Errata on Section 4.3 of "Inference under Covariate-Adaptive Randomization"

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Bugni et al. (2018, Section 4.3) studies the properties of a two sample t-statistic coupled with a permutation critical value, under covariate-adaptive randomization. The section is written for a generic value of θ_0, but the notation implicitly assumes that $\theta_0 = 0$. To allow for $\theta_0 \neq 0$, one should modify the notation in Section 4.3 in two ways:

1. Define $Y_i^\dagger = Y_i - \theta_0 A_i$ and re-define $X^{(n)}$ as
$$X^{(n)} = \{(Y_i^\dagger, A_i, Z_i) : 1 \leq i \leq n\} \ , \tag{0.1}$$
and $gX^{(n)}$ below (38) as
$$gX^{(n)} = \{(Y_i^\dagger, A_{g(i)}, Z_i) : 1 \leq i \leq n\} \ . \tag{0.2}$$

2. Wherever it reads "... the absolute value of $T_n^{\text{t-stat}}(X^{(n)})$ in (11)", replace with "... the absolute value of $T_n^{\text{t-stat}}(X^{(n)})$ in (11) with $\theta_0 = 0$".

In summary, the correct implementation of the permutation test when $\theta_0 \neq 0$ involves re-centering the outcome variable Y_i before doing the permutations (here denoted by $Y_i^\dagger$) and using the following test statistic
$$|T_n^{\text{t-stat}}(X^{(n)})| = \frac{|\bar{Y}_{n,1}^\dagger - \bar{Y}_{n,0}^\dagger|}{\sqrt{\frac{\hat{\sigma}_{n,1}^{2,\dagger}}{n_1} + \frac{\hat{\sigma}_{n,0}^{2,\dagger}}{n_0}}} \ . \tag{0.3}$$

We note that the formal results in Bugni et al. (2018, Section 4.3) are correct as these actually consider the test with the above modifications. In particular, the proofs of Theorems 4.5 and 4.6 both read "... we assume without loss of generality that $\theta_0 = 0$; the general case then follows from the same arguments with Y_i replaced by $Y_i - \theta_0 A_i$".

References

Bugni, F. A., I. A. Canay, and A. M. Shaikh (2018): "Inference under Covariate Adaptive Randomization," *Journal of the American Statistical Association*, 113:524, 1784–1796.