Search Intensity

Robert Shimer[*]
Department of Economics
University of Chicago
`shimer@uchicago.edu`

April 8, 2004

Abstract

Standard theories of labor market search predict that workers should search less when the returns to search are low, yielding the counterfactual prediction that labor market participation and other measures of search intensity should be strongly pro-cyclical and unemployment should be acyclical or even procyclical. I argue that this is a consequence of how search intensity is modelled. In a discrete time setting, I model search intensity as a worker's choice of the number of simultaneous applications to make. My main result is that when the cost of making an application is small, a worker who has at least an eighty percent probability of getting a job responds to an adverse shock by increasing his search intensity. Workers who are less likely to get jobs become discouraged, reducing their search intensity.

1   Introduction

Modern theories of the labor market recognize unemployment to be an equilibrium phenomenon that serves a productive purpose: in an ever-changing economy, unemployment is an efficient method of moving workers from one productive activity to another.[1] According to the textbook model, employment relationships are continually hit by idiosyncratic match-specific, firm-specific, or industry-specific shocks. When a worker-firm match becomes sufficiently unproductive, the pair agrees to terminate the relationship, and the newly-unemployed worker begins to search for a more productive job. In some treatments of this model, the worker must decide on the 'intensity' of his job search. This decision depends on three factors: the marginal increase in the likelihood of obtaining a job in response to an increase in search intensity; the increase in the expected present value of his income in response to obtaining a job; and the marginal cost of search effort. An increase in either of the first two factors or a decrease in the third raises the equilibrium search intensity of unemployed workers.

While this may provide an adequate theory of steady state search intensity, the theory runs into some difficulties in attempting to explain business cycle fluctuations in unemployment (Veracierto 2002). During economic downturns, the marginal product of search intensity will likely fall both because of a decline in the probability of obtaining a job conditional on a given search intensity and because of a decline in the expected present value of income from a job. This decline in search intensity should be observable in aggregate data as a decrease in labor market participation, an increase in the number of workers who indicate that they would like a job but are actively searching, or a decrease in the search intensity of active searchers. Section 2 of this paper shows that this is counterfactual.

The theory has similar difficulties in explaining the European unemployment phenomenon. In most European countries it is hard to get a job and, given the large subsidies to nonemployment, the return to getting job is small. If workers have an opportunity to drop out of the labor force, the theory predicts that they will do so in great numbers in Europe. While it is true that labor force participation is low in Europe compared to the United States, unemployment is also high most countries. For example, during a severe recession in the early 1990s, the Swedish labor force participation rate fell from 67.4 to 63.7 percent, a sizable decline. But this was not accompanied by a fall in unemployment. Instead, the unemployment rate increased by more than a factor of five, from 1.8 to 9.6 percent of the labor force.[2]

---

[1]Pissarides (2000) provides a textbook treatment of the modern theory of unemployment.

[2]These data refer to the five years between 1990 and 1994 and come from the Bureau of Labor Statistics

The goal of this paper is to explain why unemployment does not decline when the labor market prospects of unemployed workers is dim. One possible answer is that we live in a dichotomous world of unemployment and nonparticipation. Some fraction of the population is interested in working and the remaining fraction has no intention of ever accepting a job. When worker loses his job, he almost always looks for another one, so there is little movement between participation and nonparticipation. There is overwhelming evidence against such an explanation. Blanchard and Diamond (1990) document that in a typical month, as many workers move from unemployment into employment as from nonparticipation into employment—1.6 million during an average month between 1968 and 1986. The flows between unemployment and nonparticipation are also substantial, jointly accounting for another 1.8 million transitions per month. These flows are easily explained by a model in which a statistical agency coarsely measures an underlying continuous search intensity decision and classifies workers as unemployed or nonparticipating, a reasonably good description of the actual data collection process.[3]

This naturally leads me to write down a model in which the cost of search intensity is relatively flat, and so intensity varies with the returns to search. Fortunately, such a model already exists in Chapter 5 of Pissarides (2000). In that model, a worker continually chooses his search intensity s with increasing and strictly convex disutility $c(s)$. A worker who searches with intensity s finds a job according to a continuous time Poisson process with arrival rate $s\mu(\theta)$, where θ measures labor market tightness and μ is an increasing function. An increase in θ raises the proportionality constant and therefore raises the marginal product of search intensity, encouraging more search activity. If this is accompanied by an increase in the utility gain from finding a job, possibly due to higher wages, search intensity unambiguously increases. The model therefore predicts that any variable that raises labor market tightness and raises the utility gain from finding a job will raise equilibrium search intensity. This will in turn raise the measured unemployment rate relative to nonparticipation. In other words, unemployment is likely to be procyclical.

Critical to this logic is the assumption that search intensity and the ease of finding a job

---

program on Foreign Labor Statistics.

[3]In the United States, an individual is unemployed if he is not currently working, is available for work, and is actively seeking a job. Active search methods include contacting an employer directly or having a job interview; contacting a public or private employment agency; contacting friends or relatives; contacting a school or university employment center; sending out resumes or filling out applications; placing or answering advertisements; or checking union or professional registers. Passive search includes attending a job training program or course or reading help-wanted advertisements. See Bureau of Labor Statistics (2001) for more details.

are complements.[4] But intuitively this need not be the case. One can imagine that during booms, a worker knows that a minimal search intensity will suffice in order to get a job, while during recessions it is necessary to search much harder in order to secure employment. It seems difficult to model this possibility in continuous time, but straightforward to do so in discrete time. In fact, Stigler (1961) has already done so. Let search intensity s denote the number of applications that a worker makes during a single time period and $\mu(\theta)$ be the probability that a single application results in a job offer, independent across applications. Then the probability that a worker receives at least one job offer is $1 - \big(1 - \mu(\theta)\big)^s$ and the marginal increase in the probability of obtaining a job from the $s + 1^{\text{st}}$ application is $\mu(\theta)\big(1 - \mu(\theta)\big)^s$. Crucially, the impact of μ on the marginal product of search is now ambiguous, increasing when $\mu < \frac{1}{s+1}$ but decreasing for higher values of μ. This means that an increase in the probability of success may reduce the incentive to search for a job, particularly for workers who are already likely to obtain a job.

When I embed this idea in an equilibrium search model, I find support for this intuition. A worker's search intensity is a non-monotone function of labor market conditions. Under general conditions, workers who have at least an eighty percent probability of finding a job within a period respond to an improvement in economic conditions by reducing their search intensity.[5] This is not a rule of thumb, but rather a general result that holds as long as the cost of making an application is small. It is independent of the discount factor, job duration, search frictions, and all other details of the economic environment, as long as shocks are caused by fluctuations in labor productivity or the job destruction rate.

This begs the question, "how long is a period?" The discrete time model offers a natural answer: a period is the length of time it takes to process an application. In the market for day laborers, a period is probably a few seconds and the probability that a worker is successful in obtaining a job within a period is small. The model predicts that day laborers should reduce their search intensity when labor market conditions are weak, say during the winter for agricultural workers. But in the market for new assistant professors, a period may be closer to a year. During years when universities make relatively few offers, a new Ph.D. would be well-advised to send out more job applications and accept more job interviews than in a period in which each application is quite likely to result in an offer. Similarly, a strong

---

[4]Complementarity is hard-wired into some empirical estimates of search intensity, e.g. Barron and Mellow (1979).

[5]To be precise, a worker responds to an improvement in economic conditions by reducing his search intensity if and only if the probability of obtaining a job p exceeds $\bar{p}$, the positive solution to $2\bar{p}+\log(1-\bar{p}) = 0$, so $\bar{p} \approx 0.796812$.

candidate who knows that each application is likely to generate a job offer should apply for fewer positions than a weak candidate for whom the probability that an application results in an offer is small but positive. More generally, it seems plausible that it is more time consuming to evaluate an application in more skilled labor markets, and so the 'perverse' results emphasized here should be most important in those markets.

This paper proceeds as follows. Section 2 presents some evidence indicating that search intensity did not fall in the U.S. during the 2001 recession. Section 3 shows that a standard continuous time search model predicts a strong negative comovement of search intensity and unemployment duration in response to a variety of different shocks. Section 4 formulates a discrete time search model with a choice of search intensity and develops the main predictions of that model: search intensity, as measured by the number of job applications, is positively correlated with unemployment duration for workers who are likely to get a job within a period and negatively correlated for workers who are unlikely to get a job. For notational simplicity, both of these sections analyze a deterministic steady state equilibrium; however, Section 5 shows that the main results carry over to a discrete time model in which aggregate productivity and the job destruction rate follow a first order Markov process.

2   The Cyclicality of Search Intensity

A basic tenet of this paper is that nonemployed workers' search intensity is not strongly procyclical. One way to examine this is to use U.S. data from the Current Population Survey (CPS) to compare the cyclical behavior of the number of 'searchers' with the behavior of the number of 'non-searchers'. The former category includes nonemployed workers who are available for and actively seeking work. That is, searchers are unemployed workers who are not on temporary layoff. A non-searcher is any other nonemployed worker who indicates that he would like to have a job, a subset of the not-in-the-labor-force (NILF) population.[6] I refer jointly to searchers and non-searchers and 'attached workers'.

Because of data limitations that I discuss in more detail later, I focus here on the period from January 1994 to January 2004. The U.S. labor market grew strongly during the first seven years of this data set and then suffered a sharp contraction in 2001, which persisted through the remainder of the sample period. Figure 1 shows that while the number of

---

[6]Formally, a searcher is defined as an individual who is "unemployed and looking" according to the "monthly labor force recode" variable in the CPS, located in columns 180–181 of the public use files. A non-searcher is defined as an individual who "wants a job" according to the "NILF recode—want a job or other NILF' variable in the CPS, located in columns 418–419 of the public use files.

searchers increased by over 2 million during 2001 and remained high in 2002 and 2003, there was virtually no change in the number of non-searchers. Put differently, this suggests that relatively few workers quit searching for a job as a consequence of the increased difficulty in finding one.

Part of this may reflect a change in the demographic characteristics of attached workers over the business cycle. Using a linear probability model,[7] I can estimate the probability that an attached worker searches for a job conditional on month dummies and on the individual's demographic characteristics. The latter include a quartic function of age, dummy variables for marital status, race, sex, and four education categories. In order to focus on people who have mostly finished their education, I look at those who are 25 years or older at the time of the survey. Since there are few workers over age 70, I truncate the sample at this older level. This gives me 382,082 individual observations during the 121 month period. Column I in Table 1 shows the effect of changes in demographic characteristics. Women are significantly less likely to be searchers, while more educated workers and blacks are slightly more likely. Since age is entered as a quartic, it is difficult to interpret the individual coefficients. Instead, Figure 2 shows that age has little effect on the probability of being a searcher for those under 50, but a 70 year old worker is 46 percent less likely than a 50 year old to be a searcher rather than a non-searcher. But I am primarily interested in the month dummies. Figure 3 shows that between August 2001 and February 2002, the probability of being a searcher jumped by almost 8 percent, with little change before or after that date. Again, there is no evidence that the deterioration of the U.S. labor market led attached workers to stop searching for a job.

Dividing attached workers only according to whether they are counted as unemployed may mask some important heterogeneity in search behavior. I therefore next examine the search behavior of attached workers more directly. The CPS asks workers who are searching for a job what they are doing to find one. The responses are coded into twelve categories.[8] I count the number of job search methods used by each attached worker. The raw data shown in Figure 4 indicates that between 1994 and 2000, attached workers used on average about 1.13 search methods. This increased dramatically in 2001, and levelled off at about 1.43

---

[7]A logit or probit gives qualitatively similar answers, although the coefficients are harder to interpret.

[8]The categories are: 1. contacted an employer directly or interviewed; 2. contacted a public employment agency; 3. contacted a private employment agency; 4. contacted friends or relatives; 5. contacted a school or university employment center; 6. sent out resumes or filled out applications; 7. checked union or professional registers; 8. placed or answered advertisements; 9. other active job search; 10. looked at advertisements; 11. attended job training programs or courses; and 12. other passive job search. The first 9 categories count as active job search. The data are contained in columns 296–331 of the public use files.

methods per worker in 2002 and 2003. By this measure, the search intensity of attached workers increased by 27 percent during this recession. Again, it is possible to condition the number of job search methods on workers' demographic characteristics. Among those who are at least 25 years old, demographic factors have essentially the same effect on the number of search methods as on the decision to be a searcher, while the trend still increased by 0.3 methods when comparing 1994–2000 with 2002–2003.

There are three potential problems with this calculation. First, 94.6 percent of non-searchers report zero search methods, so to some extent this number is measuring the number of non-searchers rather than than the search intensity of searchers. Second, I only control for workers' characteristics imperfectly. Search intensity may change systematically with a workers' industry or occupation. And third, the number of search methods may change with the duration of an unemployment spell, which itself increased dramatically in 2001. I can address all of these issues by focusing my attention on searchers and controlling for industry, occupation, and unemployment duration directly. Formally, I regress the number of search methods used on the usual demographic controls (a quartic in age, marital status, race, sex, and education), on 23 industry dummies and 14 occupation dummies, on a quartic function of unemployment duration, and on the time period. I restrict my sample to searchers age 25 to 70 between 1994 and 2002; the latter restriction is necessitated by a change in the categorization of industries and occupations in January 2003. The sample now includes 184,111.

It is worth stressing that while the first regression exploited only the difference between searchers and non-searchers, this regression uses only within-searcher variation, an entirely different source of information. Nevertheless, the results are reassuringly similar. Column II in Table 1 shows that women and married people search less, while more educated people search more intensely. Only the coefficient on 'black' changes sign, but that coefficient is small in both instances. Figure 5 shows that the age pattern of job search is also virtually unchanged with this identification scheme, while Figure 6 shows humped-shaped pattern in the response of search intensity to unemployment duration. Workers with very short unemployment durations use at least 0.30 search methods less than workers with 10 to 30 weeks of unemployment, perhaps reflecting the use of additional search methods by workers whose 'standard' methods have failed them.[9] Industry and occupation also affect search

---

[9]The appearance of a hump peaking near 20 weeks appears to be due to the quartic specification. With a higher order polynomial, the response of search intensity to unemployment duration is much flatter near the peak.

duration, although the pattern is hard to summarize. Workers in executive, administrative, and managerial occupations use more search methods than others, while those in farming, forestry, and fishing occupations use the fewest. In terms of industry, workers in durable goods manufacturing use the most methods, while those who work in forestry and fisheries use the least.

Again, the main interest lies in the behavior of the period dummies. Figure 7 shows that there was a slight increase in the number of search methods used in 2001, which possibly began in the second quarter of 2000. But the magnitude of this increase is much smaller than was suggested by the previous results. To some extent, this is due to the demographic controls and especially the controls for unemployment duration. But the primary reason for these modest results is the change in sample, focusing exclusively on searchers rather than all attached workers. In fact, the dotted line in Figure 7 shows the results from a regression of the number of search methods used on time dummies alone, with January 1994 normalized to zero. The demographic, industry, occupation, and unemployment duration controls have virtually no effect on the first seven years of data, and then mute the response by about 0.07 search methods in 2001 and especially 2002. The bottom line is that there is no evidence that search intensity fell as the U.S. labor market weakened in 2001, and some evidence that search intensity actually increased.

Ideally one could perform a similar analysis over a longer time period, but unfortunately the 1994 redesign of the CPS makes this difficult. In particular, the structure of the questions related to search methods changed dramatically. Before 1994, the BLS only recognized six search methods, while it allowed for twelve after the redesign. Nevertheless, one can use the basic CPS to construct an internally consistent measure of the number of search methods used by searchers from 1976 to 1993. Figure 8 shows that the dramatic increase in search intensity observed in 2001 appears to be unprecedented. Instead, during the eighteen years between 1976 and 2003, search intensity experienced at least two low frequency cycles. But curiously the dates of those cycles are not orthogonal to the business cycle. Search intensity increased during all three NBER recessions dates (1980, 1981–82, and 1990-91) and fell during most of the long boom in the mid-1980s. Again, there is no evidence that search intensity is procyclical.

3   Continuous Time Model

This section describes a standard continuous time search model in which workers can vary their search intensity. I argue that most driving processes will induce a stable positive correlation between search intensity and measures of labor market conditions, such as job finding rates.

3.1   Assumptions

Time is continuous. There are two types of economic agents, workers and firms. All agents are risk-neutral and discount future payoffs at rate $r > 0$.

There is a measure 1 of workers who may be employed or unemployed at any point in time. Unemployed workers get leisure $z(1 - hs^\nu)$, where 1 is their endowment of time, s is their search intensity and $h > 0$ and $\nu > 1$ are parameters. A worker who searches with intensity s finds a job at a rate proportional to s, where the proportionality constant μ is endogenous and depends on aggregate labor market conditions. An employed worker receives a wage w determined by bargaining between the worker and firm. Jobs are identical but end exogenously at rate $\lambda > 0$, leaving the worker unemployed.

Firms may open vacancies by paying a flow cost c. Each vacancy contacts a worker at an endogenous rate η that depends on aggregate labor market conditions, yielding a filled job. A filled job generates a flow profit $x - w$ until the job ends, at which point the firm is left with nothing. Free entry drives the value of a vacancy to zero.

Wages are determined by Nash bargaining between employed workers and firms. Let $E(w)$ denote the expected present value of lifetime income for a worker employed at a current wage w, U denote the expected present value of income for an unemployed worker, and $J(w)$ denote the expected present value of profit from a particular filled job. The wage is set to maximize the weighted product of surplus from the match in excess of threat points,

$$w^* = \arg\max_w \left(E(w) - U\right)^\gamma J(w)^{1-\gamma}, \tag{1}$$

where γ is workers' bargaining power.

The rate at which workers and firms meet is described by a constant returns to scale matching function. Let v denote the measure of vacancies in the economy, $\bar{s}$ denote unemployed workers' average search intensity, and u denote the unemployment rate. Then the flow of matches in the economy is given by an increasing function $m(\bar{s}u, v)$. Let

$\theta \equiv \frac{v}{\bar{s}u}$ measure labor market tightness. A worker who searches with intensity s meets a firm at rate $s\mu(\theta) = \frac{sm(\bar{s}u,v)}{\bar{s}u} = sm(1,\theta)$, increasing in θ. A firm meets a worker at rate $\eta(\theta) = \frac{m(\bar{s}u,v)}{v} = m(\theta^{-1}, 1) = \frac{\mu(\theta)}{\theta}$.

Note that I work in a representative agent economy throughout this paper, and so the model cannot explain why different workers search with different intensities. But it is straightforward to extend the analysis to heterogeneous worker environment in which different types of workers search in different sub-markets, with no interaction across markets. In that case, the model can explain the cross-sectional relationship between job finding probability and search intensity.

3.2 Characterization of Equilibrium

I describe the economic environment precisely through a series of Bellman equations. For workers,

$$rU = \max_s \left(z\big(1 - hs^\nu\big) + s\mu(\theta)\big(E(w^*) - U\big)\right) \tag{2}$$

$$rE(w) = w + \lambda\big(U - E(w)\big). \tag{3}$$

The first equation gives the flow value of an unemployed worker. He chooses his search intensity s in order to maximize his leisure $z\big(1 - hs^\nu\big)$ plus the returns from search, equal to the matching probability $s\mu(\theta)$ times the capital gain upon getting a job. An employed worker gets a wage w but the job ends at rate λ, leaving the worker unemployed. The equations for firms are similar:

$$rV = -c + \eta(\theta)\big(J(w^*) - V\big) \tag{4}$$

$$rJ(w) = x - w - \lambda J(w). \tag{5}$$

The first equation gives the value of a vacancy, equal to minus the posting cost c plus the probability of contacting a worker times the resulting capital gain. The second equation directly parallels (3), except that a firm is left with nothing when a job ends. Note that free entry drives the value of a vacancy to $V = 0$.

Substituting equations (3) and (5) into the Nash bargaining solution (1) implies

$$\frac{E(w^*) - U}{\gamma} = \frac{J(w^*)}{1 - \gamma} = \frac{x - rU}{r + \lambda}.$$

Substituting that into (2) and (4) gives

$$rU = \max_s \left(z\left(1 - hs^{\nu}\right) + \frac{s\mu(\theta)\gamma(x - rU)}{r + \lambda} \right) \tag{6}$$

$$c = \frac{\eta(\theta)(1 - \gamma)(x - rU)}{r + \lambda}, \tag{7}$$

where I simplify the second equation using the free entry condition $V = 0$. Now combine equations (6) and (7) using the matching function identity $\eta(\theta) = \mu(\theta)/\theta$ to get

$$rU = \max_s \left(z\left(1 - hs^{\nu}\right) + \frac{s\theta\gamma c}{1 - \gamma} \right). \tag{8}$$

From this we can compute the first order condition for a worker's choice of search intensity s, assuming an interior solution. Since all workers use the same search intensity, I denote this using the average intensity $\bar{s}$:

$$\nu h \bar{s}^{\nu-1} = \frac{\theta c \gamma}{z(1 - \gamma)}. \tag{9}$$

Since $\nu > 1$, this defines an increasing relationship between search intensity and the measure of labor market tightness θ. To close the model, eliminate U between equations (7) and (8) and simplify using the fact that all workers search with a common intensity given by (9):

$$\left(x - z\left(1 - (\nu - 1)h\bar{s}^{\nu}\right)\right)\eta(\theta)(1 - \gamma) = c(r + \lambda). \tag{10}$$

Since $\nu > 1$ and η is decreasing, this defines a decreasing relationship between $\bar{s}$ and θ. There is at most one solution to equations (9) and (10), and that solution defines an equilibrium; standard conditions ensure existence.

3.3   Discussion

An increase in productivity x or a decrease in the job destruction rate λ shifts equation (10) without affecting (9). It is therefore straightforward to show that this results in a simultaneous increase in both equilibrium search intensity and equilibrium labor market tightness. The duration of an unemployment spell, $\frac{1}{\bar{s}\mu(\theta)}$ therefore falls. Although this is not (yet) a stochastic model, one would expect productivity or job destruction shocks to induce a negative correlation between search intensity and unemployment duration, i.e. workers should

10

search harder during booms.

It is worth noting that endogenous search intensity exacerbates the shift in the steady state vacancy-unemployment locus resulting from a change in job destruction. In steady state, job creation and destruction balance, so

$$m(\bar{s}u, v) = \lambda(1 - u).$$

An increase in λ reduces search intensity $\bar{s}$, raising the number of vacancies required to sustain a given unemployment rate. Even without the reduced search intensity, Shimer (2003) argued that this caused an increase in both unemployment and vacancies, inconsistent with the empirical observation that vacancies and unemployment are strongly negatively correlated at business cycle frequencies. This model of endogenous search intensity exacerbates the problem of countercyclical vacancies since procyclical search intensity further amplifies movements in the steady state vacancy-unemployment locus in response to shocks.

I can quantify the relationship between the time cost of search $h\bar{s}^\nu$ and the vacancy-unemployment ratio in response to shifts in variables like labor productivity x and the job destruction rate λ. Equation (9) implies

$$h\bar{s}^\nu = \frac{kc\gamma}{\nu z(1 - \gamma)},$$

where k is the vacancy-unemployment ratio. I can quantify this using the BLS's Job Openings and Labor Turnover Survey (JOLTS) and Current Population Survey (CPS). According to the JOLTS, the stock of job openings declined from an average of 4.1 million during the winter quarter (December to February) of 2000–01 to 2.8 million one year later and 2.6 million in 2002–03. At the same time, the CPS indicates that the number of unemployed workers increased from 6.8 million to 8.2 million and then to 8.6 million, so all together the vacancy-unemployment ratio fell in half during this two year period. Despite this, there is no evidence that the time cost of search for the average unemployed worker fell at all during this period.

The root cause of the procyclicality in search intensity can be seen directly from equation (2). The marginal product of search increases when either the efficiency of search $\mu(\theta)$ or the capital gain upon getting a job $E(w^*) - U$ increases. The first variable is, almost by definition, procyclical. The second variable is procyclical in a bargaining model; bargaining ensures that this is proportional to firms' capital gain $J(w^*)$, while free entry by firms implies

11

this must rise in booms to offset the increased duration of a vacancy $\eta(\theta)$. But even without the bargaining assumption, one would require a strongly countercyclical wage to offset the increase in workers' hiring probability during booms.

4   Discrete Time Model

I now turn to a discrete time version of the same model. The main technical difference lies in the interpretation of s: I assume that a worker who searches with intensity s applies for s jobs, each of which hires the worker with independent probability $\mu \in [0, 1]$, itself a function of labor market tightness. The worker's goal is to get at least one job offer, but multiple offers are no more valuable than a single offer.

Now think of the returns to search in two extreme cases. First, if $\mu = 0$, there is no reason to search, since applications are always rejected. Second, if $\mu = 1$, there is no reason to apply for more than one job, since any application is accepted. But at intermediate values of μ, a worker may be willing to make multiple applications in order to raise the probability that he receives at least one offer. This implies that at low values of μ, an increase in μ raises the returns to search, while the opposite is true at high values of μ. This possibility was absent from the continuous time model.

This model is essentially an equilibrium version of Benhabib and Bull (1983) or Morgan and Manning (1985). In those papers, a worker samples wages from a fixed distribution, here either 0 with probability $1 - \mu$ or w^* with probability μ. In each period, the worker chooses his sample size s, but may sample again the next period if the result of this period's search is unsatisfactory. A large per-period sample size is attractive because it increases the probability of obtaining a job sooner rather than later, but receiving multiple acceptable job offers within a period is wasteful, putting a limit on the optimal fixed sample size. Some combination of fixed sample size and sequential search is typically optimal.

4.1   Model

The basic structure of the model is unchanged from the continuous time model. Workers and firms are risk-neutral and discount future payoffs at rate $\beta \in (0, 1)$.

An unemployed worker gets leisure $z\left(1 - hs^{\nu}\right)$, where $h > 0$ and $\nu \geq 1$. In the discrete time model, I do not require strict convexity of search costs, since the marginal return to search is concave in the discrete time model. More precisely, a worker who makes s applications gets

a job offer with probability $1 - (1 - \mu)^s$, where μ is the endogenous probability that any one application is successful, with a realization independent across applications. If the worker receives multiple offers, he must turn all but one down before bargaining over the wage w. In other words, multiple job offers does not permit him to obtain a higher wage, an issue that I return to later.[10] The job starts at the beginning of the next period and subsequently ends with probability $\lambda \in (0, 1]$ in any future period.

An open vacancy costs a firm c per period and attracts an applicant who is willing to accept the job with endogenous probability η. A filled job generates profit $x - w$ until the job ends, at which point the firm is left with nothing. Free entry drives the value of a vacancy to zero.

Wages are determined by Nash bargaining between employed workers and firms. The wage is set to maximize the weighted product of the surplus from the match in excess of threat points, exactly as in equation (1).

The main change from the continuous time model lies in the matching function. Let $m(\bar{s}u, v) \leq \min\{\bar{s}u, v\}$ denote the measure of job offers generated in each period. An application results in an offer with probability $\mu(\theta) = \frac{m(\bar{s}u,v)}{\bar{s}u} = m(1, \theta)$, increasing in θ and lying between zero and one. But recall that this is not the probability that a worker accepts a job. I denote that by

$$p(\theta, s) = 1 - \big(1 - \mu(\theta)\big)^s. \tag{11}$$

In a symmetric equilibrium, the measure of workers who get job within a period is $up(\theta, \bar{s})$, and so the probability that a typical vacancy is filled is

$$\eta(\theta, \bar{s}) = \frac{1 - \big(1 - \mu(\theta)\big)^{\bar{s}}}{\bar{s}\theta} = \frac{p(\theta, \bar{s})}{\theta\bar{s}}. \tag{12}$$

If $\bar{s} = 1$, so workers make only one application pre period, this equals $\mu(\theta)/\theta$, but otherwise it is a strictly decreasing function of $\bar{s}$. For a given labor market tightness θ, an increase in workers' search effort reduces the efficiency of the matching process. Additionally, if $\mu(\theta)$ is concave and $s > 1$, η is a strictly decreasing function of θ.[11]

Equation (12) is important for what follows, so is worth deriving in a different manner.

---

[10]See also Albrecht, Gautier, and Vroman (2003a).

[11]Proof: The partial derivative of η with respect to θ is proportional to

$$\theta\bar{s}(1 - \mu(\theta))^{\bar{s}-1}\mu'(\theta) - 1 + (1 - \mu(\theta))^{\bar{s}} = \theta g'(\theta) - g(0) + g(\theta),$$

where $g(\theta) = (1 - \mu(\theta))^{\bar{s}}$ and $g(0) = 1$ because $\mu(0) = 0$. This is negative if g is convex, and convexity of g is ensured by the assumptions that $\bar{s} \geq 1$ and μ is concave.

The probability that a firm hires a worker is the probability that it receives at least one application, $\mu(\theta)/\theta$ times the probability that the firm's offer is accepted. If workers make one application each, $\bar{s} = 1$, the latter probability is 1, so the firm hires a worker with probability $\mu(\theta)/\theta$. If workers make two applications each, the firm hires the worker if either the worker's other application is rejected (probability $1 - \mu(\theta)$) or the worker's other application is accepted but he decides to take this job offer (probability $\frac{1}{2}\mu(\theta)$). Combining this, the firm hires a worker with probability

$$\frac{\mu(\theta)}{\theta}\left(1 - \mu(\theta) + \tfrac{1}{2}\mu(\theta)\right) = \eta(\theta, 2),$$

where the equality follows directly from the definition of η in (12). The same logic carries over to the case of $\bar{s} = 3$. Now the probability that a firm hires a worker is

$$\frac{\mu(\theta)}{\theta}\left((1 - \mu(\theta))^2 + \mu(\theta)(1 - \mu(\theta)) + \tfrac{1}{3}\mu(\theta)^2\right),$$

which one can again verify equals $\eta(\theta, 3)$.

It is also useful to provide a concrete interpretation of the function μ. Suppose that workers randomly send their applications to vacancies, and each vacancy that receives at least one application accepts one worker. This is a standard 'urn-ball' matching function, originally explored by Butters (1977) and more recently in a similar context by Albrecht, Gautier, and Vroman (2003b). With a standard abuse of the law of large numbers, the fraction of firms receiving no applications is $e^{-\bar{s}u/v}$, while the remaining firms make one offer each. This implies that the total number of offers is $m(\bar{s}u, v) = \left(1 - e^{-\bar{s}u/v}\right)v$ and the probability that an application generates an offer is $\mu(\theta) = \theta\left(1 - e^{-1/\theta}\right)$. The fraction of vacancies that are filled is

$$\eta(\theta, \bar{s}) = \frac{1 - \left(1 - \theta\left(1 - e^{-1/\theta}\right)\right)^{\bar{s}}}{\bar{s}\theta},$$

decreasing in search intensity $\bar{s}$ and in labor market tightness θ. An increase in search intensity does not have an obvious effect on the fraction of vacancies that are filled, and hence on the fraction of unemployed workers who find a job because it does not alleviate the coordination problem in the matching market. For example,

$$\eta(\tfrac{v}{u}, 1) = 1 - e^{-u/v} \text{ and } \lim_{\bar{s} \to \infty} \eta(\tfrac{v}{\bar{s}u}, \bar{s}) = \frac{u}{v}\left(1 - e^{-v/u}\right).$$

14

When $\bar{s} = 1$, workers randomly apply for one job, leaving some firms without applicants. When $\bar{s} = \infty$, virtually all firms get applications, but firms randomly accept one applicant, leaving some workers without offers. Which is more efficient depends on whether $u \gtrless v$.

There is one important issue that merits discussion: given the motivation for the functional forms, it is natural to assume that search intensity must be a positive integer $s \in \{0, 1, \ldots\}$. But it is mathematically easier if I instead view the motivation as one possible explanation for these functional forms and allow s to be any positive real number. I follow that approach initially, focusing on the case in which search intensity is a large number so integer constraints are unlikely to be important. In Section 4.4, I show that restricting s to be a positive integer introduces a peculiar multiplicity of equilibria into the model.

4.2 Characterization of Equilibrium

I characterize the equilibrium through a series of discrete time Bellman equations. Starting with workers,

$$U = \max_s \left(z\big(1 - hs^\nu\big) + \beta\Big(\big(1 - p(\theta, s)\big)U + p(\theta, s)E(w^*)\Big) \right) \tag{13}$$

$$E(w) = w + \beta\big((1 - \lambda)E(w) + \lambda U\big) \tag{14}$$

An unemployed worker chooses his search effort s to maximize the value of leisure this period, $z\big(1 - hs^\nu\big)$, plus the discounted continuation value; he will be unemployed with probability $1 - p(\theta, s)$ next period and otherwise employed at the bargained wage w^*. An employed worker receives the wage, but the match ends the following period with probability λ, leaving him unemployed. There are similar equations for firms:

$$V = -c + \beta\big(\eta(\theta, \bar{s})J(w^*) + \big(1 - \eta(\theta, \bar{s})\big)V\big) \tag{15}$$

$$J(w) = x - w + \beta(1 - \lambda)J(w). \tag{16}$$

Combining the Nash bargaining solution (1) with equations (14) and (16) gives

$$\frac{E(w^*) - U}{\gamma} = \frac{J(w^*)}{1 - \gamma} = \frac{x - (1 - \beta)U}{1 - \beta(1 - \lambda)}.$$

15

Use this and (14) to eliminate $E(w^*)$ from (13):

$$(1 - \beta)U = \max_s \left(z(1 - hs^\nu) + \frac{\beta p(\theta, s)\gamma(x - (1 - \beta)U)}{1 - \beta(1 - \lambda)} \right) \quad (17)$$

Similarly, use the Nash bargaining solution and (16) to eliminate $J(w^*)$ from (15). Also set $V = 0$ using the free entry condition:

$$c = \frac{\beta\eta(\theta, \bar{s})(1 - \gamma)(x - (1 - \beta)U)}{1 - \beta(1 - \lambda)} \quad (18)$$

Combining this with (17) gives

$$(1 - \beta)U = \max_s \left(z(1 - hs^\nu) + \frac{p(\theta, s)\gamma c}{\eta(\theta, \bar{s})(1 - \gamma)} \right). \quad (19)$$

Note that s^ν is weakly convex and, from equation (11), that $p(\theta, s)$ is strictly concave function of s, so for a fixed value of $\bar{s}$ and θ, all workers will choose the same search intensity s satisfying the necessary and sufficient first order condition. Since in equilibrium that value will be the same as $\bar{s}$, I can write the optimal search intensity condition as

$$\frac{(1 - \gamma)z\nu h\bar{s}^{\nu-1}}{\gamma c} = -\frac{(1 - \mu(\theta))^{\bar{s}} \log(1 - \mu(\theta))\bar{s}\theta}{1 - (1 - \mu(\theta))^{\bar{s}}}, \quad (20)$$

where I replace the partial derivative of p and the level of η using their definitions (11) and (12).

To close the model, simplify (19) using symmetry of search intensities, $s = \bar{s}$, with $p(\theta, \bar{s}) = \eta(\theta, \bar{s})\theta\bar{s}$:

$$(1 - \beta)U = z(1 - h\bar{s}^\nu) + \frac{\gamma c\bar{s}\theta}{1 - \gamma} \quad (21)$$

Substitute this into (18) and simplify:

$$\frac{c}{\beta\eta(\theta, \bar{s})} = \frac{(1 - \gamma)(x - z(1 - h\bar{s}^\nu)) - \gamma c\bar{s}\theta}{1 - \beta(1 - \lambda)} \quad (22)$$

An equilibrium is then a search intensity $\bar{s}$ and labor market tightness θ that solves equations (20) and (22).

16

4.3 Small Search Cost

This section focuses on limiting results when the shift parameter for the marginal time cost of search, h, is very small. I derive the 'eighty percent' rule described in the introduction. It is perhaps not too surprising that the number of applications $\bar{s}$ tends towards infinity as h converges to zero. But I show that there is a well-behaved limit in which total search expenditures $S \equiv h\bar{s}^{\nu}$, the probability of obtaining a job $p(\theta, \bar{s})$, the probability of hiring a worker $\eta(\theta, \bar{s})$, and the vacancy-unemployment ratio $k \equiv v/u$ converge to finite numbers. This limit is particularly easy to analyze.

I start by looking at the behavior of the probability that a worker gets a job when $\bar{s}$ is large for a given vacancy-unemployment ratio. One can show that this is[12]

$$\tilde{p}(k) \equiv \lim_{\bar{s} \to \infty} p(k/\bar{s}, \bar{s}) = 1 - e^{-\mu'(0)k}.$$

The matching function enters only through $\mu'(0)$, which can be interpreted as the probability that an application generates an offer at a firm that receives no other applications. This is bounded above by 1, and in fact equals 1 if $\lim_{v \to \infty} m(\bar{s}u, v) = \bar{s}u$, a common assumption. Similarly, the probability that a firm hires a worker is $\tilde{\eta}(k) = \tilde{p}(k)/k$.

The next step is to rewrite the equilibrium conditions in terms of these limiting functions. Equation (22) is easily rewritten in the limiting case when $\bar{s}$ is large:

$$x - z(1 - S) = \frac{\left(1 - \beta(1 - \lambda)\right)ck}{\beta(1 - \gamma)\left(1 - e^{-\mu'(0)k}\right)} + \frac{\gamma ck}{1 - \gamma}. \tag{23}$$

This describes a monotonically increasing relationship between the vacancy-unemployment ratio k and the total time cost of search S.

Now turn to the first order condition (20). Use equation (11) to rewrite the expression in terms of p rather than $\mu(\theta)$ and then take the limit as $\bar{s}$ gets large:

$$S = \left(\frac{\gamma c}{(1 - \gamma)\nu z}\right)\left(-\frac{(1 - \tilde{p}(k))\log(1 - \tilde{p}(k))k}{\tilde{p}(k)}\right).$$

Since $k = -\log(1 - \tilde{p}(k))/\mu'(0)$, we may write this more usefully as

$$S = \left(\frac{\gamma c}{(1 - \gamma)\nu\mu'(0)z}\right)\left(\frac{(1 - \tilde{p}(k))\log(1 - \tilde{p}(k))^2}{\tilde{p}(k)}\right). \tag{24}$$

---

[12]Proof: Take the limit of $\log(1 - p(\varepsilon k, \varepsilon)) = \frac{1}{\varepsilon}\log(1 - \mu(\varepsilon k))$ as ε converges to zero. This can be evaluated using L'Hôpital's rule. Since $\mu(0) = 0$, we get $\log(1 - \tilde{p}(k)) = -\mu'(0)k$.

This defines a non-monotone single-peaked relation between the probability of getting a job $\tilde{p}(k)$ and the time cost of search S. When $\tilde{p}(k) = 0$ or $\tilde{p}(k) = 1$, so the vacancy-unemployment ratio k is zero or infinite, equation (24) states that workers choose to search with minimal intensity. The time cost of search is maximized when $2\tilde{p}(k) + \log(1 - \tilde{p}(k)) = 0$, or equivalently when the worker has a $\tilde{p}(k) = 0.797$ probability of obtaining a job within a period.[13] This result is independent of any other details of the economic environment, beyond the assumption that the marginal time cost of search is arbitrarily small.

An equilibrium is a time cost of search S and a vacancy-unemployment ratio that solves equations (23) and (24). A sufficient condition for an equilibrium to exist is that the value of search intensity S associated with $k = 0$ in equation (23) is negative, or equivalently

$$c < \frac{\beta\mu'(0)(1 - \gamma)\big(x - z\big)}{1 - \beta(1 - \lambda)}. \tag{25}$$

This condition has a logical interpretation. The cost of search must be sufficiently low so that if the vacancy-unemployment ratio were zero, a firm would be willing to open a vacancy, hire a worker next period with probability $\mu'(0)$, and keep a fraction $1 - \gamma$ of the surplus from producing rather than enjoying full leisure, $x - z$, discounted until the match eventually ends.

If condition (25) is satisfied, it is possible to prove that there is a unique equilibrium. More precisely, think of equations (23) and (24) as defining search intensity S as functions of the vacancy-unemployment ratio k; an equilibrium is a value of k where both expressions give the same value of S. Then it is possible to prove that in equilibrium, equation (23) defines a steeper relationship than equation (24).[14] This not only establishes uniqueness of the equilibrium, but also allows for unambiguous comparative statics. An increase in labor productivity or a decrease in the job destruction rate shifts equation (24) to the right, raising the vacancy-unemployment ratio associated with any search intensity. This therefore raises the equilibrium vacancy-unemployment ratio and moves the economy along the non-monotone curve (24); the impact on search intensity is ambiguous. If k exceeds $1.59/\mu'(0)$,

---

[13]The time cost of search is always less than approximately $\frac{0.648\gamma c}{(1-\gamma)\nu\mu'(0)z}$, so the time endowment 1 must exceed this number to ensure the existence of an interior equilibrium.

[14]Differentiate both expressions with respect to k. The difference in derivatives is positive if

$$\frac{\beta\gamma}{1 - \beta(1 - \lambda)} > \frac{\tilde{p}(k) + (1 - \tilde{p}(k))\log(1 - \tilde{p}(k))}{-\tilde{p}(k)^2 - 2\tilde{p}(k)(1 - \tilde{p}(k))\log(1 - \tilde{p}(k)) - (1 - \tilde{p}(k))\log(1 - \tilde{p}(k))^2}$$

It is easy to verify numerically that the right hand side is negative for all values of $\tilde{p}(k)$, and so the condition is satisfied.

18

so $\tilde{p}(k)$ exceeds 0.8, the increase in k reduces search intensity, while the opposite is true when k is small.

In other words, in a labor market where workers have at least an 80 percent probability of getting a job within a period, an increase in labor productivity or a reduction in the job destruction rate that further increases the probability of getting a job (a 'boom') reduces the time expenditure on search. Workers are sufficiently confident that they can obtain a job that they do not work very hard towards getting one. Conversely, if the probability of getting a job is less than 80 percent, the time expenditure falls along with the probability of getting a job falls. This is the standard discouraged worker phenomenon, exactly as in the continuous time model.

Another way to interpret this result is to note that over a fairly wide range of values of the probability of getting a job, search intensity is unresponsive to that probability. For example, if workers have either a 70 percent or an 87 percent probability of getting a job, their search intensity is just four percent lower than its maximal value. To be more concrete, consider the following numerical example, designed to reflect a time period that lasts one month: The elasticity of the search cost function is $\nu = 1$, the marginal utility of leisure is $z = 1$, the discount factor is $\beta = 0.995$, the job destruction rate is $\lambda = 0.04$, workers' bargaining power is $\gamma = 0.5$, the cost of a vacancy is $c = 1$, and the slope of the matching function is $\mu'(0) = 1$. As productivity x varies from approximately 1.60 to 2.85, over a substantial range, the vacancy-unemployment ratio k doubles from 1.05 to 2.23 and search intensity initially increases from 0.593 to 0.648 and then falls back to 0.600 (Figure 9). It would be difficult to observe such small variation in intensity in any empirical analysis.

4.4   Discrete Search Choice

In the previous section, I focused on the limiting case with small search costs. This section explores the opposite case, examining how the results change when the discrete choice problem is important. Assume now that the worker must select $s \in \{0, 1, 2, \ldots\}$, a non-negative integer. For simplicity, assume that the time cost of search is constant, $\nu = 1$. The return to search is still concave, so all workers will choose the same search intensity, $s = \bar{s}$, as in the continuous choice model. However, that search intensity is described by a pair of inequalities,

$$\frac{\mu(\theta)\big(1 - \mu(\theta)\big)^{\bar{s}-1}\bar{s}\theta}{1 - \big(1 - \mu(\theta)\big)^{\bar{s}}} \geq \frac{(1 - \gamma)z}{\gamma c} \geq \frac{\mu(\theta)\big(1 - \mu(\theta)\big)^{\bar{s}}\bar{s}\theta}{1 - \big(1 - \mu(\theta)\big)^{\bar{s}}}, \tag{26}$$

rather than the first order condition (20). The remainder of the model is unchanged. In particular, an equilibrium is a labor market tightness θ and search intensity $\bar{s}$ that satisfies equations (22) and the inequalities in condition (26).

Oddly, condition (26) does not uniquely pin down workers' search intensity given the labor market tightness, due to a general equilibrium feedback. Economically, recall that equation (26) embodies worker optimization as well as the Nash bargaining and free entry conditions. An increase in workers's search intensity reduces the rate at which firms match $\eta(\theta, \bar{s})$, which must be offset by an increase in total surplus. This encourages workers to search with still greater intensity.

Mathematically, the problem is that

$$\frac{\mu(\theta)\big(1 - \mu(\theta)\big)^{\bar{s}}(\bar{s} + 1)\theta}{1 - \big(1 - \mu(\theta)\big)^{\bar{s}+1}} > \frac{\mu(\theta)\big(1 - \mu(\theta)\big)^{\bar{s}}\bar{s}\theta}{1 - \big(1 - \mu(\theta)\big)^{\bar{s}}}.$$

This implies that the second inequality in (26) may hold at $\bar{s}$, but the first one also holds at $\bar{s} + 1$, so both search intensities are consistent with optimization. In fact, one can show that this source of multiplicity can generate at most two equilibria.

Because of this feedback, it is easy to construct discrete choice examples with multiple equilibria. Fix the same parameter values as in Figure 9, with productivity fixed at $x = 2$ and the time cost of search function $h = 0.25$. Also specify the urn-ball matching technology $\mu(\theta) = \theta(1 - \exp(-1/\theta))$. There is an equilibrium with $\bar{s} = 2$ and $\theta = 0.671$ and another equilibrium with $\bar{s} = 3$ and $\theta = 0.526$. The vacancy-unemployment ratio $k = \theta\bar{s}$ is higher in the equilibrium with more search, as is the probability that a worker is hired (0.831 versus 0.770).

Although this multiplicity never disappears, it is economically inconsequential when the cost of search is low. If $h = 0.001$, there are two equilibria $(\bar{s}, k) = (644, 1.4721)$ and $(645, 1.4730)$. Moreover, these equilibria are virtually identical to those in the limiting economy, where $S = h\bar{s}^{\nu} = 0.645$ and $k = 1.4732$. I conclude that unless each application is extremely costly, the discrete choice problem is cumbersome and does not add much that is of economic interest to the problem.

5   Aggregate Shocks

I have so far examined a deterministic steady state equilibrium. This section shows how to extend the discrete time model to an environment in which aggregate productivity x and the job destruction rate λ follow a first order Markov process. This is important for interpreting the results: in a stochastic environment, we may examine how an individual worker varies his search intensity in response to a transitory change in economic conditions, such as a recession.

All the asset values and wages are now state contingent, e.g. $U_{x,\lambda}$ is the value of an unemployed worker as a function of the current aggregate state. Let $\mathbb{E}_{x,\lambda}$ denote the one-period-ahead expectations operator conditional on the current state, so $\mathbb{E}_{x,\lambda}U_{x',\lambda'}$ is the expected value of an unemployed worker next period as a function of the current state $\{x, \lambda\}$.

Using this notation, we may write the Bellman equations for workers in the stochastic model as

$$U_{x,\lambda} = \max_s \left(z\big(1 - hs^\nu\big) + \beta\Big(\big(1 - p(\theta_{x,\lambda}, s)\big)\mathbb{E}_{x,\lambda}U_{x',\lambda'} + p(\theta_{x,\lambda}, s)\mathbb{E}_{x,\lambda}E_{x',\lambda'}(w_{x',\lambda'})\Big)\right) \quad (27)$$

$$E_{x,\lambda}(w) = w + \beta\big((1 - \lambda)\mathbb{E}_{x,\lambda}E_{x',\lambda'}(w_{x',\lambda'}) + \lambda\mathbb{E}_{x,\lambda}U_{x',\lambda'}\big). \quad (28)$$

Here $\theta_{x,\lambda}$ is the state-contingent measure of labor market tightness and $w_{x,\lambda}$ is the state contingent equilibrium wage. There is a similar pair of equations for firms. Using the free entry condition to set $V_{x,\lambda} = 0$ for all x, λ, we may write this as

$$c = \beta\eta(\theta_{x,\lambda}, \bar{s}_{x,\lambda})\mathbb{E}_{x,\lambda}J_{x',\lambda'}(w_{x',\lambda'}) \quad (29)$$

$$J_{x,\lambda}(w) = x - w + \beta(1 - \lambda)\mathbb{E}_{x,\lambda}J_{x',\lambda'}(w_{x',\lambda'}). \quad (30)$$

Here $\bar{s}_{x,\lambda}$ is state-contingent average search intensity. As usual, I close the model with the Nash bargaining solution:

$$w_{x,\lambda} \in \arg\max_w \left(E_{x,\lambda}(w) - U_{x,\lambda}\right)^\gamma J_{x,\lambda}(w)^{1-\gamma}$$

for all $\{x, \lambda\}$, a natural generalization of equation (1). Since equations (28) and (30) imply $E'_{x,\lambda}(w) = -J'_{x,\lambda}(w) = 1$, the Nash bargaining solution is equivalent to a surplus-splitting rule:

$$\frac{E_{x,\lambda}(w_{x,\lambda}) - U_{x,\lambda}}{\gamma} = \frac{J_{x,\lambda}(w_{x,\lambda})}{1 - \gamma}. \quad (31)$$

21

for all $\{x, \lambda\}$. Moreover, since the surplus-splitting rule holds state-by-state, it holds in expectation one period ahead:

$$\mathbb{E}_{x,\lambda} \frac{E_{x',\lambda'}(w_{x',\lambda'}) - U_{x',\lambda'}}{\gamma} = \mathbb{E}_{x,\lambda} \frac{J_{x',\lambda'}(w_{x',\lambda'})}{1 - \gamma}.$$

Use this to simplify the Bellman equation for unemployed workers (27):

$$U_{x,\lambda} - \beta \mathbb{E}_{x,\lambda} U_{x',\lambda'} = \max_s \left(z(1 - hs^\nu) + \beta p(\theta_{x,\lambda}, s) \mathbb{E}_{x,\lambda}(E_{x',\lambda'}(w_{x',\lambda'}) - U_{x',\lambda'}) \right)$$

$$= \max_s \left(z(1 - hs^\nu) + \frac{\beta \gamma p(\theta_{x,\lambda}, s)}{1 - \gamma} \mathbb{E}_{x,\lambda} J_{x',\lambda'}(w_{x',\lambda'}) \right)$$

Eliminate the expectation of the value of a filled job next period using the free entry condition (29):

$$U_{x,\lambda} - \beta \mathbb{E}_{x,\lambda} U_{x',\lambda'} = \max_s \left(z(1 - hs^\nu) + \frac{p(\theta_{x,\lambda}, s)\gamma c}{\eta(\theta_{x,\lambda}, \bar{s}_{x,\lambda})(1 - \gamma)} \right) \tag{32}$$

Once again, convexity of s^ν and concavity of p in s ensures that all workers choose the same search intensity $\bar{s}_{x,\lambda}$, in the differentiable case satisfying the necessary and sufficient first order condition

$$\frac{(1 - \gamma)z\nu h\bar{s}_{x,\lambda}^{\nu-1}}{\gamma c} = -\frac{\left(1 - \mu(\theta_{x,\lambda})\right)^{\bar{s}_{x,\lambda}} \log(1 - \mu(\theta_{x,\lambda})) \bar{s}_{x,\lambda} \theta_{x,\lambda}}{1 - \left(1 - \mu(\theta_{x,\lambda})\right)^{\bar{s}_{x,\lambda}}}, \tag{33}$$

a state-contingent version of equation (20). In particular, for the case of a small time cost of search, search expenditures are related to the contemporaneous probability of getting a job via a stochastic version of equation (24)

$$S_{x,\lambda} = \left(\frac{\gamma c}{(1 - \gamma)\nu \mu'(0)z} \right) \left(\frac{(1 - \tilde{p}(k_{x,\lambda})) \log(1 - \tilde{p}(k_{x,\lambda}))^2}{\tilde{p}(k_{x,\lambda})} \right),$$

where $k_{x,\lambda}$ is the state-contingent vacancy-unemployment ratio. Search expenditures remain independent of the stochastic process for productivity and the job destruction rate and are maximized during periods when the probability of getting a job is approximately eighty percent.

I close the model in the usual way. In equilibrium, all workers use the same search

22

strategy. Since furthermore $p(\theta, s) = s\theta\eta(\theta, s)$, equation (33) becomes

$$U_{x,\lambda} - \beta\mathbb{E}_{x,\lambda}U_{x',\lambda'} = z(1 - h\bar{s}^\nu_{x,\lambda}) + \frac{\gamma c\bar{s}_{x,\lambda}\theta_{x,\lambda}}{1 - \gamma}.$$

Next solve equations (28), (30) and (31) for the wage, further simplifying with the preceding equation.

$$\begin{aligned} w_{x,\lambda} &= \gamma x + (1 - \gamma)\big(U_{x,\lambda} - \beta\mathbb{E}_{x,\lambda}U_{x',\lambda'}\big) \\ &= \gamma\big(x + c\bar{s}_{x,\lambda}\theta_{x,\lambda}\big) + (1 - \gamma)z(1 - h\bar{s}^\nu_{x,\lambda}). \end{aligned}$$

Substitute this into equations (29) and (30) and simplify again:

$$\begin{aligned} \frac{c}{\beta\eta(\theta_{x,\lambda}, \bar{s}_{x,\lambda})} = (1 - \gamma)\big(\mathbb{E}_{x,\lambda}x' - z(1 - \mathbb{E}_{x,\lambda}h\bar{s}^\nu_{x',\lambda'})\big) \\ - \gamma c\mathbb{E}_{x,\lambda}\bar{s}_{x',\lambda'}\theta_{x',\lambda'} + \mathbb{E}_{x,\lambda}\frac{c(1 - \lambda')}{\eta(\theta_{x',\lambda'}, \bar{s}_{x',\lambda'})}. \quad (34) \end{aligned}$$

This is a stochastic analog of equation (24). In the case of a small time cost of search, this reduces to

$$\frac{ck_{x,\lambda}}{\beta\big(1 - e^{-\mu'(0)k_{x,\lambda}}\big)} = (1 - \gamma)\big(\mathbb{E}_{x,\lambda}x' - z(1 - \mathbb{E}_{x,\lambda}S_{x',\lambda'})\big) - \gamma c\mathbb{E}_{x,\lambda}k_{x',\lambda'} + \mathbb{E}_{x,\lambda}\frac{c(1 - \lambda')k_{x',\lambda'}}{1 - e^{-\mu'(0)k_{x',\lambda'}}},$$

the stochastic analog of equation (23). An equilibrium is a state-contingent search intensity $\bar{s}_{x,\lambda}$ and labor market tightness $\theta_{x,\lambda}$ satisfying equations (33) and (34).

The important point is that the contemporaneous values of x and λ, as well as the stochastic process for those variables, determine the vacancy-unemployment ratio and workers' search intensity. This in turn determines the probability that a worker obtains a job within a period. When the equilibrium probability of obtaining a job exceeds eighty percent, workers respond to an increase in that probability by 'relaxing', i.e. reducing their search intensity. When the equilibrium probability of getting a job is lower, workers respond to a decrease in the probability by getting discouraged, i.e. reducing their search intensity. This is independent of the stochastic process for x and λ.

6   Additional Evidence and Future Plans

The discrete time model predicts that workers who have more than an eighty percent chance of getting a job within a period should search harder when labor market conditions worsen. Since a period reflects the length of time it takes to evaluate an application, this is more likely to apply to more educated workers than less educated workers. This section presents some preliminary results that, unfortunately, do not provide much support for this conclusion. First I regress the binary search decision on demographic controls and period dummies separately for four education categories for workers age 25 to 70. Figure 10 shows that all four groups show a significant increase in this measure of search intensity during 2001. Then I regress the number of search methods on demographic controls, industry, occupation, and unemployment duration for searchers age 25 to 70. Figure 11 is a little more supportive of the theory proposed here. Although no group shows a sharp decrease in search intensity in 2001, the increase is strongest for more educated workers and negligible for workers with less than a high school diploma.

What goes wrong? A possibility that I am currently exploring is that when wages are determined via Nash bargaining, they are too flexible. In the Nash bargaining model, an improvement in labor market conditions raises the surplus from obtaining a job increases, which encourages workers to search harder. If wages were fixed, the surplus from obtaining a job would actually fall, since future jobs are easier to come by. This effect may imply that an increase in the probability of obtaining a job reduces search intensity even if the probability of finding a job within a period is already quite low.

References

ALBRECHT, JAMES, PIETER GAUTIER, AND SUSAN VROMAN (2003a): "Equilibrium Directed Search with Multiple Applications," Georgetown University mimeo.

——— (2003b): "Matching with Multiple Applications," *Economics Letters*, 78(1), 67–70.

BARRON, JOHN, AND WESLEY MELLOW (1979): "Search Effort in the Labor Market," *Journal of Human Resources*, 14(3), 389–404.

BENHABIB, JESS, AND CLIVE BULL (1983): "Job Search: The Choice of Intensity," *Journal of Political Economy*, 91(5), 747–764.

BLANCHARD, OLIVIER, AND PETER DIAMOND (1990): "The Cyclical Behavior of the Gross Flows of U.S. Workers," *Brookings Papers on Economic Activity*, 2, 85–143.

BUREAU OF LABOR STATISTICS (2001): "How the Government Measures Unemployment," ⟨`http://www.bls.gov/cps/cps_htgm.htm`⟩, modified October 16, 2001.

BUTTERS, GERARD (1977): "Equilibrium Distribution of Sales and Advertising Prices," *Review of Economic Studies*, 44, 465–491.

MORGAN, PETER, AND RICHARD MANNING (1985): "Optimal Search," *Econometrica*, 53(4), 923–944.

PISSARIDES, CHRISTOPHER (2000): *Equilibrium Unemployment Theory*. MIT Press, Cambridge, MA, second edn.

SHIMER, ROBERT (2003): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies: Evidence and Theory," NBER working paper 9536.

STIGLER, GEORGE (1961): "The Economics of Information," *Journal of Political Economy*, 69, 213–225.

VERACIERTO, MARCELO (2002): "On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation," Federal Reserve Bank of Chicago working paper 2002-12.

	Column I	Column II
Dependent Variable	Searcher	No. of Methods
Black	0.009 (0.002)	−0.025 (0.007)
Woman	−0.141 (0.002)	−0.129 (0.006)
Married	−0.031 (0.002)	−0.034 (0.006)
Less than HS	omitted	omitted
HS Diploma	0.041 (0.002)	0.146 (0.008)
Some College	0.061 (0.002)	0.300 (0.009)
College Plus	0.065 (0.003)	0.437 (0.010)
Age	quartic	quartic
Unemployment duration	no	quartic
Industry controls	no	yes
Occupation controls	no	yes
Time dummies	yes	yes
No. of Observations	382,082	184,111
R^2	0.0649	0.0642

Table 1: Linear regression coefficients. In Column I, the dependent variable is equal to 1 if the individual is a searcher and 0 if he or she is a non-searcher. In column II, the dependent variable is equal to the number of search methods used and the sample includes only searchers. The six independent variables with coefficient estimates and standard errors shown are dummy variables equal to one if the condition holds and zero otherwise. The quartic terms for age and unemployment duration are shown in Figures 2, 5, and 6. The sample includes individuals age 25–70.
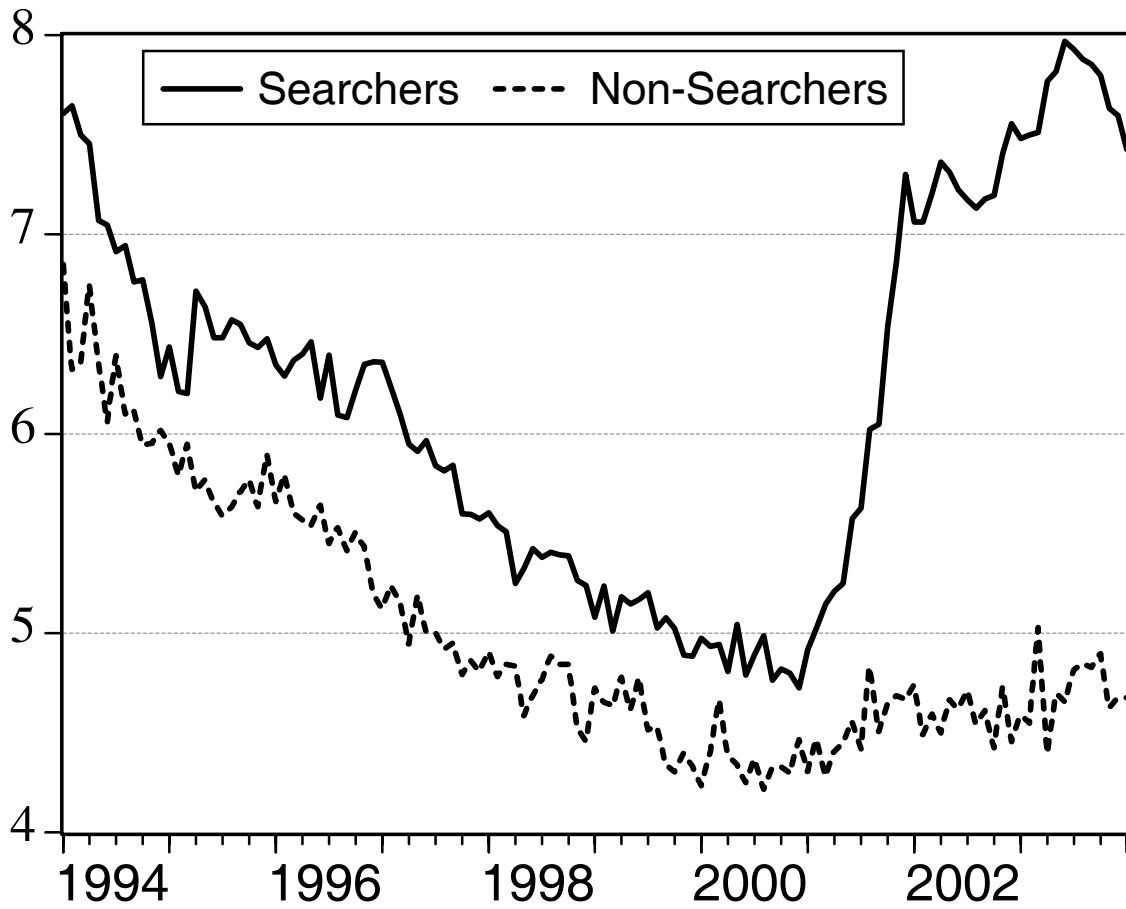
Figure 1: The number of job searchers and job wanters, in millions and seasonally adjusted. Author's calculations from the monthly CPS.
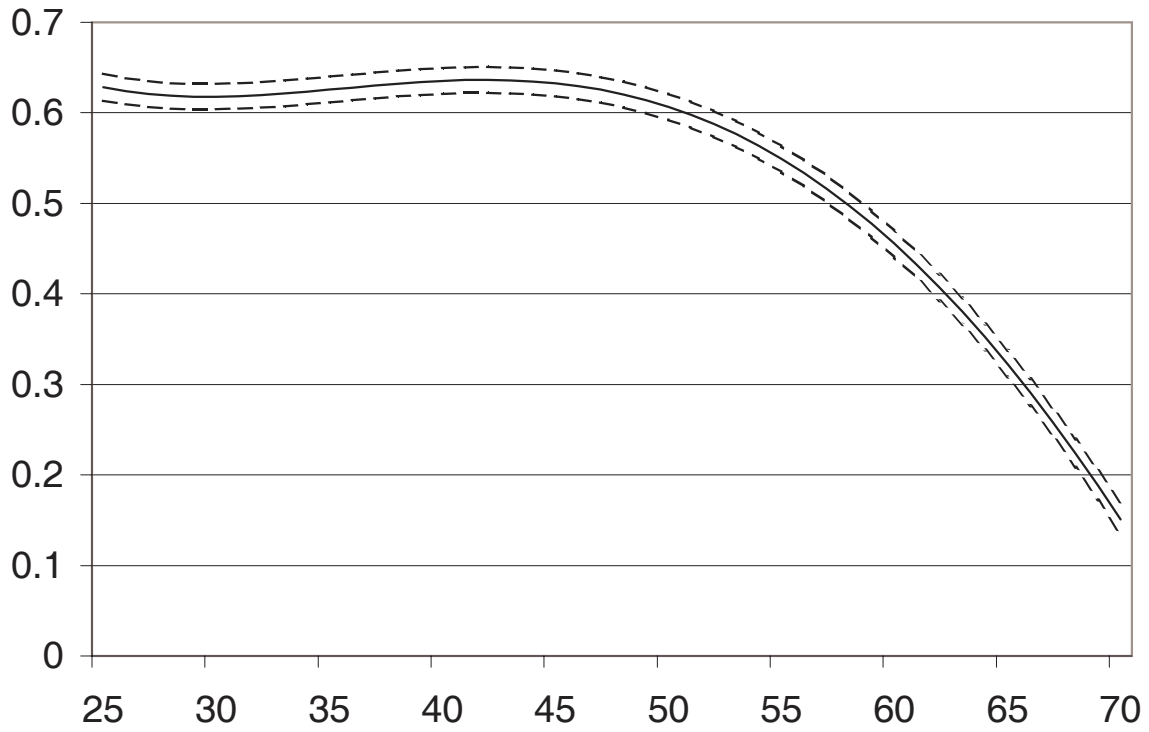
Figure 2: The effect of age on the probability of being a searcher rather than a non-searcher for the regression in Column I of Table 1. The dashed lines indicate the 95 percent confidence interval.
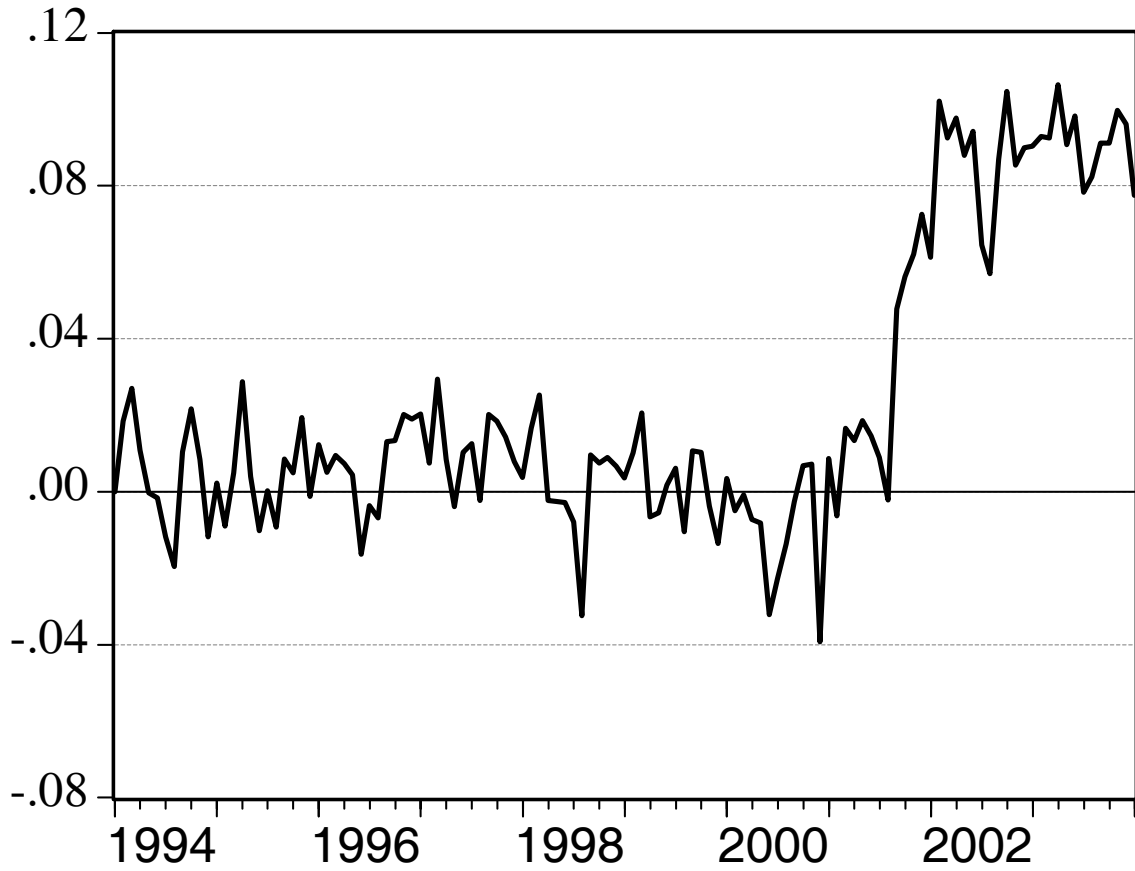
Figure 3: The probability of searching for a job among attached workers (searchers and non-searchers) as a function of time, after conditioning on age, marital status, race, sex, and education, for workers age 25–70. January 1994 normalized to 0. Author's calculations from the monthly CPS.
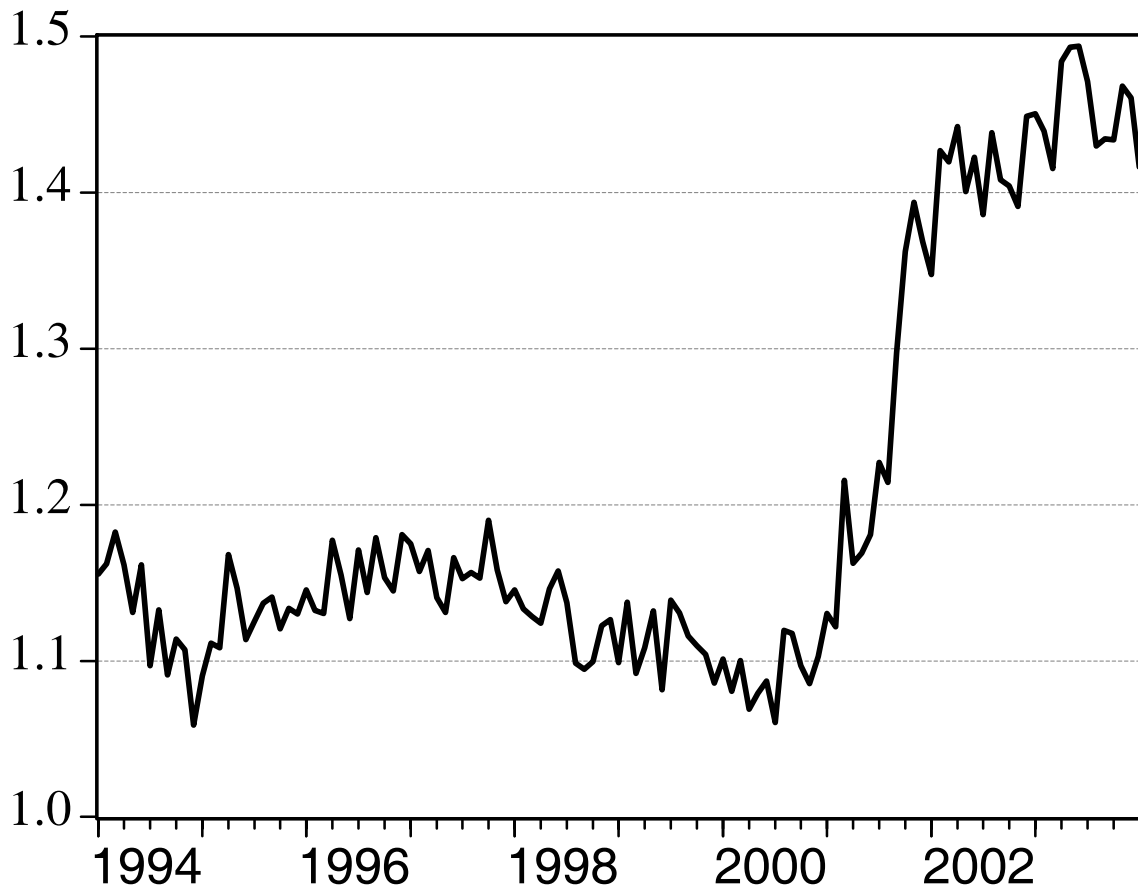
Figure 4: The average number of job search methods used by attached workers, seasonally adjusted. Author's calculations from the monthly CPS.
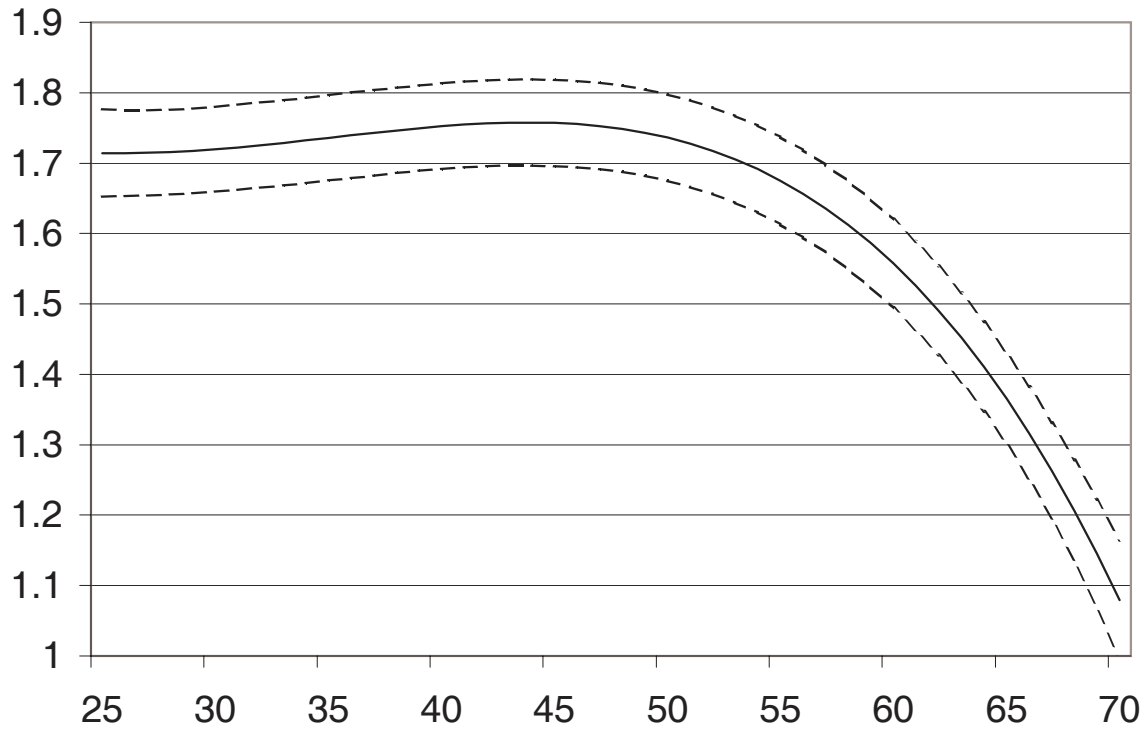
Figure 5: The effect of age on the number of search methods used by searchers in the regression in Column II of Table 1. The dashed lines indicate the 95 percent confidence interval.
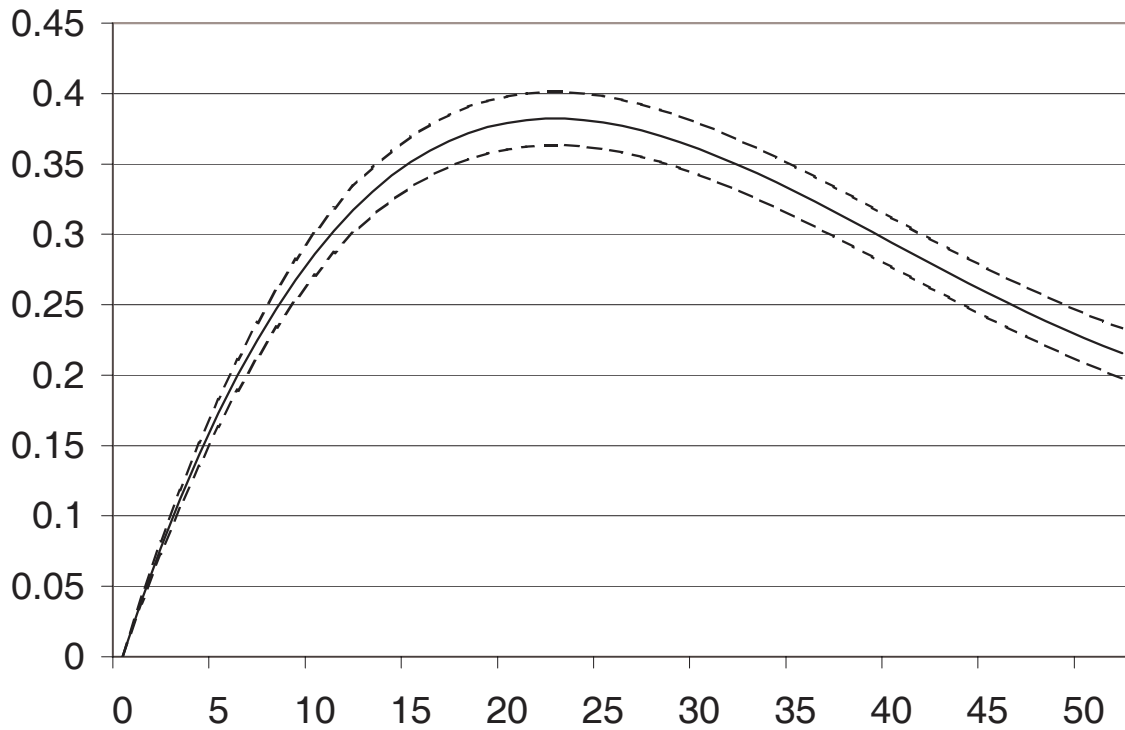
Figure 6: The effect of unemployment duration (in weeks) on the number of search methods used by searchers in the regression in Column II of Table 1. The dashed lines indicate the 95 percent confidence interval.
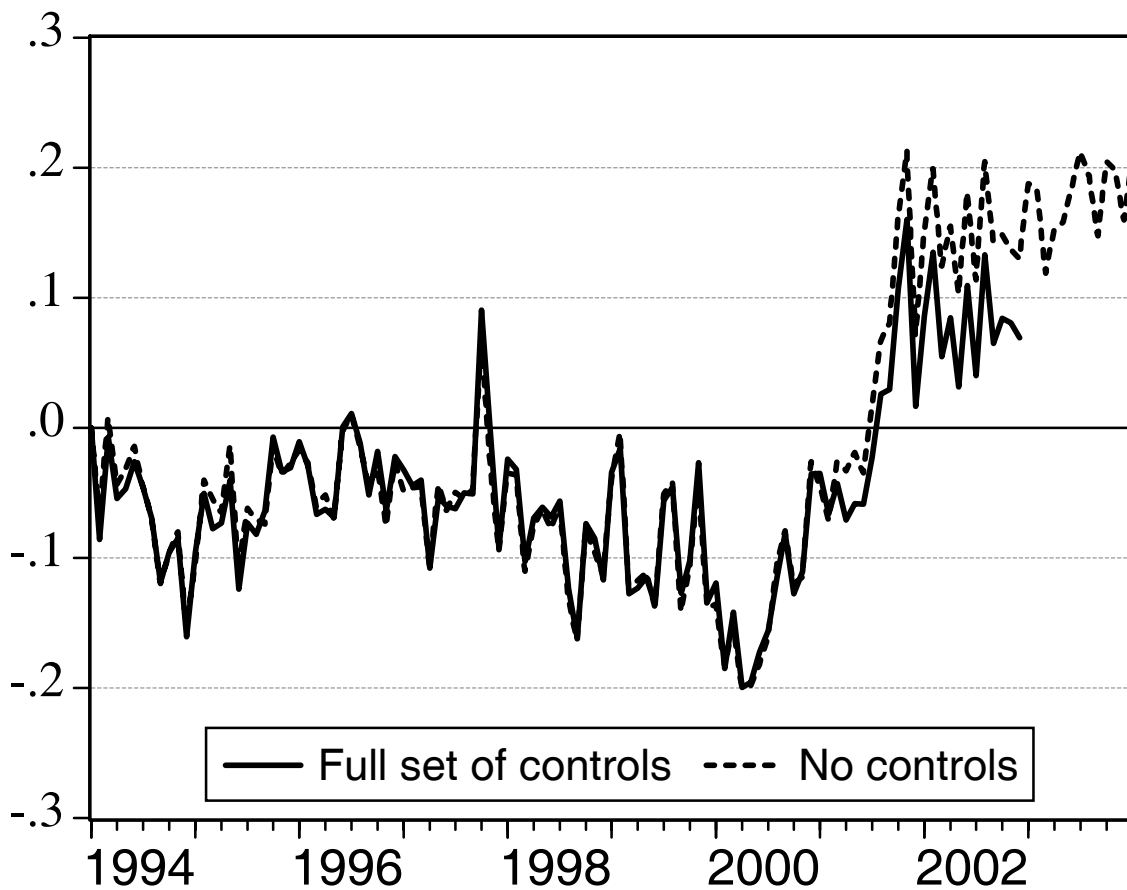
Figure 7: The number of search methods used by searchers as a function of time for workers age 25–70. The solid line conditions on age, marital status, race, sex, education, industry, occupation, and unemployment duration. The dotted line conditions on nothing. In both cases, January 1994 is normalized to 0. Author's calculations from the monthly CPS.
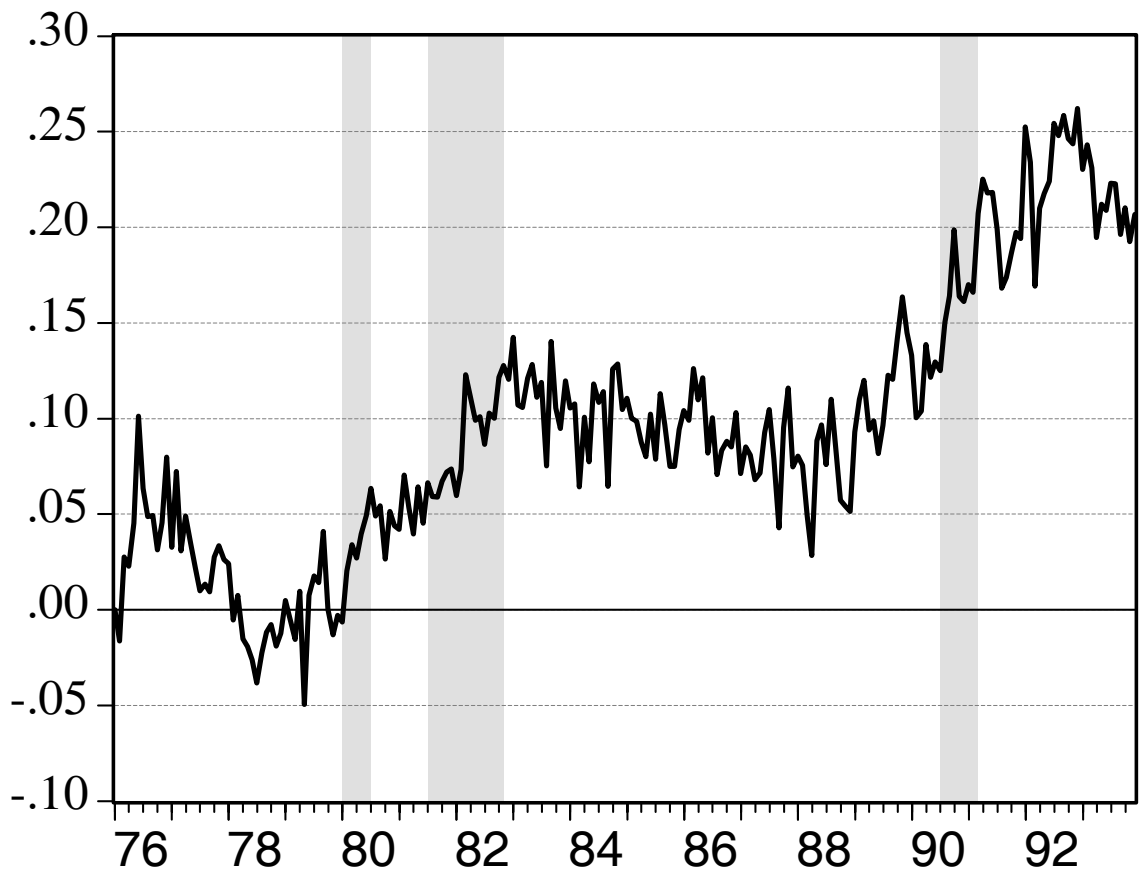
Figure 8: The number of search methods used by searchers for all workers. January 1976 is normalized to 0. NBER recession dates are shaded. Author's calculations from the monthly CPS.
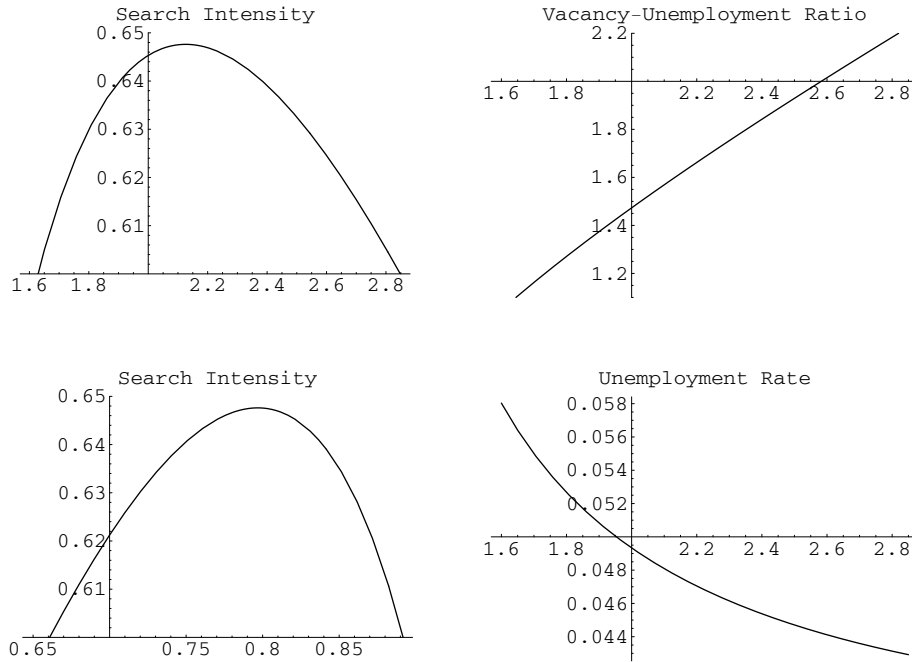
Figure 9: The top left panel shows the fraction of leisure time devoted to search as a function of labor productivity. The top right panel shows the vacancy-unemployment ratio as a function of labor productivity. The bottom left panel shows the fraction of leisure time devoted to search as an implicit function of the job finding rate (the eighty percent rule), with changes in labor productivity as the driving force. The bottom right panel shows the unemployment rate as a function of labor productivity. Parameterization: $\nu = 1$, $z = 1$, $\beta = 0.995$, $\lambda = 0.04$, $\gamma = 0.5$, $c = 1$, $\mu'(0) = 1$.
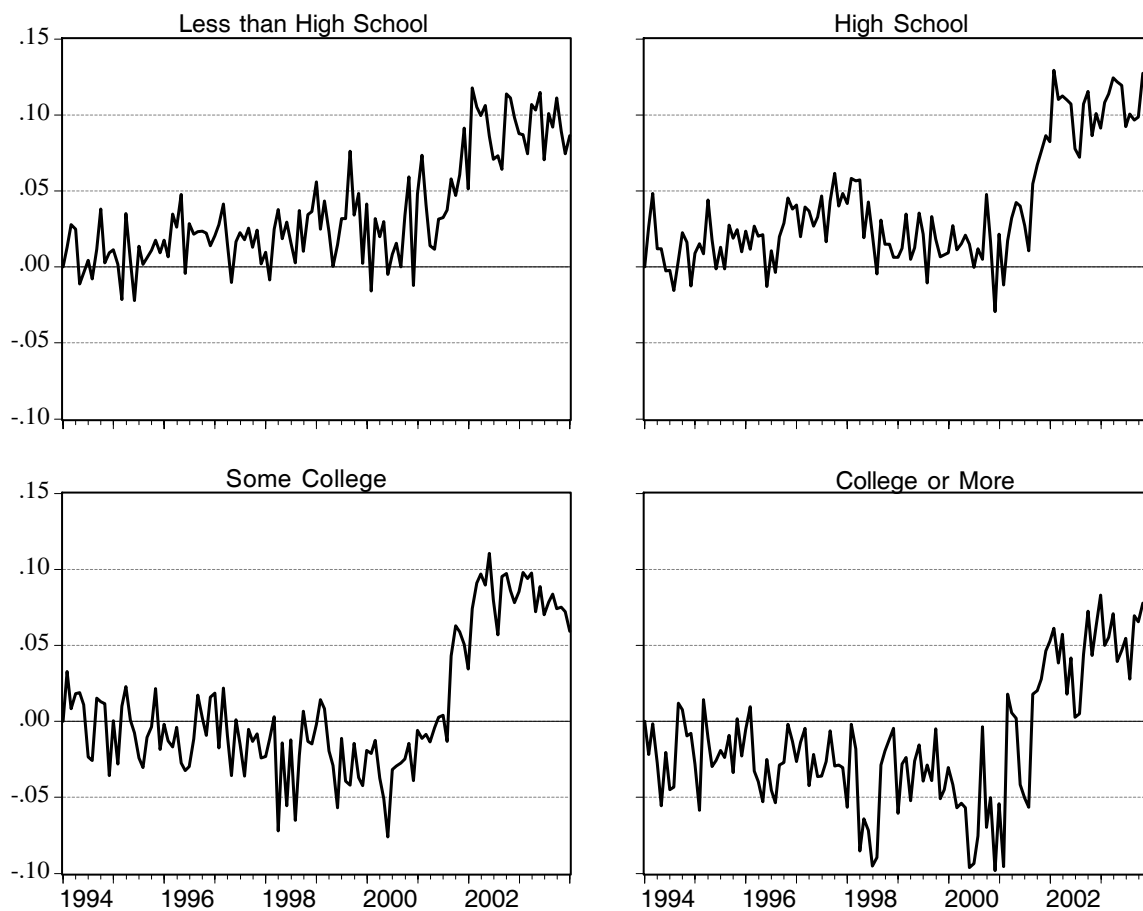
Figure 10: The probability of searching for a job among attached workers (searchers and non-searchers) as a function of time, after conditioning on age, marital status, race, and sex, for workers age 25–70. January 1994 normalized to 0. Author's calculations from the monthly CPS.
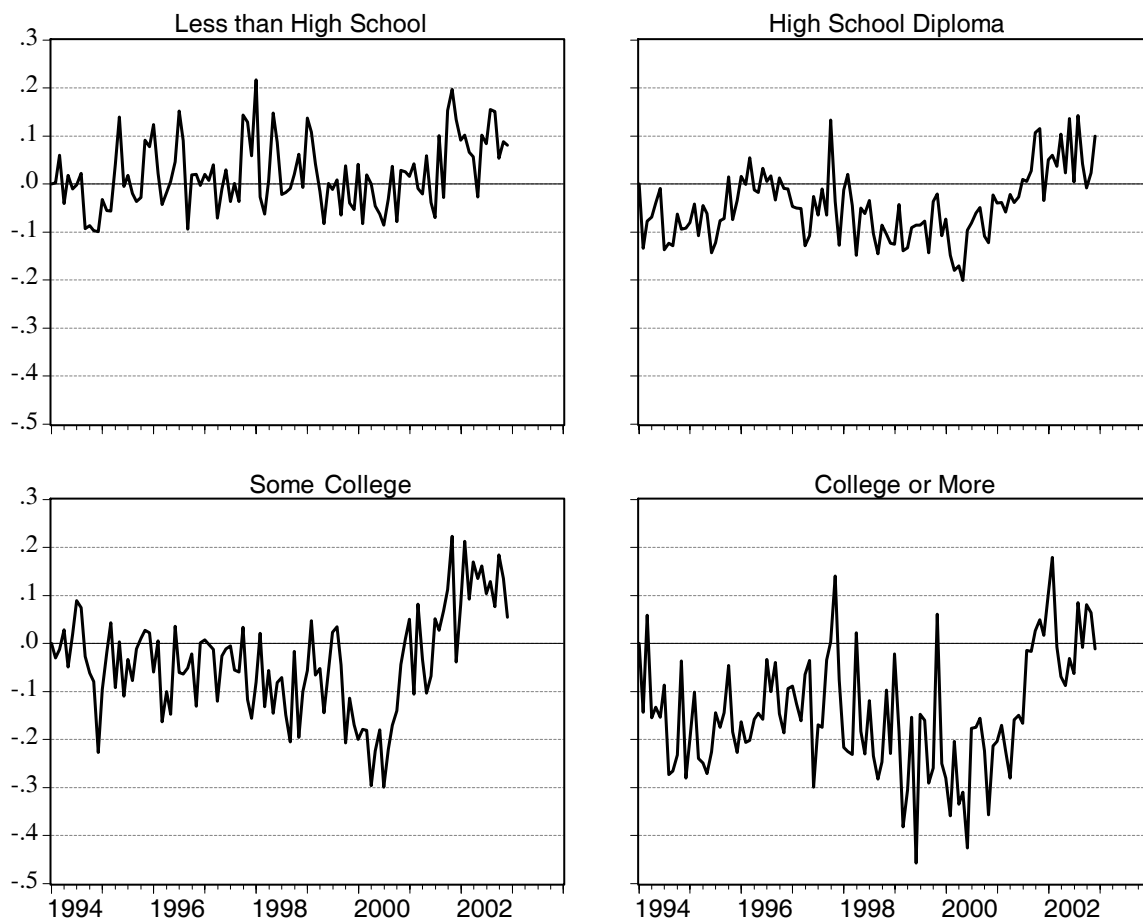
Figure 11: The number of search methods used by searchers as a function of time for workers age 25–70. The solid line conditions on age, marital status, race, sex, industry, occupation, and unemployment duration. January 1994 is normalized to 0. Author's calculations from the monthly CPS.