Job Auctions*

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Abstract

This paper proposes job auctions as a theory of wage determination in models of labor markets with search frictions. Workers apply for jobs by bidding in auctions, and firms reward the job to the applicant who offers it the most profit. In equilibrium, more productive applicants always outbid less productive ones, but the threat of competition holds down the former's wage demand. The equilibrium of the job auction model is always efficient, in contrast to standard search models. The model produces a distinctive and empirically testable relationship between the wage-productivity schedule, the unemployment-productivity schedule, and the underlying labor productivity distribution. The model also predicts that the minimum wage will have a ripple effect on workers for whom it is not binding, by reducing the competition from less productive workers.

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1 Introduction

How do firms choose between heterogeneous job applicants? According to neoclassical economic theory, wages adjust so that firms are indifferent about whom to hire. But reality is much richer than this simple model. Firms spend considerable resources recruiting the most desirable workers, evidence that wage differentials do not fully offset productivity differentials. A likely reason is that labor market frictions mitigate competition between firms, reducing wages below workers' marginal product.

This paper explores the interaction between heterogeneous workers in frictional labor markets. Workers apply for job openings by bidding in auctions. Firms reward the job to the applicant who promises it the most profit. The model has a unique equilibrium in which firms strictly prefer to hire more productive workers. As a result, these workers enjoy shorter unemployment spells. In equilibrium, however, they do not necessarily receive higher wages. When the productivity distribution is concentrated near a point, more productive workers may demand lower wages in order to fend off competition from their less productive peers.

The model yields strong predictions about the interaction between heterogeneous workers. An increase in one worker's productivity from x to x' will reduce the wage of more productive workers as competition becomes more fierce; but will have no effect on the wage of less productive workers, since they are only hired (hence only receive the wage) when x does not apply for the job. It will also raise the unemployment rate of workers in the interval $[x, x']$, since they will lose the job competition with x.

Finally, I consider the normative behavior of the model, showing that the equilibrium is efficient along a number of dimensions. For example, if workers can make investments in order to boost their productivity, the private and social returns to investment are equal.

At a superficial level, job auctions may seem irrelevant, since a formal auction is rarely, if ever, observed. Nevertheless, firms do engage in activities that are analogous to auctions. When workers apply for jobs, employers ask them their wage expectation. Other aspects of the job are also typically discussed at that time, e.g., willingness to work flexible hours or overtime, and fringe benefits. Workers do not know who else, or indeed how many other people, are applying for the job. When they make concessions to potential employers, they recognize that each concession increases the probability that they are hired, but reduces the value of the job to them if they are hired. Finally, a firm hires the most attractive applicant, or it hires no one if it would rather remain vacant for another period. I show that this is essentially a sealed bid first price auction with a (credible) reserve bid equal to the value of a vacant firm.

Related Literature. Job auctions are related to the search and matching literature (e.g. Pissarides, 1990), which has become the standard theory of frictional labor markets. Unemployed workers and vacant firms periodically meet and have an opportunity to match. The meeting process, the source of search frictions, is buried in a black box 'matching function', giving the number of meetings per unit of time as a function of the number of searchers. When two agents meet and match, they bargain over the division of output. If they choose not to match, they continue searching for partners.

Although this model has been adapted for many purposes, it unfortunately does not yield an interesting theory of the interaction between heterogeneous workers. Consider again the effect of an increase in one worker's productivity from x to x'. According to the standard model, this will not affect any other worker, since it will not change the number of meetings per unit of time either for workers or for firms.[1]

In contrast, this paper uses an explicit model of the matching function, designed to capture interactions between heterogeneous agents. Search frictions exist because of the difficulty of coordinating job search in a large market economy. Unemployed workers search sequentially for jobs, applying for one job opening in each period. By chance, some jobs attract multiple applicants, while other identical-looking ones attract none. A job that receives no applications remains vacant, while a job that attracts more that one applicant ranks them according to profitability and hires the best one, leaving the remaining applicants unemployed.

[1]The standard model admits indirect effects. The productivity increase will raise firms' profitability, which will tend to reduce all bargained wages. Higher profitability may also lead to some job creation, which will reduce unemployment and raise wages.

Suppose firms prefer to hire workers who have been unemployed for less time (Blanchard and Diamond, 1994; Blanchard and Diamond, 1996). These 'coordination frictions' imply that an increase in the short-term unemployment rate will reduce the rate at which the long-term unemployed find jobs. This is consistent with evidence that the average duration of unemployment rises sharply during recessions, even though hiring rates are relatively constant. Similarly, coordination frictions can explain why the unemployment rate of less educated workers has sharply increased in the United States since 1970, during a period when the fraction of workers with a college education has also risen dramatically (Shimer, 1998). The standard model does not admit these interactions.

A number of other papers model the matching function as a coordination friction. These can be divided into two groups, depending on how wages are determined. Blanchard and Diamond (1996) assumes that employed workers and firms bargain over wages, as in the standard model. This contrasts with my assumption that unemployed applicants and firms bargain over wages. One of the main goals of this paper is to show that this modification has both positive and normative implications. On the normative side, the equilibrium of the job auction model is always efficient, while the equilibrium of the bargaining model is inefficient along a number of dimensions. On the positive side, Blanchard and Diamond (1996) find that firms may sometimes hire less productive workers in preference to more productive ones, in order to hold down the latters' wage demand. I show that with job auctions, such situations cannot arise. Competition from less productive bidders is sufficient to hold down the wage demanded by more productive ones. I also show that the job auction model predicts that at points where the underlying labor productivity density is particularly high, the wage-productivity schedule will be relatively flat or even declining. The bargaining model predicts that the opposite relationship. Finally, I show that the job auction model predicts that a minimum wage will affect the wage-productivity schedule even at points where it is not binding. The standard model predicts that the minimum wage will only affect wages where it is binding.

Other papers that model the matching function as a coordination friction assume firms commit to and advertise wages in order to attract job applicants (Montgomery, 1991; Peters, 1991; Burdett, Shi, and Wright, 1997). More recently, some authors have noted that firms could generally make more profit by committing to auctions mechanisms, competing along the dimensions of the reserve bid (McAfee, 1993; Peters, 1997). In the equilibrium of such models, the reserve bid is equal to the value of a vacant firm — the credible reserve bid that I use in this paper. As a result, the equilibrium of these models is identical to the equilibrium of the job auction model. My approach offers at least two advantages. First, the informational assumption is much closer in spirit to the standard search model. I assume workers are randomly matched with firms, rather than seeking out the most desirable firm. In the environment of this paper, that has no practical significance. However, in a model with heterogeneous firms, the job auction model recognizes that workers must sample a number of jobs before finding an appropriate one. The wage posting model presumes that workers can immediately apply for the most suitable job (see especially Peters, 1997). Thus standard search frictions are absent from the wage posting model. Second, my model is much simpler to solve. While this is not inherently good, the simplicity allows me to look at a number of issues that have been neglected in the wage posting literature, e.g. the normative behavior of the model, and the testable positive implications of this model and its bargaining counterpart.

Outline. Section 2 develops a model of job auctions. I solve for the equilibrium in Section 3, and provide some positive implications of the model. Section 4 shows that the equilibrium is efficient. Section 5 solves a version of the standard search model, and contrasts the positive and normative conclusions of the two environments. Section 6 concludes.

2 A Model of Job Auctions

This section develops a discrete time, infinite horizon search and matching model. During each period, unemployed workers apply for one vacant job. Employers observe workers' productivity, solicit wage demands, and then hire at most one applicant. Employed workers produce a homogeneous consumption good.

Agents. There are large numbers (formally continua) of wealth-maximizing workers and firms. All agents discount the future with a common factor $\theta < 1$.

Normalize the measure of workers to one. Workers are distinguished by their productivity x with a time-invariant cumulative distribution G with density g, strictly positive on its support $[0, 1]$. At the start of each time period, each worker may be in one of two states, employed or unemployed. Let $U_t(x)$ denote the endogenous measure of workers with productivity less than x who are unemployed at the start of period t, and $G(x) - U_t(x)$ denote the measure who are employed.

Let M denote the measure of firms.[2] I assume that a firm can only hire one worker, and so can be thought of as a 'job'. At the start of each period, a firm may either have a vacancy or a filled job. The measure of filled jobs is identical to the measure of employed workers $1 - U_t(1)$. The remaining firms are vacant, $V_t = M - 1 + U_t(1)$.

Search. Each unemployed worker applies for a job at one random vacant firm in every period. These applications are independent over time and across workers, so some vacancies get many applications, while others get none. This is the coordination friction. Each worker knows her own type, but does not know the other applicants' types, or even how many other workers are applying for the job. The applicants make wage demands, then firms observe their applicants' types and wage demands. They decide whether to hire an applicant, and if so, which one. The chosen worker begins her job the following period. A worker who is not hired remains unemployed, and a firm that does not receive any applications or that chooses not to hire any applicants remains vacant.

Production. An employed type x worker produces output x each period, getting the negotiated wage w, which leaves the firm with profit $\pi \equiv x - w$.

Match Destruction. At the end of each period, all matches, new or old, are destroyed with exogenous probability $\delta \in (0, 1)$, leaving the worker unemployed and

[2]The assumption that the measure of firms is inelastic is made for simplicity. All of the substantive conclusions of this paper would carry through in a model where the measure of firms responds elastically to profits.

the firm vacant.[3]

3 Equilibrium

I look for a steady state equilibrium of this model. This imposes three restrictions: (i) workers make wage demands to maximize their expected wealth; (ii) firms hire the applicant who yields the highest profit π, or no one if that is more profitable; and (iii) all distributions and prices are time-invariant. For this reason, I omit time-subscripts throughout the remainder of this analysis.

The Number of Applications. The expected number of applications per vacancy is equal to the ratio of unemployed workers to vacancies, $U(1)/V$. Similarly, when a worker applies to a particular vacancy, she anticipates that the vacancy will receive on average $U(1)/V$ other applications. However, the actual number of applications is a random variable with a Poisson distribution. In particular, there are no applications with probability $\exp(-U(1)/V)$, where 'exp' is the exponential function.

Job Auctions. Firms choose which applicant to hire through a job auction. Each worker x makes a bid π, and retains a payoff $w = x - \pi$. This is a symmetric private value auction with an unusual and very convenient feature. Regardless of the strategies that workers employ, all bidders face the same distribution of opposing bids, since they face the same distribution of opponents' types and strategies.[4] Let $H(\pi)$ denote the measure of unemployed workers who bid at least π, almost everywhere continuous since H is monotonic. Then the probability that a worker who bids π is the high bidder is $\exp(-H(\pi)/V)$, independent of that worker's identity.

A firm has a credible reserve bid in this auction — it will not hire a worker if she promises it less profit than it expects to get by keeping the vacancy open. Let

[3]It simplifies the algebra to allow newly formed matches to be destroyed before they ever produce.

[4]Contrast this with a standard auction. Suppose there are two bidders. If one bids aggressively and the other conservatively, the aggressive bidder faces a low distribution of bids and the conservative bidder faces a high distribution of bids. This may lead to multiple equilibria in certain types of auctions (Milgrom, 1981).

$J^F(\pi)$ denote the expected value of a firm with a filled job earning per-period profit π and J^V denote the value of a firm with a vacancy. Then a high bid of π will be accepted if $J^F(\pi) \geq J^V$. To understand when this is true, write down the Bellman equation for a firm with a filled job earning profit π:

$$(1-\theta)J^F(\pi) = \pi + \theta\delta(J^V - J^F(\pi)).$$

The flow value of a firm is equal to its 'dividend' π plus the probability of a capital loss next period: the match ends with probability δ, leaving the firm with a vacancy and continuation value J^V. Manipulate this to obtain the firm's surplus in a match:

$$J^F(\pi) - J^V = \frac{\pi - (1-\theta)J^V}{1-\theta(1-\delta)}. \tag{1}$$

A firm will accept a high bid of $\pi \geq (1-\theta)J^V$. Lower bids are always rejected, as the firm would rather maintain a vacancy. Thus $j^V \equiv (1-\theta)J^V$ is the firm's (endogenous) reserve bid. In summary, a worker who promises the firm profit of π is hired with probability $P(\pi)$:

$$P(\pi) = \begin{cases} \exp(-H(\pi)/V) & \text{if } \pi \geq j^V \text{ and } H \text{ is continuous} \\ 0 & \text{otherwise.} \end{cases}$$

Using standard arguments, I can place some restrictions on P: it is continuous at $\pi > j^V$, and it is strictly increasing if $P(\pi) \in (0,1)$. For example, a discontinuity in P at π corresponds to a positive measure of workers bidding π. If any such worker is getting a positive wage, she could do better by reducing her wage demand (raising her bid), thereby discretely increasing her employment probability. Otherwise, she could cut her bid and make a positive profit. Conversely, P is flat at π if no workers bid π. A worker making a slightly higher bid could cut her bid without reducing her employment probability.

To put further restrictions on P, write down workers' Bellman equations:

$$(1-\theta)J^E(x,\pi) = x - \pi + \theta\delta(J^U(x) - J^E(x,\pi)), \qquad (2)$$
$$(1-\theta)J^U(x) = \max_\pi \theta(1-\delta)P(\pi)(J^E(x,\pi) - J^U).$$

$J^E(x,\pi)$ is the value of an employed type x worker who has committed to a wage $w = x - \pi$. The first equation exactly mirrors the equation for J^F. A worker gets a dividend equal to her wage. Next period, her match ends with probability δ, leaving her with the value of an unemployed worker J^U. An unemployed type x worker must choose her bid π. She is hired, and her match is not immediately destroyed, with probability $(1-\delta)P(\pi)$, leaving her with a capital gain $J^E(x,\pi) - J^U$ next period. Otherwise she remains unemployed. Solve these equations for $j^U(x) \equiv (1-\theta)J^U(x)$:

$$j^U(x) = \frac{\theta(1-\delta)}{1-\theta(1-\delta)} \max_\pi P(\pi)\big(x - \pi - j^U(x)\big). \qquad (3)$$

Now if $x < j^V$, the last term is negative whenever $P(\pi)$ is positive, so the best x can do is earn zero value. On the other hand, $j^U(x)$ is positive for $x > j^V$. I focus on these types. Let $\Pi : [j^V, 1] \to \mathbb{R}_+$ denote the equilibrium bid as a function of the worker's type.

Lemma 1. $\Pi(x)$ *is strictly increasing.*

Proof. Write down equation (3) for a type x_1 worker:

$$j^U(x_1) = \frac{\theta(1-\delta)}{1-\theta(1-\delta)} P(\Pi(x_1))\big(x_1 - \Pi(x_1) - j^U(x_1)\big).$$

By revealed preference, $x_2 > x_1$ gets less than $j^U(x_2)$ by bidding $\Pi(x_1)$:

$$j^U(x_2) \geq \frac{\theta(1-\delta)}{1-\theta(1-\delta)} P(\Pi(x_1))\big(x_2 - \Pi(x_1) - j^U(x_2)\big).$$

Subtract these inequalities and rearrange them:

$$\frac{j^U(x_2) - j^U(x_1)}{x_2 - x_1} \geq \frac{\theta(1-\delta)P(\Pi(x_1))}{1-\theta(1-\delta)\big(1-P(\Pi(x_1))\big)}.$$

Replicate the logic with the optimal bid for a type x_2 worker, $\Pi(x_2)$:

$$\frac{j^U(x_2) - j^U(x_1)}{x_2 - x_1} \leq \frac{\theta(1-\delta)P(\Pi(x_2))}{1 - \theta(1-\delta)\big(1 - P(\Pi(x_2))\big)}.$$

Together these inequalities imply $P(\Pi(x_2)) \geq P(\Pi(x_1))$. As P is increasing, this implies the bid function is nondecreasing, $\Pi(x_2) \geq \Pi(x_1)$. Extending this argument, $\Pi(x_2) \geq \Pi(x) \geq \Pi(x_1)$ for all $x \in [x_1, x_2]$. If $\Pi(x_1) = \Pi(x_2)$, a positive measure of workers would make this bid, implying H, hence P, is discontinuous, a contradiction. Therefore Π is strictly increasing, $\Pi(x_2) > \Pi(x_1)$. □

Unemployment Density. The monotonic bid function implies that an unemployed worker $x > j^V$ who makes her equilibrium bid is hired whenever she is the most productive applicant. More precisely, she is hired with probability $P(\Pi(x)) = \exp(-Q(x))$, where $Q(x) \equiv \int_x^1 u(y)dy/V$ is the expected number of more productive applicants for the job. The density (across worker types) of new jobs created is the product of the hiring probability, the density of unemployed workers $u(x)$, and the probability that a new job is not destroyed $1-\delta$. On the other hand, a fraction δ of employed workers lose their job in any period. Equate these flows in steady state:

$$(1-\delta)\exp(-Q(x))u(x) = \delta(g(x) - u(x)) \text{ for } x > j^V. \tag{4}$$

Since $Q'(x) = -u(x)/V$, we can solve this differential equation to get an implicit definition of Q for $x > j^V$:

$$(1-\delta)V\big(1 - \exp(-Q(x))\big) = \delta(1 - G(x) - VQ(x)). \tag{5}$$

The left hand side is the measure of vacancies that get an application from a worker at least as productive as x, multiplied by the probability that the match is not immediately destroyed. Analogous with the 'aggregate matching function' in the standard search and matching model (Pissarides, 1990), this is a constant returns to scale function of the measure of unemployed, but employable workers, $\int_{j^V}^1 u(x)dx$, and the measure of vacancies V. The right hand side is the measure of employed workers with productivity at least x, multiplied by the destruction probability. Equation (5) implicitly defines $Q(x)$ as a continuously decreasing function of $V > 0$.

Vacancy Rate. Next I calculate the measure of vacancies in the economy, equal to the measure of firms M minus the measure of employed workers. Equation (4) yields the density of employed type $x > j^V$ workers at any point in time, $g(x) - u(x)$. Integrate this over $x > j^V$ to get the employment rate. Thus

$$V = M - \int_{j^V}^{1} \frac{(1-\delta)\exp(-Q(x))}{\delta + (1-\delta)\exp(-Q(x))}\, g(x)dx. \tag{6}$$

When the threshold j^V is lower, there is more employment, hence fewer vacancies. This makes it harder for workers to find jobs, raising queue lengths. Formally:

Lemma 2. *Equations (5) and (6) implicitly define V and $Q(x)$, $x \in [j^V, 1]$, as continuous functions of $j^V \in [0, 1]$, with V increasing and Q decreasing.*

Proof. The right hand side of (6) is a continuously increasing function of j^V. Moreover, it is implicitly a continuously decreasing function of V, since (5) defines $\exp(-Q(x))$ as a continuously increasing function of V for all x. Thus I can write the right hand side of (6) as $\phi(j^V, V)$. For a given value of j^V, a solution to the two equations is a fixed point $V = \phi(j^V, V)$. I must show that there is a unique fixed point, which is continuously increasing in j^V; (5) then implies $Q(x)$ is continuously decreasing in j^V for all x.

For a given $j^V \in [0, 1]$, $\phi(j^V, 0) = M$, since (4) implies $\exp(-Q(x)) = 0$ for all x when $V = 0$. As V increases, $\phi(j^V, V)$ continuously decreases, so there exists a unique fixed point $V = \phi(j^V, V)$ for given j^V. Moreover, since ϕ is continuously increasing in its first argument, a small increase in j^V slightly increases the value of the fixed point, as desired. □

Value of Unemployment. Use the envelope theorem to differentiate j^U in (3), yielding the marginal value of an unemployed worker:

$$j^{U\prime}(x) = \frac{\theta(1-\delta)\exp(-Q(x))}{1 - \theta(1-\delta)\big(1 - \exp(-Q(x))\big)}. \tag{7}$$

To interpret this, assume $\theta = 1$, so there is no discounting. Then the marginal value of an unemployed worker is the fraction of the worker's lifetime that she spends employed, as this is the time that she makes use of her productivity. When $\theta < 1$, this equation changes slightly, since employment status in the near future matters relatively more. I can also integrate (7) using the terminal condition $j^U(j^V) = 0$ to solve for $j^U(x)$:

$$j^U(x) = \int_{j^V}^{x} \frac{\theta(1-\delta)\exp(-Q(y))}{1-\theta(1-\delta)\big(1-\exp(-Q(y))\big)}\,dy. \tag{8}$$

Equilibrium. To close the model, calculate the value of a vacancy:

$$j^V = \theta(1-\delta)\int_{j^V}^{1} (J^F(\Pi(x)) - J^V)\exp(-Q(x))(-Q'(x))dx.$$

If a firm hires a type x worker and the job is not immediately destroyed, it gets a capital gain $J^F(\Pi(x)) - J^V$ the following period. The density of the most productive applicant is $\exp(-Q(x))(-Q'(x))$, a positive number since Q is decreasing. Equation (1) implies $J^F(\pi) - J^V = (\Pi(x) - j^V)/(1-\theta(1-\delta))$. Substitute this into the equation for j^V, then replace $\Pi(x)$ by inverting (3) with $P(\Pi(x)) = \exp(-Q(x))$:

$$\begin{aligned} j^V &= \int_{j^V}^{1}\left(\frac{\theta(1-\delta)}{1-\theta(1-\delta)}(x - j^U(x) - j^V)\exp(-Q(x)) - j^U(x)\right)(-Q'(x))dx \\ &= \int_{j^V}^{1}\left(\frac{\theta(1-\delta)}{1-\theta(1-\delta)}\left(1 - j^{U\prime}(x)\right)\big(1-\exp(-Q(x))\big) - j^{U\prime}(x)Q(x)\right)dx. \end{aligned}$$

The second equation is obtained from the first using integration by parts. Finally, replace $j^{U\prime}$ using (7) and simplify:

$$j^V = \theta(1-\delta)\int_{j^V}^{1}\frac{1-(1+Q(x))\exp(-Q(x))}{1-\theta(1-\delta)\big(1-\exp(-Q(x))\big)}\,dx. \tag{9}$$

The numerator of the integrand is the probability that a firm receives at least two applications from workers of type x or greater. This is multiplied by the marginal value of hiring a type x worker at her reservation wage j^U, $(1-j^{U\prime}(x))/(1-\theta(1-\delta))$, then integrated over acceptable types x and appropriately discounted.

To understand this formula, think of a sealed bid second price auction with a reserve bid of j^V. The value of a vacancy is equal to the expected value of hiring the second highest bidder at her reservation wage j^U:

$$j^V = \theta(1-\delta)\int_{J^V}^{1} \frac{x - j^U(x) - j^V}{1-\theta(1-\delta)} Q(x)\exp(-Q(x))(-Q'(x))dx.$$

Here $Q(x)\exp(-Q(x))(-Q'(x))$ is the density of the the second highest bidder. Integrating this formula by parts yields (9). This implies that the value of sealed bid first and second price auctions are the same, the familiar Revenue Equivalence Theorem.

Finally we can characterize an equilibrium:

Proposition 1. *An equilibrium is a tuple $\{j^V, V, Q\}$, such that the queue length Q satisfies (5) given V; vacancies V satisfy (6) given Q and j^V; the value of vacancies j^V satisfies (9) given Q. There is a unique equilibrium.*

Proof. The characterization of equilibrium follows from the preceding text. To prove existence and uniqueness, recall that Lemma 2 established Q as a continuously decreasing function of j^V. Since the integrand of (9) is increasing in Q, the right hand side is a continuously decreasing function $\psi(j^V)$. Since $\psi(1) = 0$, there is a unique fixed point $j^V = \psi(j^V) \in (0, 1)$. Such a point represents a solution to (5), (6), and (9). □

The Interaction Between Workers. The model makes predictions about the interaction between heterogeneous workers. First, the unemployment rate of less productive workers does not directly affect the hiring rate or unemployment rate of more productive ones, since the latter are always hired in preference to the former (equation (4). Of course, there may be indirect interactions; by taking jobs, for example, less productive workers may reduce the supply of vacancies. Thus an increase in a worker's productivity from y_1 to y_2, with $j^V < y_1 < y_2 < x$, will not affect x's unemployment rate. However, it will reduce x's value $j^U(x)$ (equation (8), as the increase in productivity forces x to bid more aggressively.

Second, an increase in the unemployment rate of more productive workers reduces the hiring rate and raises the unemployment rate of less productive workers.

By raising her unemployment rate, it also reduces a worker's value. However, the structure of unemployment among more productive workers is of no consequence to x; the queue length $Q(x)$ is a sufficient statistic for this. The reason is that x always loses job auctions against more productive workers, but does not care to whom she loses.

Bidding Strategies. We are finally in a position to characterize the equilibrium bid function Π. Invert equation (3) to solve for $\Pi(x)$. Replace $P(\Pi(x)) = \exp(-Q(x))$ and $j^U(x)$ with equation (8). Simplifying yields:

$$\Pi(x) = j^V + \int_{j^V}^{x} \frac{\big(1-\theta(1-\delta)\big)\big(1-\exp(Q(x)-Q(y))\big)}{1-\theta(1-\delta)\big(1-\exp(-Q(y))\big)} dy. \tag{10}$$

The equilibrium bid function depends on the pressure created by less productive applicants, $y \in [j^V, x]$. More productive workers do not affect x's bid, since she is never hired when they apply for a job. Less productive workers affect the bid through x's need to outbid them. For example, an increase in $Q(y)$ for some positive measure of $y < x$ will raise x's optimal bid. The more such workers apply for a job, the more aggressively x must bid. That will, of course, raise the equilibrium bid of still more productive workers.

One can confirm from equation (10) that the bid function is nondecreasing. Interestingly, however, the 'wage function' $W(x) \equiv x - \Pi(x)$ may be decreasing. This is likely to occur at points where the slope of the queue function is very large, or equivalently at points where the productivity distribution is concentrated,[5] since $\Pi'(x)$ is proportional to $-Q'(x) \equiv u(x)/V$. Raising the wage demand leads to a sharp reduction in the employment probability, so competition forces these workers to accept a low wage.

This model predicts, however, that exactly at those points where the wage schedule is relatively flat or declining, the unemployment rate $u(x)/g(x)$ is sharply de-

[5] If the productivity distribution has a mass point at x, then type x workers will make a range of bids. As a result, the wage *correspondence* will no longer be lower hemicontinuous at x. Slightly less productive workers will get a much higher wage than slightly more productive workers. A formal treatment of mass points complicates the analysis unnecessarily, and so is omitted from this paper.

clining (see equation (4)). Moreover, these two effects should offset each other. Equation (7) shows that lifetime income should be a relatively smooth function of productivity, as ${j^U}'(x)$ does not depend on the concentration of the productivity distribution at x. I discuss in Section 5 how these predictions can be used to test the model.

Minimum Wage. Another distinct prediction of the model, is the response of the wage schedule W to a mandatory minimum wage $\bar{w} > 0$. With a minimum wage, the productivity threshold will rise to $z(\bar{w}) = j^V + \bar{w}$, leaving workers with lower productivity unemployed. More interesting is what happens to workers above the threshold. The minimum wage effectively negates the existence of workers with productivity between j^V and $z(\bar{w})$. The value of the marginal worker $z(\bar{w})$ satisfies (see equation (3))

$$j^U(z(\bar{w})) = \frac{\theta(1-\delta)\exp(-Q(z(\bar{w})))}{1-\theta(1-\delta)\big(1-\exp(-Q(z(\bar{w})))\big)}\,\bar{w}.$$

This is equal to her value in the absence of a minimum wage, given by equation (8), in the case where there are no workers with productivity $y \in [j^V, z(\bar{w})]$. The minimum wage eliminates competition from less productive workers, and so allows $z(\bar{w})$ to reduce her equilibrium bid.

For still more productive workers, the slope of the value function ${j^U}'$ still satisfies equation (7). For a given value of j^V and V, all workers with productivity $x > z(\bar{w})$ enjoy a higher value when the minimum wage is higher. Equivalently, they make lower bids Π and demand higher wages W, due to the reduced competition. Of course, there will be additional indirect effects through V and j^V. The increase in the hiring threshold raises the number of vacancies, further shortening queue lengths and raising wages. This may be partially offset by a decline in the value of a vacancy, hence in the productivity threshold. Nevertheless, the first order predictions of the model are that an increase in the minimum wage will raise the wage of all workers who keep their job. This is consistent with existing empirical evidence on the 'ripple effect' (Welch, 1978). However, the source of these ripple effects is both plausible and original: minimum wages reduces competition from less productive workers.

This is distinct from existing explanations, which generally rely on the minimum wage being binding for the worker in some states of the world.

4 Social Optimum

To characterize the normative behavior of this economy, I consider a hypothetical social planner who wishes to maximize the present value of output in the economy[6]

$$\sum_{t=0}^{\infty} \theta^t \int_0^1 (g(x) - u_t(x)) x dx.$$

The planner has the ability to decide whether matches are created or destroyed. However, he cannot affect the fundamental coordination friction in the economy. Formally, the planner faces a pair of inequality constraints on the unemployment density of type x workers at time $t+1$, $t \geq 0$.

$$u_{t+1}(x) \geq \delta(g(x) - u_t(x)) + \big(1 - (1-\delta)\exp(-Q_t(x))\big) u_t(x), \tag{11a}$$

$$u_{t+1}(x) \leq g(x). \tag{11b}$$

$Q_t(x) = \int_x^1 u_t(y) dy / V_t$ is again the queue length faced by a type t worker, and $V_t = M - \int_0^1 (g(y) - u_t(y)) dy$ is the measure of vacancies. The lower bound on the unemployment density is the sum of the fraction of employed type x workers who lost their job and the fraction of unemployed type x workers who did not find themselves at the front of job queues at time t. But by destroying jobs, the planner can achieve an upper bound of $g(x)$. This formulation presumes that the planner always wants firms to hire the most productive applicant; in this model, there is no reason for him to do anything else.

Represent this constrained optimization problem as a Lagrangian with multiplier

[6] This is a reasonable objective if the planner has access to lump-sum transfers, and is standard in the search literature (Diamond, 1982; Hosios, 1990).

$-\theta^t \mu_t(x)$ on inequality (11a) and $\theta^t \lambda_t(x)$ on inequality (11b) at time t for type x:

$$L(u,\lambda,\mu) = \sum_{t=0}^{\infty} \theta^t \int_0^1 \Bigg((g(x) - u_t(x))x + \lambda_t(x)\big(u_{t+1}(x) - g(x)\big) \\ + \mu_t(x)\Big(u_{t+1}(x) - \delta(g(x) - u_t(x)) - \big(1 - (1-\delta)\exp(-Q_t(x))\big)u_t(x)\Big)\Bigg)dx.$$

I characterize the steady state of this model through the first order conditions of the Lagrangian. Because of the focus on steady states, I drop time subscripts.

The necessary first order conditions include a pair of Kuhn-Tucker conditions for (almost) every x: Inequality (11a) and $\mu(x) \geq 0$ with complementary slackness; and inequality (11b) and $\lambda(x) \leq 0$ with complementary slackness. Since both constraints (11) cannot simultaneously bind, $\lambda(x)\mu(x) = 0$ for all x as well.

The other first order condition is that the derivative of the Lagrangian with respect to (almost) every $u(x)$ is zero:

$$\begin{aligned} \theta x = \lambda(x) + \mu(x)\Big(&1 - \theta(1-\delta)\big(1 - \exp(-Q(x))\big) \\ &+ \theta(1-\delta)\int_0^x \mu(y)\exp(-Q(y))Q'(y)dy \\ &- \theta(1-\delta)\int_0^1 \mu(y)Q(y)\exp(-Q(y))Q'(y)dy\Big). \end{aligned} \tag{12}$$

The first line represents the direct effect of a change in $u(x)$, while the remaining two lines represent indirect effects operating through the queue length: an increase in $u(x)$ increases $Q(y)$ for $y < x$ (the second line); and decreases $Q(y)$ for all y by raising the number of vacancies (the third line).

Differentiate (12) with respect to x. If $\mu(x) = 0$, $\lambda'(x) = \theta > 0$. Thus there is a threshold $\bar{x}_\lambda$ such that $\lambda(x)$ is negative when $x < \bar{x}_\lambda$ and equal to zero otherwise. Alternatively, if $\lambda(x) = 0$,

$$\mu'(x) = \frac{\theta}{1 - \theta(1-\delta)\big(1 - \exp(-Q(x)\big)} > 0.$$

There is another threshold $\bar{x}_\mu$ such that $\mu(x)$ is positive when $x > \bar{x}_\mu$ and equal to zero otherwise. Since $\lambda(x)\mu(x) = 0$, $\bar{x}_\lambda \leq \bar{x}_\mu$.

Finally, take $z \in [\bar{x}_\lambda, \bar{x}_\mu]$, so $\lambda(z) = \mu(z) = 0$. Equation (12) implies

$$\begin{aligned} z &= -(1-\delta) \int_{\bar{x}_\mu}^1 \mu(x) Q(x) \exp(-Q(x) Q'(x) dx \\ &= (1-\delta) \int_{\bar{x}_\mu}^1 \mu'(x) \big(1 - (1 + Q(x)) \exp(-Q(x))\big) dx. \end{aligned}$$

The second line follows from the first through repeated use of integration by parts. Replace $\mu'(x)$ using the expression above. This pins down z uniquely, which implies $\bar{x}_\mu = \bar{x}_\lambda = z$:

$$z = \theta(1-\delta) \int_z^1 \frac{1 - (1 + Q(x)) \exp(-Q(x))}{1 - \theta(1-\delta)\big(1 - \exp(-Q(x)\big)} dx. \tag{13}$$

Workers with productivity more than z are always hired, while those with productivity less than z are never hired. Thus z is the socially optimal productivity threshold. To summarize:

Proposition 2. *The social optimum is a tuple $\{z, V, Q\}$, such that the queue length Q satisfies (5) given V; vacancies V satisfy (6) given Q and $\bar{x}$; and the productivity threshold z satisfies (13) given Q. A tuple is a social optimum if and only if it is an equilibrium.*

The equivalence of the equilibrium and social optimum follows immediately by comparing the characterizations in Propositions 1 and 2.

Note also that the social shadow value of unemployed workers $\mu(x)$ is equal to its private counterpart $j^U(x)$. This implies that in an extension to the model in which workers were offered the opportunity to make productivity-enhancing investments, they would choose the socially efficient investment level. Thus the equilibrium is efficient along a number of dimensions.

Intuitively, the equilibrium is efficient because on average, each worker receives her expected marginal product. Consider an unemployed worker applying for a job. When she is not the most productive applicant, her marginal product is zero; she brings nothing to the match and she is not hired. When she is the most productive applicant, she has a positive marginal product, equal to the gap between (the present value of) her productivity and the productivity of the second best applicant, or to the gap between her productivity and the firm's option of remaining vacant, whichever is smaller. This is exactly the wage she would get in a sealed bid second price auction, which is revenue-equivalent to the first price job auction. Thus, despite the incomplete markets created by search frictions, labor markets are competitive.

5 A Version of the Standard Search Model

To illustrate the importance of job auctions for the positive and normative behavior of this model, I consider an alternative wage-setting arrangement. Following Blanchard and Diamond (1996), I assume that $\Pi(x)$ is determined by bilateral bargaining between employed workers and firms. Profits are divided according to the Nash bargaining solution, with threat point equal to the value of being unmatched.[7] Workers keep a share $\beta \in (0,1)$ of match surplus:

$$(1-\beta)(J^E(x,\Pi(x)) - J^U(x)) = \beta(J^F(\Pi(x)) - J^V) \text{ for all } x.$$

Eliminate J^F and J^E using equations (1) and (2) respectively:

$$\Pi(x) = j^V + (1-\beta)(x - j^V - j^U(x)). \tag{14}$$

Firms' bargained profit is their outside option j^V, plus a share $1-\beta$ of the flow match surplus $x - j^V - j^U$. Firms prefer to hire the most productive applicant if the profit function is nondecreasing. This requires that the value of unemployment be increasing more slowly than productivity.

To see whether this is the case, suppose firms always hire the most productive applicant. Then $Q(x)$ will be determined, as in the job auction model, by the solution to equations (5) and (6). Also, the hiring rate will satisfy $P(\Pi(x)) =$

[7] The results in this section do not depend on the Nash bargaining assumption. What is crucial, is that the bargained profit $\Pi(x)$ depends only on the worker's productivity and outside option, as will be the case if workers and firms bargain over wages after the firm decides whom to hire. This ensures that if firms always hire the most productive applicant, less productive applicants do not affect a worker's wage demand. A similar argument to the one below obtains.

$\exp(-Q(x))$. Plug the profit function (14) into equation (3):

$$j^U(x) = \frac{\theta(1-\delta)\beta\exp(-Q(x))}{1-\theta(1-\delta)\big(1-\beta\exp(-Q(x))\big)}\,(x-j^V) \tag{15}$$

If $-Q'(x) = u(x)/V$ is large, $j^{U\prime}(x)$ will exceed unity, contradicting the existence of an equilibrium in which firms hire the most productive applicant.

Intuitively, the value of unemployment j^U depends on how frequently a worker is hired and on her wage $W(x) = x - \Pi(x) = j^U(x) + \beta(x - j^V - j^U(x))$ when she is hired. It does not depend on how many workers are 'just behind her' on the job queue. If there is a large measure of workers with productivity approximately equal to x, a slightly less productive worker is much less likely to be hired, and hence will have a much lower value of unemployment.

When there is no equilibrium in which firms hire the most productive applicant, a 'stochastic ranking equilibrium' (Blanchard and Diamond, 1996) will exist. In such an equilibrium, firms sometimes hire less productive workers in preference to more productive ones. This depresses the outside option of more productive workers, increasing the appeal of hiring them. In equilibrium, firms are indifferent about whom to hire, as $\Pi'(x) = 0$ and $W'(x) = 1$.

This contrasts with the bidding function in the job auction model, equation (10). In that case, a worker alters her bid depending on the competition from less productive applicants. This is the reason that the job auction model predicts the wage will be flat or even declining at points where the productivity density is high. The bargaining model predicts that the slope of the wage-productivity schedule will be highest, possibly as high as unity, at such points. This distinction between job auctions and the standard search model is testable, possibly by using the observable education distribution to proxy for the unobservable productivity distribution.

The standard model also predicts that a minimum wage will not directly affect the value of unemployment or the level of wages at points where it is not binding. Again, the reason is that wages do not depend on pressure from less productive workers. The model does admit indirect effects on wages through an increase in the number of vacancies and a decline in the value of a vacancy. However, the total effects are smaller in the standard model than in the job auction model, another testable distinction.

A stochastic ranking equilibrium is clearly inefficient, since firms do not take advantage of the most productive opportunities. But even in cases where firms always hire the most productive applicant, the equilibrium of the standard model will still be inefficient. One way to see this is to compare the value of an unemployed worker x in the efficient equilibrium of the job auction model, $j^U(x)$ in equation (8), with the value of an unemployed worker in this model, $j^U(x)$ in equation (15). The former depends on the whole distribution of workers with productivity less than x, while the latter depends only on $Q(x)$. Hence the two will differ unless the underlying productivity distribution has exactly the right shape. Alternatively, one can construct the value of a vacancy in the standard model, and show that there is no reason for this to equal the efficient threshold z in equation (13). Thus the known inefficiency of the standard search model with heterogeneous agents (Acemoglu, 1997; Masters, 1998) extends to this setting.

6 Conclusion

This paper has developed a model in which applicants compete for jobs via an auction mechanism. It contrasted the positive and normative behavior of the job auction and standard search models. In solving the model, I found that firms always hired the most productive job applicant (Lemma 1). In reality, firms sometimes hire less productive workers, finding more productive ones to be overqualified. This might appear to be evidence in favor of the standard model with its stochastic ranking equilibrium. However, what firms are typically afraid of, is that an overqualified worker might soon leave to take a better position. This suggests the need for models with two-sided heterogeneity and realistic interactions between heterogeneous workers and firms.

In the environment of this paper, wage posting models (Peters, 1991; McAfee, 1993; Peters, 1997) deliver identical predictions to the job auction model. Firms commit to and publicize wages or auction mechanisms in order to attract applicants. This creates a competitive market for applicants, who are therefore paid their marginal product, as in the job auction model. One attractive feature of job auctions, is that they makes more modest informational demands than the wage posting model. In the latter, workers must know the wage schedule offered or auction mechanism used by each firm. In the job auction model, workers simply have rational beliefs about their opponents' bidding strategies. Thus in an economy with two-sided heterogeneity, the wage posting model presumes that workers are immediately able to contact the most suitable firm (Peters, 1997; Shi, 1998). Job auctions recognize that workers have to search for the right employer.

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