, p_j\right) \\ \eta N\left(\frac{1}{8} + \frac{p_j - p_D}{2t}\right) & \text{if } p_D \in \left(p_j, p_j + \frac{t}{4}\right) \\ 0 & \text{if } p_D \geq p_j + \frac{t}{4}. \end{cases} \tag{6}$$

For territory $j \in \{A, D\}$, demand from the part of the population between j and D is the complement.

B.1.1 AB, CD

There are four cases to consider:

1. Suppose $p_A < p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(15\eta^2 + 64\eta + 26\right)t}{6\left(15\eta^2 + 16\eta + 4\right)}$$

$$p_B = \frac{\left(27\eta^2 + 58\eta + 20\right)t}{6\left(15\eta^2 + 16\eta + 4\right)}$$

$$p_C = \frac{\left(33\eta^2 + 56\eta + 16\right)t}{6\left(15\eta^2 + 16\eta + 4\right)}$$

$$p_D = \frac{\left(30\eta^2 + 59\eta + 16\right) t}{6\left(15\eta^2 + 16\eta + 4\right)}$$

These prices are consistent with $p_A < p_D$ and $p_C \geq p_D$ for any $\eta \in [1, 2]$. Hence, this case is a candidate for an equilibrium.

2. Suppose $p_A < p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(8\eta^2 + 45\eta + 52\right) t}{6\left(8\eta^2 + 19\eta + 8\right)}$$

$$p_B = \frac{\left(14\eta^2 + 51\eta + 40\right) t}{6\left(8\eta^2 + 19\eta + 8\right)}$$

$$p_C = \frac{\left(16\eta^2 + 57\eta + 32\right) t}{6\left(8\eta^2 + 19\eta + 8\right)}$$

$$p_D = \frac{\left(16\eta^2 + 54\eta + 35\right) t}{6\left(8\eta^2 + 19\eta + 8\right)}$$

These prices are inconsistent with $p_C < p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

3. Suppose $p_A \geq p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(12\eta^2 + 67\eta + 26\right) t}{6(24\eta + 11)}$$

$$p_B = \frac{\left(12\eta^2 + 64\eta + 29\right) t}{6(24\eta + 11)}$$

$$p_C = \frac{\left(24\eta^2 + 50\eta + 31\right) t}{6(24\eta + 11)}$$

$$p_D = \frac{\left(24\eta^2 + 68\eta + 13\right) t}{6(24\eta + 11)}$$

These prices are inconsistent with $p_A \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

4. Suppose $p_A \geq p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking

first-order conditions and solving yields:

$$p_A = \frac{(19\eta + 86)t}{210}$$

$$p_B = \frac{(16\eta + 89)t}{210}$$

$$p_C = \frac{(26\eta + 79)t}{210}$$

$$p_D = \frac{(44\eta + 61)t}{210}$$

These prices are inconsistent with $p_A \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

We have only one candidate for an equilibrium (case 1). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 6. In this candidate profile of prices, the prices are ordered as follows:

$$p_C > p_D > p_B > p_A.$$

Hence, the worst-off population member is the one who is just indifferent between buying from C and D. This person is located at a distance from C given by:

$$x^*_{CD} = \frac{13\eta^2 + 18\eta + 4}{120\eta^2 + 128\eta + 32}.$$

This person prefers to buy the good as long as:

$$1 - p_C - x^*_{CD}t \geq 0.$$

This is true if and only if:

$$\frac{549\eta^2 + 490\eta + 116}{720\eta^2 + 768\eta + 192} \geq 0,$$

which is the case for any $\eta \in [1, 2]$.

Equilibrium rents are:

$$u^{\mathbf{AB},CD}(\eta) = \frac{\left(33\eta^2 + 56\eta + 16\right) Nt}{6\left(15\eta^2 + 16\eta + 4\right)}$$

and

$$u^{AB,\mathbf{CD}}(\eta) = \frac{\left(30\eta^2 + 59\eta + 16\right) Nt}{6\left(15\eta^2 + 16\eta + 4\right)}.$$

B.1.2 ABD, C

There are four cases to consider:

1. Suppose $p_A < p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(15\eta^2 + 31\eta + 22\right) t}{2(45\eta + 19)}$$

$$p_B = \frac{\left(10\eta^2 + 52\eta + 25\right) t}{4(45\eta + 19)}$$

$$p_C = \frac{\left(5\eta^2 + 28\eta + 9\right) t}{2(45\eta + 19)}$$

$$p_D = \frac{\left(30\eta^2 + 82\eta + 9\right) t}{4(45\eta + 19)}$$

   These prices are inconsistent with $p_C \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

2. Suppose $p_A < p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(4\eta + 13)t}{4(3\eta + 5)}$$

$$p_B = \frac{\left(12\eta^2 + 43\eta + 32\right) t}{16(\eta + 1)(3\eta + 5)}$$

$$p_C = \frac{\left(2\eta^2 + 10\eta + 9\right) t}{4(\eta + 1)(3\eta + 5)}$$

$$p_D = ((45 + 60\eta + 16\eta^2)t)/(16(1 + \eta)(5 + 3\eta))$$

   These prices are inconsistent with $p_A < p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

3. Suppose $p_A \geq p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(18\eta^2 + 37\eta + 13\right) t}{12(\eta+1)(3\eta+1)}$$

$$p_B = \frac{\left(15\eta^2 + 31\eta + 10\right) t}{12(\eta+1)(3\eta+1)}$$

$$p_C = \frac{(\eta+4)t}{6(\eta+1)}$$

$$p_D = \frac{\left(12\eta^2 + 31\eta + 13\right) t}{12(\eta+1)(3\eta+1)}$$

These prices are consistent with $p_C < p_D < p_A$ for all $\eta \in [1, 2]$, so this case is a candidate for an equilibrium.

4. Suppose $p_A \geq p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(4\eta + 13)t}{24}$$

$$p_B = \frac{(5\eta + 23)t}{48}$$

$$p_C = \frac{(\eta + 4)t}{12}$$

$$p_D = \frac{(11\eta + 17)t}{48}$$

These prices are inconsistent with $p_C \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case 3). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 6. In this profile, prices are ordered as follows:

$$p_A > p_B > p_D > p_C.$$

Hence, the worst-off population member is the one who is just indifferent between A and

B. This person's distance from A is

$$x^*_{AB} = \frac{\eta}{12\eta + 4}.$$

Plugging this in, the person indifferent between A and B prefers to purchase the good if:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if and only if:

$$\frac{\eta^2(36 - 21t) - 8\eta(5t - 6) - 13t + 12}{12\left(3\eta^2 + 4\eta + 1\right)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$ the inequality holds if and only if $\frac{15n^2+8n-1}{12(3n^2+4n+1)} \geq 0$, which is true for every $\eta \in [1, 2]$.

The equilibrium rents are

$$u^{\mathbf{ABD},C}(\eta) = \frac{\left(48\eta^3 + 274\eta^2 + 389\eta + 109\right) Nt}{288(\eta + 1)(3\eta + 1)}$$

$$u^{ABD,\mathbf{C}}(\eta) = \frac{(\eta + 4)^2 Nt}{72(\eta + 1)}.$$

B.1.3 B, ACD

There are four cases to consider:

1. Suppose $p_A < p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(18\eta^2 + 37\eta + 13\right) t}{12\left(3\eta^2 + 4\eta + 1\right)}$$

$$p_B = \frac{\left(15\eta^2 + 31\eta + 10\right) t}{12(\eta + 1)(3\eta + 1)}$$

$$p_C = \frac{(\eta + 4)t}{6(\eta + 1)}$$

$$p_D = \frac{\left(12\eta^2 + 31\eta + 13\right) t}{12(\eta + 1)(3\eta + 1)}$$

These prices are inconsistent with $p_C \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

2. Suppose $p_A < p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(2\eta + 5)t}{12}$$

$$p_B = \frac{(\eta + 4)t}{12}$$

$$p_C = \frac{(2\eta + 5)t}{12}$$

$$p_D = \frac{(4\eta + 13)t}{24}$$

These prices are consistent with $p_A < p_D$ and $p_C < p_D$ for all $\eta \in [1, 2]$, so this case is a candidate for an equilibrium.

3. Suppose $p_A \geq p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(9\eta^2 + 37\eta + 10\right) t}{24(3\eta + 1)}$$

$$p_B = \frac{(\eta + 4)t}{12}$$

$$p_C = \frac{\left(15\eta^2 + 31\eta + 10\right) t}{24(3\eta + 1)}$$

$$p_D = \frac{\left(15\eta^2 + 43\eta + 10\right) t}{24(3\eta + 1)}$$

These prices are inconsistent with $p_A \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

4. Suppose $p_A \geq p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(2\eta + 5)t}{12}$$

$$p_B = \frac{(\eta + 4)t}{12}$$

$$p_C = \frac{(2\eta + 5)t}{12}$$

$$p_D = \frac{(7\eta + 10)t}{24}$$

These prices are inconsistent with $p_C \geq p_D$ or $p_A \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case 2). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 6. In this profile, prices are ordered as follows:

$$p_D > p_A = p_C > p_B.$$

Hence, the worst-off population member is the one who is just indifferent between either A and D or C and D. The consumer between A and D who is indifferent's distance form A is

$$x^*_{AD} = \frac{3}{16}.$$

Plugging this in, the person indifferent between E and F prefers to purchase the good if:

$$1 - p_A - x^*_{AD} t \geq 0$$

which is true if and only if:

$$\frac{(39 - 4(2\eta + 5)t)}{48} \geq 0.$$

The left-hand side is decreasing in t and in η, so it suffices to check $t = 1$ and $\eta = 2$, where the inequality holds.

The equilibrium rents are

$$u^{\mathbf{ACD},B}(\eta) = \frac{\left(16\eta^2 + 89\eta + 100\right) Nt}{576}$$

$$u^{ACD,\mathbf{B}}(\eta) = \frac{(\eta + 4)^2 Nt}{144}.$$

B.1.4 A, BCD

There are four cases to consider:

1. Suppose $p_A < p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking

first-order conditions and solving yields:

$$p_A = \frac{(\eta+4)t}{6(\eta+1)}$$

$$p_B = \frac{\left(15\eta^2+31\eta+10\right)t}{12(\eta+1)(3\eta+1)}$$

$$p_C = \frac{\left(18\eta^2+37\eta+13\right)t}{12(\eta+1)(3\eta+1)}$$

$$p_D = \frac{\left(12\eta^2+31\eta+13\right)t}{12(\eta+1)(3\eta+1)}$$

These prices are consistent with $p_A \leq p_D$ and $p_D \geq p_C$ for any $\eta \in [1,2]$, so this is a candidate for an equilibrium.

2. Suppose $p_A < p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(\eta+4)t}{6(\eta+1)}$$

$$p_B = \frac{(2\eta+5)t}{6(\eta+1)}$$

$$p_C = \frac{(4\eta+13)t}{12(\eta+1)}$$

$$p_D = \frac{(2\eta+5)t}{6(\eta+1)}$$

These prices are inconsistent with $p_C \leq p_D$ for any $\eta \in [1,2]$, so there is no such equilibrium.

3. Suppose $p_A \geq p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(\eta+4)t}{12}$$

$$p_B = \frac{\left(9\eta^2+31\eta+16\right)t}{24(3\eta+1)}$$

$$p_C = \frac{\left(15\eta^2 + 31\eta + 22\right)t}{24(3\eta + 1)}$$

$$p_D = \frac{\left(15\eta^2 + 37\eta + 4\right)t}{24(3\eta + 1)}$$

These prices are inconsistent with $p_A \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

4. Suppose $p_A \geq p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(\eta + 4)t}{12}$$

$$p_B = \frac{(5\eta + 23)t}{48}$$

$$p_C = \frac{(4\eta + 13)t}{24}$$

$$p_D = \frac{(11\eta + 17)t}{48}$$

These prices are inconsistent with $p_A \geq p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case 1). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 6. In this profile, prices are ordered as follows:

$$p_C > p_B > p_D > p_A.$$

Hence, the worst-off population member is the one who is just indifferent between B and C. This consumer's distance from B is

$$x^*_{BC} = \frac{2\eta + 1}{12\eta + 4}.$$

Plugging this in, the person indifferent between D and E prefers to purchase the good if:

$$1 - p_B - x^*_{BC}t \geq 0,$$

which is true if and only if:

$$\frac{\eta^2(36-21t)-8\eta(5t-6)-13t+12}{12\left(3\eta^2+4\eta+1\right)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t=1$. At $t=1$, the inequality holds if $15\eta^2+8\eta-10$, which is true for any $\eta \in [1,2]$.

The equilibrium rents are

$$u^{\mathbf{A},BCD}(\eta) = \frac{(\eta+4)^2 Nt}{72(\eta+1)}$$

$$u^{A,\mathbf{BCD}}(\eta) = \frac{\left(48\eta^3+274\eta^2+389\eta+109\right) Nt}{288(\eta+1)(3\eta+1)}.$$

B.1.5 ABC, D

There are four cases to consider:

1. Suppose $p_A < p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(6\eta^2+29\eta+21\right) t}{12\left(3\eta^2+4\eta+1\right)}$$

$$p_B = \frac{\left(15\eta^2+35\eta+18\right) t}{12(\eta+1)(3\eta+1)}$$

$$p_C = \frac{\left(15\eta^2+29\eta+12\right) t}{12(\eta+1)(3\eta+1)}$$

$$p_D = \frac{(2\eta+3)t}{6(\eta+1)}.$$

   These prices are inconsistent with $p_A \leq p_D$ for any $\eta \in [1,2]$, so there is no such equilibrium.

2. Suppose $p_A < p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(\eta+6)t}{12\eta}$$

$$p_B = \frac{(5\eta + 12)t}{24\eta}$$

$$p_C = \frac{(\eta + 6)t}{12\eta}$$

$$p_D = \frac{(2\eta + 3)t}{12\eta}.$$

These prices are inconsistent with $p_A < p_D$ or $p_C < p_D$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

3. Suppose $p_A \geq p_D$ and $p_C < p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(15\eta^2 + 29\eta + 12\right) t}{12\left(3\eta^2 + 4\eta + 1\right)}$$

$$p_B = \frac{\left(15\eta^2 + 35\eta + 18\right) t}{12\left(3\eta^2 + 4\eta + 1\right)}$$

$$p_C = \frac{\left(6\eta^2 + 29\eta + 21\right) t}{12\left(3\eta^2 + 4\eta + 1\right)}$$

$$p_D = \frac{(2\eta + 3)t}{6(\eta + 1)}$$

These prices are inconsistent with $p_C \geq p_D$ for any $\eta \in [1, 2]$, so there in so such equilibrium.

4. Suppose $p_A \geq p_D$ and $p_C \geq p_D$. If demand is given by Equations 5 and 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(\eta + 6)t}{12}$$

$$p_B = \frac{(2\eta + 15)t}{24}$$

$$p_C = \frac{(\eta + 6)t}{12}$$

$$p_D = \frac{(2\eta + 3)t}{12}$$

These prices are consistent with $p_A \leq p_D$ and $p_C \geq p_D$ for any $\eta \in [1, 2]$, so this is a candidate for an equilibrium.

We have only one candidate for equilibrium (case 4). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 6. In this profile, prices are ordered as follows:

$$p_B > p_A = p_C > p_D.$$

Hence, the worst-off population member is the one who is just indifferent between B and either A or C. The distance of the indifferent consumer between A and B from A is

$$x^*_{AB} = \frac{3}{16}.$$

Plugging this in, the person indifferent between A and B prefers to purchase the good if:

$$1 - p_A - x^*_{AB} t \geq 0.$$

This is true if

$$\frac{48 - (4\eta + 33)t}{48} \geq 0.$$

which is true for any $(\eta, t) \in [1, 2] \times (0, 1]$.

The equilibrium rents are

$$u^{\mathbf{ABC},D}(\eta) = \frac{\left(4\eta^2 + 48\eta + 153\right) Nt}{576}$$

$$u^{ABC,\mathbf{D}}(\eta) = \frac{(2\eta + 3)^2 Nt}{144}.$$

B.2 Local Transportation Costs

Without loss of generality, suppose the two factions start controlling A, B and C, D. To find the incremental returns, I first characterize equilibrium in the five scenarios: AB, CD, ABD, C, B, ACD, A, BCD, and ABC, D.

For a given vector of prices, demand is the same as in Equation 5 at territory B but it may be changed at A, C, and D. Fix a vector of prices. As long as $p_D \leq \frac{\tau+1}{\tau} - \frac{p_j}{\tau} - \frac{t}{4\tau}$, for

$j \in \{A, D\}$, demand at territory D from the part of the population between D and j is:

$$\mathcal{D}_D(p_D, p_j) = \begin{cases} \frac{N}{4} & \text{if } p_D \leq p_j - \frac{\tau t}{4} \\ N\left(\frac{1}{4(\tau+1)} + \frac{p_j - p_D}{t(\tau+1)}\right) & \text{if } p_D \in \left(p_j - \frac{\tau t}{4}, p_j + \frac{\tau t}{4}\right) \\ 0 & \text{if } p_D \geq p_j + \frac{\tau t}{4}. \end{cases} \tag{7}$$

For territory $j \in \{A, D\}$, demand from the population between j and D is the complement.

B.2.1 AB, CD

Assuming that demand is interior and given by Equations 5 and 7, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(35\tau^2 + 109\tau + 66\right) Nt}{6\left(4\tau^2 + 27\tau + 39\right)}$$

$$p_B = \frac{\left(52\tau^2 + 203\tau + 165\right) Nt}{12\left(4\tau^2 + 27\tau + 39\right)}$$

$$p_C = \frac{\left(22\tau^2 + 107\tau + 81\right) Nt}{6\left(4\tau^2 + 27\tau + 39\right)}$$

$$p_D = \frac{\left(91\tau^2 + 404\tau + 345\right) Nt}{24\left(4\tau^2 + 27\tau + 39\right)}$$

For this to be an equilibrium, it must be that demand is interior and that the worst off consumer prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 7.

For demand to be interior, we need

$$p_D \in \left(p_j - \frac{\tau t}{4}, p_j + \frac{\tau t}{4}\right),$$

for $j \in \{A, C\}$. This follows from direct comparison.

The order of prices is $p_A > p_B > p_D > p_C$. Hence, there are two candidates for the worst-off consumer: the consumer indifferent between buying from A and B and the consumer indifferent between buying from A and D.

The consumer indifferent between A and B is located at a distance from A given by

$$x^*_{AB} = \frac{1}{8} + \frac{p_B - p_A}{2t}..$$

We need the following:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if

$$\frac{-67t\tau^2 - 251t\tau - 207t + 48\tau^2 + 324\tau + 468}{48\tau^2 + 324\tau + 468} \geq 0.$$

This is true if and only if the numerator is positive. The numerator is decreasing in t, so it suffices to check at $t = 1$, where the numerator is clearly positive.

The consumer indifferent between A and D is located at a distance from A given by

$$x^*_{AD} = \frac{\tau}{4(\tau+1)} + \frac{p_D - p_A}{t(\tau+1)}.$$

We need the following:

$$1 - p_A - x^*_{AD} t \geq 0$$

which is true if

$$\frac{24\left(4\tau^3 + 31\tau^2 + 66\tau + 39\right) - t\left(164\tau^3 + 689\tau^2 + 902\tau + 345\right)}{24(\tau+1)\left(4\tau^2 + 27\tau + 39\right)} \geq 0.$$

Again, it suffices to check at $t = 1$, where the inequality clearly holds.

The rents at these equilibrium prices are:

$$u^{\mathbf{AB},CD}(\tau) = \frac{= \left(1514\tau^5 + 17240\tau^4 + 70049\tau^3 + 127707\tau^2 + 104709\tau + 31581\right) Nt}{144(\tau+1)\left(4\tau^2 + 27\tau + 39\right)^2}$$

and

$$u^{AB,\mathbf{CD}}(\tau) = \frac{\left(1936\tau^5 + 24913\tau^4 + 115576\tau^3 + 242742\tau^2 + 234456\tau + 85977\right) Nt}{288(\tau+1)\left(4\tau^2 + 27\tau + 39\right)^2}.$$

B.2.2 ABD, C

Assuming that demand is interior and given by Equations 5 and 7, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{t(35\tau + 33)}{24(\tau+3)}$$

$$p_B = \frac{t(13\tau + 15)}{12(\tau+3)}$$

$$p_C = \frac{t(11\tau + 9)}{12(\tau + 3)}$$

$$p_D = \frac{t(13\tau + 15)}{12(\tau + 3)}$$

For this to be an equilibrium, it must be that demand is interior and that the worst off consumer prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 7.

For demand to be interior, we need

$$p_D \in \left(p_j - \frac{\tau t}{4}, p_j + \frac{\tau t}{4}\right),$$

for $j \in \{A, C\}$. This follows from direct comparison.

The order of prices is $p_A > p_B = p_D > p_C$. Hence, there are two candidates for the worst-off consumer: the consumer indifferent between buying from A and B and the consumer indifferent between buying from A and D.

The consumer indifferent between A and B is located at a distance from A given by

$$x^*_{AB} = \frac{1}{8} + \frac{p_B - p_A}{2t}..$$

We need the following:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if

$$\frac{48(\tau + 3) - t(67\tau + 81)}{48(\tau + 3)} \geq 0.$$

This is true if and only if the numerator is positive. The numerator is decreasing in t, so it suffices to check at $t = 1$, where the numerator is equal to $63 - 19\tau$, which is positive for any $\tau \in [1, \frac{27}{16}]$.

The consumer indifferent between A and D is located at a distance from A given by

$$x^*_{AD} = \frac{\tau}{4(\tau + 1)} + \frac{p_D - p_A}{t(\tau + 1)}.$$

We need the following:

$$1 - p_A - x^*_{AD} t \geq 0$$

which is true if

$$\frac{24\left(\tau^2+4\tau+3\right)-t\left(41\tau^2+77\tau+30\right)}{24(\tau+1)(\tau+3)} \geq 0.$$

Again, it suffices to check at $t=1$, where the numerator is $-17\tau^2+19\tau+42$, which is positive for any $\tau \in [1, \frac{27}{16}]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{ABD},C}(\tau)=\frac{t\left(757\tau^2+1614\tau+909\right)}{1152(\tau+1)(\tau+3)}$$

and

$$u^{ABD,\mathbf{C}}(\tau)=\frac{t(11\tau+9)^2}{288(\tau+1)(\tau+3)}.$$

B.2.3 B, ACD

Assuming that demand is interior and given by Equations 5 and 7, taking first-order conditions and solving gives the following prices:

$$p_A=\frac{7t}{12}$$

$$p_B=\frac{5t}{12}$$

$$p_C=\frac{7t}{12}$$

$$p_D=\frac{17t}{24}$$

For this to be an equilibrium, it must be that demand is interior and that the worst off consumer prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 7.

For demand to be interior, we need

$$p_D \in \left(p_j-\frac{\tau t}{4}, p_j+\frac{\tau t}{4}\right),$$

for $j \in \{A, C\}$. This follows from direct comparison.

The order of prices is $p_D > p_A = p_C > p_B$. Hence, there are two candidates for the worst-off consumer: the consumer indifferent between buying from A and B and the consumer indifferent between buying from A and D.

The consumer indifferent between A and B is located at a distance from A given by

$$x^*_{AB} = \frac{1}{8} + \frac{p_B - p_A}{2t}..$$

We need the following:

$$1 - p_A - x^*_{AB}t \geq 0$$

which is true if

$$1 - \frac{5t}{8} \geq 0,$$

which is true for any $t \in (0, 1)$.

The consumer indifferent between A and D is located at a distance from A given by

$$x^*_{AD} = \frac{\tau}{4(\tau+1)} + \frac{p_D - p_A}{t(\tau+1)}.$$

We need the following:

$$1 - p_A - x^*_{AD}t \geq 0$$

which is true if

$$\frac{-20t\tau - 17t + 24\tau + 24}{24\tau + 24} \geq 0.$$

This is decreasing in t, so it suffices to check at $t = 1$, where it is clearly positive for any $\tau \in [1, \frac{27}{16}]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{ACD},B}(\tau) = \frac{(98\tau + 107)Nt}{288(\tau+1)}$$

and

$$u^{ACD,\mathbf{B}}(\tau) = \frac{25t}{144}.$$

B.2.4 A, BCD

Assuming that demand is interior and given by Equations 5 and 7, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{t(11\tau + 9)}{12(\tau+3)}$$

$$p_B = \frac{t(13\tau + 15)}{12(\tau+3)}$$

$$p_C = \frac{t(35\tau + 33)}{24(\tau + 3)}$$

$$p_D = \frac{t(13\tau + 15)}{12(\tau + 3)}$$

For this to be an equilibrium, it must be that demand is interior and that the worst off consumer prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 7.

For demand to be interior, we need

$$p_D \in \left(p_j - \frac{\tau t}{4}, p_j + \frac{\tau t}{4}\right),$$

for $j \in \{A, C\}$. This follows from direct comparison.

The order of prices is $p_C > p_B = p_D > p_A$. Hence, there are two candidates for the worst-off consumer: the consumer indifferent between buying from B and C and the consumer indifferent between buying from C and D.

The consumer indifferent between B and C is located at a distance from B given by

$$x^*_{BC} = \frac{1}{8} + \frac{p_C - p_B}{2t}..$$

We need the following:

$$1 - p_B - x^*_{BC} t \geq 0$$

which is true if

$$\frac{24\left(\tau^2 + 4\tau + 3\right) - t\left(41\tau^2 + 77\tau + 30\right)}{24(\tau + 1)(\tau + 3)} \geq 0.$$

The left-hand side is decreasing in t, so it suffices to check at $t = 1$, where the left-hand side equals $\frac{63-19\tau}{48(\tau+3)}$, which is clearly positive for any $\tau \in [1, \frac{27}{16}]$.

The consumer indifferent between C and D is located at a distance from C given by

$$x^*_{CD} = \frac{\tau}{4(\tau + 1)} + \frac{p_D - p_C}{t(\tau + 1)}.$$

We need the following:

$$1 - p_C - x^*_{CD} t \geq 0$$

which is true if

$$\frac{24\left(\tau^2 + 4\tau + 3\right) - t\left(41\tau^2 + 90\tau + 45\right)}{24(\tau + 1)(\tau + 3)} \geq 0.$$

This is decreasing in t, so it suffices to check at $t = 1$, where it is clearly positive for any $\tau \in [1, \frac{27}{16}]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{A},BCD}(\tau) = \frac{t(11\tau + 9)^2}{288(\tau + 1)(\tau + 3)}$$

and

$$u^{A,\mathbf{BCD}}(\tau) = \frac{t\left(757\tau^2 + 1614\tau + 909\right)}{1152(\tau + 1)(\tau + 3)}.$$

B.2.5 ABC, D

Assuming that demand is given by Equations 5 and 7, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{(4\tau + 3)t}{12}$$

$$p_B = \frac{(8\tau + 9)t}{24}$$

$$p_C = \frac{(4\tau + 3)t}{12}$$

$$p_D = \frac{(2\tau + 3)t}{12}$$

For this to be an equilibrium, it must be that demand is interior and that the worst off consumer prefers to purchase the good, so that demand is in fact characterized by Equations 5 and 7.

For demand to be interior, we need

$$p_D \in \left(p_j - \frac{\tau t}{4}, p_j + \frac{\tau t}{4}\right),$$

for $j \in \{A, C\}$. This follows from direct comparison.

The order of prices is $p_B > p_A = p_C > p_D$. Hence, the worst-off consumer is either the one indifferent between buying from A and B or the one indifferent between C and D.

The consumer indifferent between A and B is located at a distance from A given by

$$x^*_{AB} = \frac{1}{8} + \frac{p_B - p_A}{2t}.$$

We need the following:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if

$$1 - \frac{(16\tau + 21)t}{48} \geq 0.$$

The left-hand side is decreasing in t, so it suffices to check at $t = 1$, where the left-hand side is positive if $\tau \leq \frac{27}{16}$.

The consumer indifferent between C and D is located at a distance from C given by

$$x^*_{CD} = \frac{\tau}{4(\tau+1)} + \frac{p_D - p_C}{t(\tau+1)}.$$

We need the following:

$$1 - p_C - x^*_{CD} t \geq 0$$

which is true if

$$\frac{12(\tau+1) - t\left(4\tau^2 + 8\tau + 3\right)}{12(\tau+1)} \geq 0.$$

This is decreasing in t, so it suffices to check at $t = 1$, where the left-hand side equals $\frac{-4\tau^2+4\tau+9}{12\tau+12}$, which is clearly positive for any $\tau \in [1, \frac{27}{16}]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{ABC},D}(\tau) = \frac{\left(128\tau^2 + 201\tau + 81\right) Nt}{576(\tau+1)}$$

and

$$u^{ABC,\mathbf{D}}(\tau) = \frac{(2\tau+3)^2 Nt}{72(\tau+1)}.$$

C Local Comparative Statics

In this appendix I provide proofs of results for the local comparative statics. Characterization of the economic equilibria are in Appendix B.

C.1 Proofs for Local Market Size Shocks

Proof of Proposition 5.1. From Appendix B.1, rents under AB, CD and under ABD, C are:

$$u^{\mathbf{AB},CD}(\eta) = \frac{\left(33\eta^2 + 56\eta + 16\right) Nt}{6\left(15\eta^2 + 16\eta + 4\right)}$$

$$u^{AB,\mathbf{CD}}(\eta) = \frac{\left(30\eta^2 + 59\eta + 16\right) Nt}{6\left(15\eta^2 + 16\eta + 4\right)}.$$

$$u^{\mathbf{ABD},C}(\eta) = \frac{\left(48\eta^3 + 274\eta^2 + 389\eta + 109\right) Nt}{288(\eta+1)(3\eta+1)}$$

$$u^{ABD,\mathbf{C}}(\eta) = \frac{(\eta+4)^2 Nt}{72(\eta+1)}.$$

Hence, if conflict is over D, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{pop}}_{\text{att},D}(\eta) &= u^{\mathbf{ABD},C}(\eta) - u^{\mathbf{AB},CD}(\eta) \\ &= \frac{\eta\left(8100\eta^6 + 47574\eta^5 + 97137\eta^4 + 91449\eta^3 + 43528\eta^2 + 10184\eta + 928\right) Nt}{288(\eta+1)(3\eta+1)(3\eta+2)^2(5\eta+2)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\text{pop}}_{\text{def},F}(\eta) &= u^{AB,\mathbf{CD}}(\eta) - u^{ABD,\mathbf{C}}(\eta) \\ &= \frac{\eta\left(675\eta^5 + 3258\eta^4 + 4941\eta^3 + 3392\eta^2 + 1081\eta + 128\right) Nt}{72(\eta+1)(3\eta+2)^2(5\eta+2)^2}.\end{aligned}$$

1. Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{pop}}_{\text{att},D}(\eta)}{\partial \eta} = \frac{\left(364500\eta^{10}+2138400\eta^9+5974479\eta^8+10702746\eta^7+13101759\eta^6+10922808\eta^5+6108492\eta^4+2238880\eta^3+513600\eta^2+66624\eta+3712\right)Nt}{288(\eta+1)^2(3\eta+1)^2(3\eta+2)^3(5\eta+2)^3}$$

   and

$$\frac{\partial \mathrm{IR}^{\text{pop}}_{\text{def},D}(\eta)}{\partial \eta} = \frac{\left(10125\eta^8+52650\eta^7+135711\eta^6+202008\eta^5+183039\eta^4+103058\eta^3+35172\eta^2+6600\eta+512\right)Nt}{72(\eta+1)^2(3\eta+2)^3(5\eta+2)^3},$$

   both of which are clearly positive for any $\eta \in [1,2]$.

2. First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, we have:

$$\mathrm{IR}^{\text{pop}}_{\text{att},D}(\eta) - \mathrm{IR}^{\text{pop}}_{\text{def},D}(\eta) = \frac{\eta\left(5778\eta^5 + 24813\eta^4 + 30981\eta^3 + 16988\eta^2 + 4324\eta + 416\right) Nt}{288(\eta+1)(3\eta+1)(3\eta+2)^2(5\eta+2)^2},$$

   which is clearly positive for any $\eta \in [1,2]$.

   Thus, expected observed violence is

$$\frac{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},D}(\eta)^2}{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},D}(\eta)} = \frac{\eta\left(675\eta^5 + 3258\eta^4 + 4941\eta^3 + 3392\eta^2 + 1081\eta + 128\right)Nt}{72(\eta+1)(3\eta+2)^2(5\eta+2)^2}.$$

Differentiating, we have:

$$\frac{\partial}{\partial\eta}\frac{\mathrm{IR}^{\mathrm{trans}}_{\mathrm{def},D}(\eta)^2}{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},D}(\eta)} = \frac{\left(10125\eta^8 + 52650\eta^7 + 135711\eta^6 + 202008\eta^5 + 183039\eta^4 + 103058\eta^3 + 35172\eta^2 + 6600\eta + 512\right)Nt}{72(\eta+1)^2(3\eta+2)^3(5\eta+2)^3},$$

which is clearly positive for any $\eta \in [1, 2]$.

■

Proof of Proposition 5.2. From Appendix B.1, rents under B, ACD are:

$$u^{\mathbf{B},ACD}(\eta) = \frac{\left(16\eta^2 + 89\eta + 100\right)Nt}{576}$$

$$u^{B,\mathbf{ACD}}(\eta) = \frac{(\eta+4)^2 Nt}{144},$$

and rents under AB, CD are reported in the proof of Proposition 5.1.

Hence, if conflict is over A, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},A}(\eta) &= u^{B,\mathbf{ACD}}(\eta) - u^{AB,\mathbf{CD}}(\eta)\\ &= \frac{\left(3600\eta^6 + 20505\eta^5 + 34132\eta^4 + 18560\eta^3 + 536\eta^2 - 2160\eta - 448\right)Nt}{576(3\eta+2)^2(5\eta+2)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},A}(\eta) &= u^{\mathbf{AB},CD}(\eta) - u^{\mathbf{B},ACD}(\eta)\\ &= -\frac{\left(225\eta^6 + 1830\eta^5 + 2230\eta^4 - 4912\eta^3 - 8272\eta^2 - 3960\eta - 616\right)Nt}{144(3\eta+2)^2(5\eta+2)^2}\end{aligned}$$

1. Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},A}(\eta)}{\partial \eta} = \frac{Nt}{576(3\eta+2)^3(5\eta+2)^3}\Big[108000\eta^7 + 537975\eta^6 + 1070640\eta^5 + 1223924\eta^4 + 826992\eta^3 + 319920\eta^2 + 65728\eta + 5696\Big],$$

which is clearly positive for any $\eta \in [1, 2]$, and

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},A}(\eta)}{\partial \eta} = \frac{-Nt}{72(3\eta+2)^3(5\eta+2)^3}\Big[3375\eta^7 + 20925\eta^6 + 46620\eta^5 + 90820\eta^4 + 102624\eta^3 + 59628\eta^2 + 17072\eta + 1936\Big],$$

which is clearly negative for any $\eta \in [1, 2]$.

2. Point (i) of this proposition shows the defender's incremental return is strictly decreasing. Thus, to show that an $\hat{\eta} \in (1, 2)$ exists, it suffices to show that the defender's incremental return is positive at $\eta = 1$ and negative at $\eta = 2$. At $\eta = 1$, the defender's incremental return is $\frac{11Nt}{144} > 0$. At $\eta = 2$ the defender's incremental return is $-\frac{385Nt}{18432} < 0$.

3. Given the previous results in this proposition, it now suffices to show that the defender's incremental return is less than the attacker's. Subtracting, this is the case if:

$$\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att,\ A}}(\eta) - \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def,\ A}}(\eta) = \frac{\left(4500\eta^6 + 27825\eta^5 + 43052\eta^4 - 1088\eta^3 - 32552\eta^2 - 18000\eta - 2912\right)t}{576(3\eta+2)^2(5\eta+2)^2} > 0.$$

   Since the defender's incremental return is decreasing in η and the attacker's is increasing in η, the left-hand side is minimized at $\eta = 1$. Thus, it suffices to show that the inequality holds at $\eta = 1$. At $\eta = 1$, the inequality reduces to $\frac{17Nt}{576} > 0$.

■

Proof of Proposition 5.3. From Appendix B.1, rents under A, BCD are:

$$u^{\mathbf{A},BCD}(\eta) = \frac{(\eta+4)^2 Nt}{72(\eta+1)}$$

$$u^{A,\mathbf{BCD}}(\eta) = \frac{\left(48\eta^3 + 274\eta^2 + 389\eta + 109\right) Nt}{288(\eta+1)(3\eta+1)},$$

and rents under AB, CD are reported in the proof of Proposition 5.1.

Hence, if conflict is over B, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},B}(\eta) &= u^{A,\mathbf{BCD}}(\eta) - u^{AB,\mathbf{CD}}(\eta) \\ &= \frac{\left(14634\eta^6 + 61857\eta^5 + 98889\eta^4 + 79948\eta^3 + 34964\eta^2 + 7888\eta + 720\right) Nt}{288(\eta+1)(3\eta+1)(3\eta+2)^2(5\eta+2)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},B}(\eta) &= u^{\mathbf{AB},CD}(\eta) - u^{\mathbf{A},BCD}(\eta) \\ &= \frac{\left(738\eta^5 + 2841\eta^4 + 4712\eta^3 + 3676\eta^2 + 1328\eta + 180\right) t}{72(\eta+1)(3\eta+2)^2(5\eta+2)^2}\end{aligned}$$

1. Differentiating the incremental returns, we have:

$$\begin{aligned}\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},B}(\eta)}{\partial \eta} &= \frac{-Nt}{288(\eta+1)^2(3\eta+1)^2(3\eta+2)^3(5\eta+2)^3}\Big[500661\eta^8 + 1979694\eta^7 \\ &+ 3546621\eta^6 + 3743352\eta^5 + 2516448\eta^4 + 1090280\eta^3 + 294720\eta^2 + 45216\eta + 3008\Big],\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},B}(\eta)}{\partial \eta} &= \frac{-Nt}{72(\eta+1)^2(3\eta+2)^3(5\eta+2)^3}\Big[7929\eta^6 + 48672\eta^5 + 96336\eta^4 \\ &+ 90232\eta^3 + 44508\eta^2 + 11280\eta + 1168\Big],\end{aligned}$$

both of which are clearly negative for any $\eta \in [1,2]$.

2. Let's start by seeing that the attacker's incremental return is larger than the defender's. Subtracting, this is the case if:

$$\mathrm{IR}^{\text{pop}}_{\text{att, B}}(\eta) - \mathrm{IR}^{\text{pop}}_{\text{def, B}}(\eta) = \frac{\eta\left(5778\eta^5 + 24813\eta^4 + 30981\eta^3 + 16988\eta^2 + 4324\eta + 416\right)Nt}{288(\eta+1)(3\eta+1)(3\eta+2)^2(5\eta+2)^2} > 0,$$

which clearly holds. Hence, expected observed violence is

$$\frac{\mathrm{IR}^{\text{pop}}_{\text{def},B}(\eta)^2}{\mathrm{IR}^{\text{pop}}_{\text{att},B}(\eta)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial\eta}\frac{\mathrm{IR}^{\text{trans}}_{\text{def},B}(\eta)^2}{\mathrm{IR}^{\text{pop}}_{\text{att},B}(\eta)} = \frac{-Nt}{18(\eta+1)^2(3\eta+2)^3(5\eta+2)^3\left(14634\eta^6 + 61857\eta^5 + 98889\eta^4 + 79948\eta^3 + 34964\eta^2 + 7888\eta + 720\right)^2}$$
$$\Bigg[\left(738\eta^5+2841\eta^4+4712\eta^3+3676\eta^2+1328\eta+180\right)\Big(326710098\eta^{13}+4565055105\eta^{12}+23032083672\eta^{11}$$
$$+62761124457\eta^{10}+107874216792\eta^9+126350643456\eta^8+105143270688\eta^7+63483012724\eta^6+27976430528\eta^5$$
$$+ 8926280128\eta^4 + 2010774496\eta^3 + 303575552\eta^2 + 27581824\eta + 1140480\Big)\Bigg]$$

which is clearly negative for any $\eta \in [1,2]$.

■

Proof of Proposition 5.4. From Appendix B.1, rents under ABC, D are:

$$u^{\mathbf{ABC},D}(\eta) = \frac{\left(4n^2 + 48n + 153\right)Nt}{576}$$

$$u^{A,ABC,\mathbf{D}}(\eta) = \frac{(2n+3)^2 Nt}{144},$$

and rents under AB, CD are reported in the proof of Proposition 5.1.

Hence, if conflict is over C, the incremental returns are:

$$\begin{aligned}
\mathrm{IR}^{\text{pop}}_{\text{att},C}(\eta) &= u^{\mathbf{ABC},D}(\eta) - u^{\mathbf{AB},CD}(\eta) \\
&= \frac{\left(900\eta^6 + 10920\eta^5 + 36625\eta^4 + 29088\eta^3 + 2424\eta^2 - 4192\eta - 1040\right) Nt}{576(3\eta+2)^2(5\eta+2)^2}
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{IR}^{\text{pop}}_{\text{def},C}(\eta) &= u^{AB,\mathbf{CD}}(\eta) - u^{ABC,\mathbf{D}}(\eta) \\
&= \frac{\left(-900\eta^6 - 2820\eta^5 - 13\eta^4 + 6894\eta^3 + 7194\eta^2 + 2752\eta + 368\right) Nt}{144(3\eta+2)^2(5\eta+2)^2}
\end{aligned}$$

1. Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{pop}}_{\text{att},C}(\eta)}{\partial \eta} = \frac{\left(3375\eta^7 + 27675\eta^6 + 68220\eta^5 + 119260\eta^4 + 122336\eta^3 + 67212\eta^2 + 18608\eta + 2064\right) Nt}{72(3\eta+2)^3(5\eta+2)^3},$$

   which is clearly positive, and

$$\frac{\partial \mathrm{IR}^{\text{pop}}_{\text{def},C}(\eta)}{\partial \eta} = -\frac{\left(13500\eta^7 + 49950\eta^6 + 78480\eta^5 + 80113\eta^4 + 52862\eta^3 + 20556\eta^2 + 4280\eta + 384\right) Nt}{72(3\eta+2)^3(5\eta+2)^3},$$

   which is clearly negative.

2. Given this monotonicity, it suffices establish that $\hat{\eta} \in (1,2)$ exists, it suffices to sow that the defender's incremental return is positive at $\eta = 1$ and negative at $\eta = 2$. At $\eta = 1$, it is equal to $\frac{11Nt}{144} > 0$ and at $\eta = 2$ it is equal to $-\frac{809Nt}{18432} < 0$.

3. Let's start by seeing that the attacker's incremental return is larger than the defender's. Subtracting, this is the case if:

$$\begin{aligned}
\mathrm{IR}^{\text{pop}}_{\text{att, C}}(\eta) - \mathrm{IR}^{\text{pop}}_{\text{def, C}}(\eta) &= \frac{\left(4500\eta^6 + 22200\eta^5 + 36677\eta^4 + 1512\eta^3 - 26352\eta^2 - 15200\eta - 2512\right) Nt}{576(3\eta+2)^2(5\eta+2)^2} \\
&> 0,
\end{aligned}$$

which clearly holds. Hence, expected observed violence is

$$\frac{\mathrm{IR}^{\text{pop}}_{\text{def},C}(\eta)^2}{\mathrm{IR}^{\text{pop}}_{\text{att},C}(\eta)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial\eta}\frac{\mathrm{IR}^{\text{trans}}_{\text{def},C}(\eta)^2}{\mathrm{IR}^{\text{pop}}_{\text{att},C}(\eta)} = -\frac{\left(13500\eta^7 + 49950\eta^6 + 78480\eta^5 + 80113\eta^4 + 52862\eta^3 + 20556\eta^2 + 4280\eta + 384\right)t}{72(3\eta+2)^3(5\eta+2)^3}$$

which is clearly negative for any $\eta \in [1, 2]$.

■

C.2 Proofs of for Local Transportation Cost Shocks

Proof of Proposition 5.5. From Appendix B.2, the rents under AB, CD and under ABD, C are:

$$u^{\mathbf{AB},CD}(\tau) = \frac{\left(1514\tau^5 + 17240\tau^4 + 70049\tau^3 + 127707\tau^2 + 104709\tau + 31581\right)Nt}{144(\tau+1)\left(4\tau^2+27\tau+39\right)^2} \tag{8}$$

$$u^{AB,\mathbf{CD}}(\tau) = \frac{\left(900\tau^5 + 4638\tau^4 + 8119\tau^3 + 6089\tau^2 + 2048\tau + 256\right)Nt}{72(3\tau+2)^2(5\tau+2)^2}, \tag{9}$$

$$u^{\mathbf{ABD},C}(\tau) = \frac{\left(757\tau^2 + 1614\tau + 909\right)Nt}{1152(\tau+1)(\tau+3)}$$

$$u^{ABD,\mathbf{C}}(\tau) = \frac{t(\tau+4)^2}{72(\tau+1)}.$$

Hence, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{att, D}}(\tau) &= u^{\mathbf{ABD},C}(\tau) - u^{\mathbf{AB},CD}(\tau)\\ &= \frac{\left(15080\tau^5 + 177053\tau^4 + 767928\tau^3 + 1594110\tau^2 + 1603584\tau + 624645\right)Nt}{1152(\tau+1)(\tau+3)\left(4\tau^2+27\tau+39\right)^2}\end{aligned}$$

and

$$\mathrm{IR}^{\text{trans}}_{\text{def, D}}(\tau) = u^{AB,\mathbf{CD}}(\tau) - u^{ABD,\mathbf{C}}(\tau)$$
$$= \frac{\left(1417\tau^5 + 20290\tau^4 + 111030\tau^3 + 277332\tau^2 + 317601\tau + 134730\right) Nt}{288(\tau+1)(\tau+3)\left(4\tau^2+27\tau+39\right)^2}$$

1. Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{att,D}}(\tau)}{\partial \tau} = -\frac{\left(30160\tau^8 + 504632\tau^7 + 3422692\tau^6 + 12403647\tau^5 + 26785827\tau^4 + 36638586\tau^3 + 32926158\tau^2 + 18979407\tau + 5508891\right) Nt}{576(\tau+1)^2(\tau+3)^2\left(4\tau^2+27\tau+39\right)^3}$$

and

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{def,D}}(\tau)}{\partial \tau} = -\frac{\left(5668\tau^8 + 124061\tau^7 + 1168135\tau^6 + 5985603\tau^5 + 18030429\tau^4 + 32762979\tau^3 + 35515809\tau^2 + 21458493\tau + 5684823\right) Nt}{288(\tau+1)^2(\tau+3)^2\left(4\tau^2+27\tau+39\right)^3},$$

each of which is clearly negative for any $\tau \in [1, \frac{27}{16}]$.

2. First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, this is the case if:

$$\mathrm{IR}^{\text{trans}}_{\text{att, D}}(\tau) - \mathrm{IR}^{\text{trans}}_{\text{def, D}}(\tau) = \frac{\left(9412\tau^4 + 86481\tau^3 + 237327\tau^2 + 247455\tau + 85725\right) Nt}{1152(\tau+3)\left(4\tau^2+27\tau+39\right)^2} > 0,$$

which holds for any $\tau \in [1, \frac{27}{16}]$.

Thus, expected observed violence is $\frac{\mathrm{IR}^{\text{trans}}_{\text{def, D}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, D}}(\tau)}$. Differentiating, we have:

$$\frac{\partial}{\partial\tau}\frac{\mathrm{IR}^{\text{trans}}_{\text{def, D}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, D}}(\tau)} = \frac{-\left(109\tau^4+1435\tau^3+6885\tau^2+13389\tau+8982\right)Nt}{36(\tau+1)^2(\tau+3)^2\left(4\tau^2+27\tau+39\right)^3\left(1160\tau^4+12281\tau^3+44901\tau^2+70815\tau+41643\right)^2}$$

$$\times\Big[3287440\tau^{12}+115234980\tau^{11}+1828256661\tau^{10}+17119243693\tau^9+104584791981\tau^8$$
$$+437894661714\tau^7+1288259108064\tau^6+2686615865892\tau^5+3950096157792\tau^4+4004209269594\tau^3$$
$$+2666035507491\tau^2+1051935189471\tau+187252225227\Big]$$

which is clearly negative for any $\tau\in[1,\frac{27}{16}]$.

■

Proof of Proposition 5.6. From Appendix B.2, rents under B, ACD are:

$$u^{\mathbf{B},ACD}(\tau)=\frac{25Nt}{144}$$

$$u^{B,\mathbf{ACD}}(\tau)=\frac{(98\tau+107)Nt}{288(\tau+1)},$$

and rents under AB, CD are reported in Equations 8 and 9.

Hence, incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{att, A}}(\tau)&=u^{B,\mathbf{ACD}}(\tau)-u^{AB,\mathbf{CD}}(\tau)\\&=-\frac{\left(368\tau^5+2033\tau^4-9554\tau^3-75033\tau^2-139944\tau-76770\right)Nt}{288(\tau+1)\left(4\tau^2+27\tau+39\right)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{def, A}}(\tau)&=u^{\mathbf{AB},CD}(\tau)-u^{\mathbf{B},ACD}(\tau)\\&=\frac{\left(557\tau^5+5720\tau^4+19312\tau^3+24516\tau^2+7017\tau-3222\right)Nt}{72(\tau+1)\left(4\tau^2+27\tau+39\right)^2}\end{aligned}$$

1. Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}_{\mathrm{att},A}^{\mathrm{trans}}(\tau)}{\partial \tau} = -\frac{\left(13212\tau^6+218539\tau^5+1358015\tau^4+4179237\tau^3+6727599\tau^2+5372604\tau+1681794\right)Nt}{288(\tau+1)^2\left(4\tau^2+27\tau+39\right)^3},$$

which is clearly negative, and

$$\frac{\partial \mathrm{IR}_{\mathrm{def},A}^{\mathrm{trans}}(\tau)}{\partial \tau} = \frac{\left(9426\tau^6+131953\tau^5+715295\tau^4+1949748\tau^3+2816946\tau^2+2035323\tau+573309\right)Nt}{72(\tau+1)^2\left(4\tau^2+27\tau+39\right)^3},$$

which is clearly positive.

2. The first step is to see that there is a critical $\tilde{\tau}$ such that, for $\tau < \tilde{\tau}$, $\mathrm{IR}_{\mathrm{def},A}^{\mathrm{trans}}(\tau) < \mathrm{IR}_{\mathrm{att},A}^{\mathrm{trans}}(\tau)$ and for $\tau > \tilde{\tau}$, $\mathrm{IR}_{\mathrm{def},A}^{\mathrm{trans}}(\tau) > \mathrm{IR}_{\mathrm{att},A}^{\mathrm{trans}}(\tau)$. Subtracting, we have:

$$\mathrm{IR}_{\mathrm{att},A}^{\mathrm{trans}}(\tau) - \mathrm{IR}_{\mathrm{def},A}^{\mathrm{trans}}(\tau) = -\frac{\left[2596\tau^5+24913\tau^4+67694\tau^3+23031\tau^2-111876\tau-89658\right]Nt}{288(\tau+1)\left(4\tau^2+27\tau+39\right)^2}$$

This has the opposite sign as the term in square brackets. Hence, it suffices to show that the term in square brackets is negative at $\tau = 1$, positive at $\tau = 2$, and crosses zero only once. At $\tau = 1$, the term in square brackets is $-83300 < 0$. At $\tau = 2$, the term in square brackets is $801946 > 0$. To show that it only crosses zero once, it suffices to show that the term in square brackets is increasing in τ. The first-derivative with respect to τ of the term in square brackets is

$$12980\tau^4 + 99652\tau^3 + 203082\tau^2 + 46062\tau - 111876,$$

which is clearly positive for any $\tau \in [1, \frac{27}{16}]$.

Given this, for $\tau < \tilde{\tau}$ expected observed violence is $\frac{\mathrm{IR}_{\text{def, A}}^{\text{trans}}(\tau)^2}{\mathrm{IR}_{\text{att, A}}^{\text{trans}}(\tau)}$. We need to show that this is increasing in τ for $\tau \in [1, \tilde{\tau}]$. Differentiating, we have:

$$\frac{\partial}{\partial \tau} \frac{\mathrm{IR}^{\text{trans}}_{\text{def, A}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, A}}(\tau)} = \frac{\left(557\tau^5 + 5720\tau^4 + 19312\tau^3 + 24516\tau^2 + 7017\tau - 3222\right) Nt}{18(\tau+1)^2 \left(4\tau^2 + 27\tau + 39\right)^3 \left(-368\tau^5 - 2033\tau^4 + 9554\tau^3 + 75033\tau^2 + 139944\tau + 76770\right)^2}$$

$$\times \Big[421548\tau^{11} + 61855339\tau^{10} + 1378741569\tau^9 + 14232487411\tau^8 + 85435697113\tau^7 + 326992719885\tau^6 + 831056738289\tau^5 + 1418084747361\tau^4 + 1602116885619\tau^3 + 1145464661052\tau^2 + 467456421222\tau + 82607123592\Big],$$

which is clearly positive for any $\tau \in [1, \frac{27}{16}]$.

For $\tau > \tilde{\tau}$ expected observed violence is $\frac{\mathrm{IR}^{\text{trans}}_{\text{att, A}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{def, A}}(\tau)}$.

$$\frac{\partial}{\partial \tau} \frac{\mathrm{IR}^{\text{trans}}_{\text{att, A}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{def, A}}(\tau)} = \frac{\left(368\tau^5 + 2033\tau^4 - 9554\tau^3 - 75033\tau^2 - 139944\tau - 76770\right) Nt}{1152(\tau+1)^2 \left(4\tau^2 + 27\tau + 39\right)^3 \left(557\tau^5 + 5720\tau^4 + 19312\tau^3 + 24516\tau^2 + 7017\tau - 3222\right)^2}$$

$$\times \Big[11249400\tau^{11} + 326875964\tau^{10} + 4081782153\tau^9 + 29076260759\tau^8 + 131711205593\tau^7 + 398948129679\tau^6 + 824840387517\tau^5 + 1163931757047\tau^4 + 1097961881751\tau^3 + 658613480517\tau^2 + 225464138226\tau + 33175451394\Big],$$

which is clearly negative for any $\tau \in [1, \frac{27}{16}]$.

■

Proof of Proposition 5.7.

From Appendix B.2, rents under A, BCD are:

$$u^{\mathbf{A},BCD}(\tau) = \frac{(11\tau+9)^2 Nt}{288(\tau+1)(\tau+3)}$$

$$u^{A,\mathbf{BCD}}(\tau) = \frac{\left(757\tau^2 + 1614\tau + 909\right) Nt}{1152(\tau+1)(\tau+3)},$$

and rents under AB, CD are reported in Equations 8 and 9.

Hence, incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{att, B}}(\tau) &= u^{A,\mathbf{BCD}}(\tau) - u^{AB,\mathbf{CD}}(\tau)\\ &= \frac{\left(4368\tau^5 + 62084\tau^4 + 327861\tau^3 + 785019\tau^2 + 861003\tau + 350865\right) Nt}{1152(\tau+3)\left(4\tau^2+27\tau+39\right)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{def, B}}(\tau) &= u^{\mathbf{AB},CD}(\tau) - u^{\mathbf{A},BCD}(\tau)\\ &= \frac{\left(1092\tau^5 + 13168\tau^4 + 60345\tau^3 + 136923\tau^2 + 153387\tau + 66285\right) Nt}{288(\tau+3)\left(4\tau^2+27\tau+39\right)^2}\end{aligned}$$

1. Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{att},B}(\tau)}{\partial\tau} = \frac{\left(19976\tau^6 + 398106\tau^5 + 3261078\tau^4 + 13686597\tau^3 + 30593691\tau^2 + 34345809\tau + 15106743\right) Nt}{576(\tau+3)^2\left(4\tau^2+27\tau+39\right)^3},$$

   and

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{def},B}(\tau)}{\partial\tau} = \frac{\left(9700\tau^6 + 154242\tau^5 + 972738\tau^4 + 3160107\tau^3 + 5695281\tau^2 + 5532435\tau + 2311497\right) Nt}{144(\tau+3)^2\left(4\tau^2+27\tau+39\right)^3},$$

   both of which are clearly positive for any $\tau \in [1, \frac{27}{16}]$.

2. Let's start by seeing that the attacker's incremental return is always larger than the defender's. Subtracting, this is the case if:

$$\mathrm{IR}^{\text{trans}}_{\text{att, B}}(\tau) - \mathrm{IR}^{\text{trans}}_{\text{def, B}}(\tau) = \frac{\left(9412\tau^4 + 86481\tau^3 + 237327\tau^2 + 247455\tau + 85725\right) Nt}{1152(\tau+3)\left(4\tau^2+27\tau+39\right)^2} > 0,$$

   which holds for any $\tau \in [1, \frac{27}{16}]$.

   Thus, expected observed violence is $\frac{\mathrm{IR}^{\text{trans}}_{\text{def, B}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, B}}(\tau)}$.

   Differentiating, we have:

$$\frac{\partial}{\partial\tau}\frac{\mathrm{IR}^{\text{trans}}_{\text{def, B}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, B}}(\tau)} = \frac{\left(84\tau^4+916\tau^3+3585\tau^2+6396\tau+4419\right)Nt}{36(\tau+3)^2\left(4\tau^2+27\tau+39\right)^3\left(336\tau^4+4388\tau^3+20157\tau^2+37128\tau+23391\right)^2}$$
$$\times\Big[4840416\tau^{10}+137038904\tau^9+1688143920\tau^8+11906937678\tau^7+53250149502\tau^6$$
$$+157951113687\tau^5+315510780501\tau^4+420939372240\tau^3+361411830612\tau^2$$
$$+182064037203\tau+41379755337\Big],$$

which is clearly positive for any $\tau \in [1, \frac{27}{16}]$.

■

Proof of Proposition 5.8.

From Appendix B.2, rents under ABC, D are:

$$u^{\mathbf{ABC},D}(\tau) = \frac{\left(128\tau^2+201\tau+81\right)Nt}{576(\tau+1)}$$

$$u^{ABC,\mathbf{D}}(\tau) = \frac{(2\tau+3)^2 Nt}{72(\tau+1)},$$

and rents under AB, CD are reported in Equations 8 and 9.

Hence, incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{att, C}}(\tau) &= u^{\mathbf{ABC},D}(\tau) - u^{\mathbf{AB},CD}(\tau)\\ &= \frac{\left(2048\tau^6+24808\tau^5+109000\tau^4+216109\tau^3+191487\tau^2+57471\tau-3123\right)Nt}{576(\tau+1)\left(4\tau^2+27\tau+39\right)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{def, C}}(\tau) &= u^{AB,\mathbf{CD}}(\tau) - u^{ABC,\mathbf{D}}(\tau)\\ &= -\frac{\left(256\tau^6+2288\tau^5+2687\tau^4-24136\tau^3-79842\tau^2-85632\tau-31221\right)Nt}{288(\tau+1)\left(4\tau^2+27\tau+39\right)^2}\end{aligned}$$

1. Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{att},C}(\tau)}{\partial \tau} = \frac{\left(1024\tau^8+22784\tau^7+202926\tau^6+946607\tau^5+2539285\tau^4+4009233\tau^3+3627765\tau^2+1710900\tau+316476\right)Nt}{72(\tau+1)^2\left(4\tau^2+27\tau+39\right)^3},$$

which is clearly positive, and

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{def},C}(\tau)}{\partial \tau} = -\frac{\left(1024\tau^8+22784\tau^7+199524\tau^6+867797\tau^5+1960285\tau^4+2049474\tau^3+338382\tau^2-887175\tau-436095\right)Nt}{288(\tau+1)^2\left(4\tau^2+27\tau+39\right)^3},$$

which is clearly negative.

2. First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, this is the case if:

$$\mathrm{IR}^{\text{trans}}_{\text{att, C}}(\tau)-\mathrm{IR}^{\text{trans}}_{\text{def, C}}(\tau) = \frac{\left(2560\tau^6+29384\tau^5+114374\tau^4+167837\tau^3+31803\tau^2-113793\tau-65565\right)Nt}{576(\tau+1)\left(4\tau^2+27\tau+39\right)^2} > 0,$$

which holds for any $\tau \in \left[1, \frac{27}{16}\right]$.

Thus, expected observed violence is $\frac{\mathrm{IR}^{\text{trans}}_{\text{def, C}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, C}}(\tau)}$. Differentiating, we have:

$$\frac{\partial}{\partial \tau}\frac{\mathrm{IR}^{\text{trans}}_{\text{def, C}}(\tau)^2}{\mathrm{IR}^{\text{trans}}_{\text{att, C}}(\tau)} = \frac{\left(256\tau^6+2288\tau^5+2687\tau^4-24136\tau^3-79842\tau^2-85632\tau-31221\right)Nt}{72(\tau+1)^2\left(4\tau^2+27\tau+39\right)^3\left(2048\tau^6+24808\tau^5+109000\tau^4+216109\tau^3+191487\tau^2+57471\tau-3123\right)^2}$$
$$\times\Big[1048576\tau^{14}+39362560\tau^{13}+658145280\tau^{12}$$
$$+6459265568\tau^{11}+41493065376\tau^{10}+184657962240\tau^9$$
$$+588688406889\tau^8+1372950483068\tau^7+2374152325704\tau^6$$
$$+3065370521340\tau^5+2947956574194\tau^4+2069180825076\tau^3$$
$$+1004603141952\tau^2+299773758708\tau+40884713469\Big].$$

This has the same sign as the numerator on the first line. To see that this is negative, note that for any $\tau \in \left[1, \frac{27}{16}\right]$:

$$256\tau^6 < 24136\tau^3$$

$$2288\tau^5 < 79842\tau^2$$

$$2687\tau^4 < 85632\tau,$$

so each positive term is more than off-set by a negative term.

■