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Should We Be Afraid of Friedman's Rule?[1]

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Should one think of zero nominal interest rates as an undesirable liquidity trap or as the desirable Friedman rule? I use three different frameworks to discuss this issue. First, I restate H. L. Cole and N. Kocherlakota's (1998, Fed. Res. Bank Minn. Quart. Rev., Spring, 2–10) analysis of Friedman's rule: short run increases in the money stock—whether through issuing spending coupons, open market operations, or foreign exchange intervention—change nothing as long as the money stock shrinks in the long run. Second, two simple Keynesian models of the inflationary process with a zero lower bound on nomianl interest rates imply either that deflationary spirals should be common or that a policy close to the Friedman rule and thus some deflation is optimal. Finally, a formal baby-sitting coop model implies multiple equilibria, but does not support the injection of liquidity to restore the good equilibrium, in contrast to P. Krugman (1998, *Slate*, August 13). *J. Japan. Int. Econ.*, December 2000, **14**(4), pp. 261–303. CenER, Tilburg University; Humboldt University, Berlin, Germany; and CEPR. © 2000 Academic Press

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1. INTRODUCTION

> *Scylla: In Greek mythology, a sea monster who lived underneath a dangerous rock at one side of the Strait of Messia, opposite the whirlpool Charybdis. She threatened passing ships and in the Odyssey ate six of Odysseus' companions. Scylla was a nymph, daughter of Phorcys. The fisherman-turned-sea-god Glaucus fell madly in love with her, but she fled from him onto the land where he could not follow. Her disappearance filled his heart with sadness. He went to the sorceress Circe to ask for a love potion to melt Scylla's heart. As he told his tale of love to Circe, she herself fell in love with him. She wooed him with her sweetest words and looks, but the sea-god would have none of her. Circe was furiously angry, but with Scylla and not with Glaucus. She prepared a vial of very powerful poison and poured it in the pool where Scylla bathed. As soon as the nymph entered the water she was transformed into a frightful monster with twelve feet and six heads, each with three rows of teeth. Below the waist her body was made up of hideous monsters, like dogs, which barked unceasingly. She stood there in utter misery, unable to move, loathing and destroying everything that came into her reach, a peril to all sailors who passed near her. Whenever a ship passed, each of her heads would seize one of the crew.*
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The liquidity trap is back, or so it would seem. Of course it has always been there as part of the standard fare taught to undergraduate students of economics practically everywhere, but it was just viewed as a possibly irrelevant piece of theory rather than something one tends to observe. It used to be regarded as a strange relic of the depression years following 1929. Now, many argue that the situation in Japan fits the description of a liquidity trap only too well. Indeed, nominal interest rates along the entire yield curve have been remarkably close to zero in recent times.

It is no surprise then, that many economists have rushed forward, led by their undergraduate textbook gut reaction, and are calling upon the Japanese central bank to inflate the economy out of its misery; see, e.g., Krugman (1998, 1999a,b). The Japanese central bank is surely in no enviable position. Monetary policies in many industrial countries are probably led less by what it hopes to achieve but rather what it hopes to avoid, navigating between the Scylla of deflation on the one side and the Charybdis of run-away inflation on the other. The Great Depression[2] following 1929 as well as the hyperinflationary episodes in countries such as Germany or Hungary are deeply etched into our collective minds: no central banker wants to be remembered as having committed the same mistakes yet once again. Will the current episode in Japan serve as the future prime example of how some hapless monetary policy has once again pushed an economy over the brink? It is hard to imagine that this question does not weigh heavily on the minds of Japanese central bankers these days.

[2] It is interesting how easily even economists jump from the word "deflation," merely indicating falling prices, to thinking about "depression." By contrast, the analysis by Bernanke (1983) and the recent breakthrough by Cole and Ohanian (1999) in the analysis of the Great Depression indicate that deflation itself may have had much less to do with it than is commonly believed.

As I am an academic and not a Japanese central banker, however, I have the luxury of contemplating an altogether different interpretation of the situation of near-zero nominal interest rates. I shall ask whether this Scylla is in fact not the ugly sea monster everybody seems to think it is, but rather still the beautiful nymph one may want to fall in love with. Indeed, a regime of nominal interest rates near zero is also known by a different name: it is called Friedman's rule.

Friedman's rule, proposed by Friedman (1969), postulates creating money until the marginal costs of money production equal its market price. As it costs practically nothing to create money,[3] and as the opportunity costs of holding money are given by the short-term nominal interest rate, good monetary policy should aim at driving these interest rates to zero. One might therefore ask provocatively: has Japan's monetary policy achieved a state of bliss or should monetary theorists have been more careful in what they wished for? E.g., Sargent (1999) and Bernanke *et al.* (1999) can both be read as recommending a monetary policy which seeks low inflation.

Strangely, the literature rarely discusses the optimality of Friedman's rule on the one hand and the textbook dangers of a liquidity trap on the other in the same context. But it should. This paper is meant to be a step in that direction and therefore distinct from the currently evolving literature on the zero lower bound on nominal interest rates, such as Akerlof *et al.* (1996), Fuhrer (1997), Fuhrer and Madigan (1997), Krugman (1998a,b, 1999a,b), Rotemberg and Woodford (1997, 1999b), Orphanides and Wieland (1998), Buiter (1999), Clouse *et al.* (1999), Reifschneider and Williams (1999), and Svensson (1999) and his discussants Meltzer (1999) and Woodford (1999a). Closest in spirit is the work by Wollman (1998, 2000), which examines the consequences of the zero lower bound on nominal interest rates for optimal monetary policy in a fully specified dynimic general equilibrium model with sticky prices. His papers also contain a further, excellent review of the relevant literature, which I shall not restate here.

I should emphasize that this paper is not necessarily about the specific situation in Japan. Whether or not the problems there are due to a deteriorating banking system (see, e.g., Bayoumi, 1999), how to restore Japanese economic growth (see, e.g., Posen, 1998) and whether or not Japanese monetary policy played and has to play a key role in that process (see, e.g., Krugman, 1999) is a much-debated subject in which this author does not have a comparative advantage. The insights here may be applicable to that situation—in fact, they probably are—but I shall leave it to more informed participants of the practical debate to make that judgement. But if they are, then it may be dangerous too for policymakers to ignore these insights as an academic luxury of merely theoretical interest: instead, they may pose some serious issues for central bankers who succeeded in achieving low inflation.

[3] There may actually be an issue here, whether inside money, i.e., checking accounts at commercial banks, is costly to create at the margin: commercial banks are, after all, rather costly operations. I shall not pursue this issue further.

I shall take the optimality of Friedman's rule as my base perspective, but then examine some other aspects which are relevant from a liquidity trap perspective. I will do so using three different frameworks. In Section 2, I will first reexamine a cash-in-advance economy analyzed by Cole and Kocherlakota (1998) and restate their result of how monetary policy can actually achieve Friedman's rule. The equilibrium examined by these authors turns out to have some powerful (and, in the end, perhaps obvious) conclusions about the ineffectiveness of some policy proposals which have been made with respect to the Japanese situation. Then I will examine two very simple monetary models in Section 3, either allowing for the possibility of a runaway deflation or imposing stability. The choice of these models was motivated by the proliferation and popularity of linear "reduced form" models in the current literature and debate on monetary policy and central banking: it is hoped that this section is a suitable complement to that literature. Finally, I will draw some conclusions about the frequency, with which we should observe deflations, and about the desirability of near-zero interest rates.

Krugman (1999) has argued that injecting liquidity into an economy such as Japan is comparable to issuing coupons in an ill-functioning baby-sitting coop and thereby stimulating economic activity. While Krugman told me in private conversation that he intended this parable only as a way to explain to the general public how monetary policy might have real effects, it is hard to read his parable from a more informed perspective without giving it a multiple equilibrium interpretation of moving from a good to a bad equilibrium and back through the injection of liquidity. I examine this argument formally in Section 4, developing a formal model of a baby-sitting coop, which indeed gives rise to multiple equilibria. However, the injection of liquidity is unlikely to enable the economy to move back to a good equilibrium: rather, it may move the economy to a region where equilibria, in which money is valued, cease to exist altogether. The formal analysis here thus offers a dire warning regarding Krugman's recommendation. Given the attention that this parable and recommendations have received, more formal research should be devoted to clarify its precise logic, separating the pitfalls from the genuine insights. The analysis here should be understood as a step in that direction and as a way of pointing out that a casual treatment of the baby-sitting coop parable as a guide to monetary policy can be dangerously misleading.

I finally conclude in Section 5 with an extensive discussion of what has been learned from these three pieces of analysis. I will cautiously endorse Friedman's rule, despite some liquidity trap fears. More importantly, I will argue that our current theoretical tools, including this paper, are unsatisfactory: they do not yet allow us to adequately deal with a deflationary economy in which prices fall faster than the real rate implied by a frictionless situation. I believe that progress in models with this feature is crucial to understand more deeply the benefits and perils of near-zero interest rates and zero or negative inflation and therefore crucial to the proper conduct of monetary policy.

In this paper, I have ignored the well-known Phelps argument (1973) that the inflation tax should not be zero in an environment with distortionary taxes and that therefore zero nominal interest rates are not optimal. That conclusion has been challenged by, e.g., Chari *et al.* (1996) and Correira and Teles (1996). Walsh (1998) offers an excellent survey and discussion, not limited to this point. Another standard reference on issues of monetary policy is Cukierman (1992). I will also ignore the arguments by, e.g., Aiyagari and Braun (1998) or Khan *et al.* (2000) that optimal monetary policy should aim somewhere between slight deflation and stable prices, if, e.g., prices are sticky. To incorporate these concerns in future work, and to precisely understand the role of the zero nominal interest rate equilibrium as an approximation to an equilibrium with low nominal interest rates, see, e.g., Orphanides and Wieland (2000), is beyond the scope of this paper, but offers a promising avenue for future research. Finally, if interest can be paid on holding cash or if cash holdings can be taxed—which may now be feasible, given modern electronic means—the nominal interest rate corresponding to the zero opportunity costs of holding money is not necessarily zero, but is itself a policy variable, thereby substantially altering the analysis, as Goodfriend (1999) has pointed out. This is an interesting avenue which deserves further exploration, but is not pursued here.

2. ZERO NOMINAL INTEREST RATES AND POLICY INEFFECTIVENESS

To think about monetary policy in an environment with near-zero interest rates, it is useful to recall the following analysis by Cole and Kocherlakota (1998). These authors have analyzed the following neoclassical growth cash-in-advance economy.

There is an infinite horizon nonstochastic economy with a continuum of households. The representative household values consumption $c_t \geq 0$ and leisure $l_t \geq 0$ according to the utility function

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

where $u(\cdot)$ is assumed to be strictly concave and continuously differentiable and satisfies $u_c(0, l) = \infty$, $u_l(c, 0) = \infty$, for all c, l. The household is endowed with some initial capital k_0, one unit of time per period, which it can use for leisure or labor and cash m_{-1}.

Output y_t is produced by competitive firms, using capital $k_{t-1} \geq 0$ and labor $n_t \geq 0$,

$$y_t = f(k_{t-1}, n_t),$$

where $f(\cdot, \cdot)$ is continuously differentiable, homogeneous of degree one, and concave. The firms rent capital from the household, hire labor, and sell. Output can in turn be used for investment x_t or consumption c_t,

$$\begin{aligned} c_t + x_t &= y_t \\ k_t &= (1-\delta)k_{t-1} + x_t. \end{aligned}$$

There is a government injecting or withdrawing cash every period, making up for the difference by lump sum transfers or lump sum taxes τ_t,

$$m_t = \tau_t + m_{t-1}.$$

Finally, trading evolves subject to a cash-in-advance constraint for consumption goods. The household also has the option of purchasing bonds b_t. Let p_t be the price level at date t, w_t be nominal wages, r_t be the (nominal) return on capital, and i_{t-1} be the nominal interest rate. The household chooses $(c_t, k_t, n_t, m_t, b_t)_{t=0}^{\infty}$ to maximize its utility U subject to the constraints

$$\begin{aligned} m_{t-1} &\geq p_t c_t \\ m_t + b_t &\leq r_t k_{t-1} + w_t n_t + b_{t-1}(1 + i_{t-1}) + m_{t-1} + \tau_t - p_t(c_t + x_t) \\ k_t &= (1-\delta)k_{t-1} + x_t, \end{aligned}$$

where

$$k_t \geq 0, m_t \geq 0, b_t \geq -B.$$

The lower constraint on b_t rules out Ponzi schemes.

Equilibrium is defined in the usual way, imposing that bonds are in zero net supply,

$$b_t = 0.$$

Equilibria will be compared to the Pareto optimum of the first-best social planners solution, which obtains in a moneyless world, i.e., which obtains without the cash-in-advance constraint.

Cole and Kocherlakota prove the following three key results: they are taken verbatim from their publication and are restated here for the sake of completeness.

PROPOSITION 1. *Equilibrium quantities are Pareto optimal if and only if* $i_t = 0$ *for all* t.

This proposition states that Friedman's rule is optimal, as should be expected in this environment.

PROPOSITION 2. *An equilibrium such that $i_t = 0$ forever exists if and only if both*

$$1.\ \lim_{t\to\infty} \inf M_t = 0$$

and

$$2.\ \inf_t M_t \beta^{-t} = \kappa > 0.$$

This is an important proposition, because it sheds light on the recent policy discussions regarding Japanese monetary policy or, more generally, monetary policy in an environment with near-zero interest rates; see, e.g., Bernanke (1999) or Meltzer (1999). The proposition states, that short-run movements in money supply may have no impact on nominal interest rates: all that matters is that the money stocks eventually shrink to zero at the rate β^t. In practical terms, this means that there is no reason for the economy to move away from near-zero nominal interest rates, as long as agents believe it to be an equilibrium, in which the monetary authority justifies these zero nominal interest rates eventually. Any extra cash is simply held as an asset, which pays the equilibrium real rate of interest; because the extra cash is withdrawn from circulation eventually by imposing taxes later on, the extra cash is not regarded as net wealth. The heart of this logic is Ricardian. The effect is a liquidy trap, but understood here more deeply through the lense of dynamic general equilibrium. Furthermore, the liquidity trap is a good thing, not a disaster: to be at the Pareto optimum means keeping the economy saturated with liquidity.

It is not hard to generalize this insight to an open economy in which there are long-term bonds as well as exchange rates with some other currency (think yen and dollar), and I shall not include the formal details: in the Friedman equilibrium of the proposition above, short-term monetary injections, whether they be through outright helicopter drops as here, through open market operations in short-term or long-term bonds, or through foreign exchange markets, selling the local currency for foreign currency, are neutral, as long they do not change the long-term outlook for monetary policy, as stated in this proposition. Agents will simply offset any such move by monetary policy by a correspondingly inverse shift in their asset portfolios. Put differently, at zero nominal interest rates, simple quantity theory ceases to hold: there is no longer a systematic relationship between the quantity of money and the price level: "along equilibrium paths in which nominal interest rates are always zero, the inflation rate is independent of the growth rate of the money supply" (Cole and Kocherlakota, p. 8).

McKinnon and Ohno (1997, 1999a,b) have argued that markets have come to expect the yen to appreciate vis-a-vis the dollar at a constant speed: therefore, there has to be a constant interest rate differential in Japan vis-à-vis the U.S. federal funds rate. As the U.S. federal funds rate has continued to decline to low levels during the past decade, the nominal interest rate in Japan had nowhere to go but down as well: according to McKinnon and Ohno, Japan's liquidity trap was imported, not homemade. The framework here can be used to shed light on this view. Suppose for the sake of argument that nominal interest rates in the United States stand at 5% forever and that monetary policy is expected to conduct long-run monetary policy in such a way as to justify a 5% appreciation of the yen forever from some point onward in the future. By arbitrage, this means that nominal interest rates have to be at zero forever from that same point onward. By Proposition 2 above, monetary policy must therefore satisfy the conditions listed, implying that there is an equilibrium in which nominal interest rates are at zero, starting now. Shortrun monetary policy activism such as foreign exchange interventions or other forms of liquidity injections will then have no effect.

Arguably, the results above are knife-edge and strictly hold only at zero nominal interest rates: with nominal interest rates slightly above zero, holding money is always dominated by interest bearing assets, and the liquidity trap disappears entirely. In practice, this may be of little comfort, and it is probably reasonable to argue that the zero nominal interest rate equilibrium and the very elastic demand for money analyzed here offer a better approximation to the situation of low nominal interest rates than an analysis based on a strict interest rate domination argument. Orphanides and Wieland (2000) pursue this question in greater detail.

For an injection of money to have an effect, one of two things has to be true. Either they imply that long-run monetary policy does not satisfy the conditions in the proposition. In practical terms, this means that agents have a believe that monetary policy will be inflationary eventually, see Krugman (1998a,b, 1999a,b), or that exchange rate appreciations are not such that a nominal interest rate of zero will be justified forever; see McKinnon and Ohno (1997, 1999a,b). Or there are sunspot equilibria in this economy in addition to the Friedman rule equilibrium $i_t \equiv 0$, with the amount of liquidity in circulation acting as the coordinating sunspot variable. In Section 4, we will encounter a model which essentially has such a feature.

This analysis, while providing useful insights, is somewhat unsatisfactory in two ways. First, it does not allow me to discuss the danger of runaway deflation, in which prices decrease faster than rate β. I will turn to that issue in the next section. Second, if money is withdrawn from circulation too fast, an equilibrium may not even exist. Cole and Kocherlakota state the following third proposition

PROPOSITION 3. *If* $u(c, n) = \log(c) + v(n)$, *then if* $M_{t+1}/M_t \leq \delta < \beta$ *for all* t, *there is no equilibrium.*

This proposition is disconcenting because there are monetary policies, for which theory—so far—is no guide as to what will happen. One view of deflationary spirals may be that these are episodes with such "freak" monetary policies. Obviously, this is a gross misinterpretation of the result above: the fact that there is no equilibrium cannot be interpreted to mean that there is a deflationary spiral.

More generally, suppose that $u(c, n) = (c^{1-\eta} - 1)/(1 - \eta) + v(n)$, suppose that the cash-in-advance constraint binds in t and $t + 1$, i.e., $p_t c_t = m_t$, $p_{t+1} c_{t+1} = m_{t+1}$, and suppose that $m_{t+1}/m_t = \delta < \beta$. One can then show that

$$1 + i_t = \frac{\delta}{\beta} \left(\frac{c_{t+1}}{c_{t+2}} \right)^{1-\eta}.$$

With $i_t \geq 0$, this implies that c_t must be shrinking at the rate of at least $(\delta/\beta)^{1/(1-\eta)} < 1$, if $\eta < 1$ and that c_t must be growing at the rate of at least $(\beta/\delta)(1/(\eta - 1) > 1$, if $\eta > 1$. The case $\eta = 1$, i.e., the case $u(c, n) = \log(c) + v(n)$ turns out to be knife-edge.

In sum, this is interesting territory worthy of further study.

3. REDUCED-FORM KEYNESIAN MODELS

Nominal interest rates are widely viewed as being constrained by zero from below.[4] There are therefore two arguments that explain why zero nominal interest rates are not a good idea despite the results of the previous section: I will describe these arguments in greater detail below. Briefly: first, zero nominal interest rates and the modest deflation which goes along with them may be a cliff from which one may fall into a deflationary spiral, see 3.1, rather than the nirvana of Pareto optimality as envisioned in the previous section. Second, to the degree that it is desirable to conduct countercyclical monetary policy, the maneuvering room in recessions disappears at zero nominal interest rates; see 3.2. In this section, I wish to examine both arguments by using some extremely simple reduced form models of inflationary (or deflationary) dynamics. These models have been called Keynesian by Clarida *et al.* (1999), although one may not necessarily want to attach that interpretation. For simplicity and to explore the relevant issues, we shall assume a rather old-fashioned backward-looking inflationary process, rather than a modern, forward-looking Phillips curve. It should be therefore all the more surprising that this framework delivers the recommendation to essentially follow Friedman's rule; see Section 3.2.

3.1. Deflationary Spirals

According to the first argument, suppose that price changes are a persistent, stochastic process, somewhat controllable through real interest rates. With real interest rates above some benchmark level, prices tend to fall, whereas they tend to rise with real interest rates below the benchmark level. Then, while nominal interest rates are above zero, monetary policy has enough maneuvering room to counteract falling or rising prices. But as nominal interest rates hit the lower bound of zero, any additional deflation will raise real interest rates as a consequence, thus

[4] One may be able to get around this constraint through borrowing subsidies, a topic to which I will return in the discussion in Section 5.

reinforcing the downward pressure on prices: a deflationary spiral results; see Krugman (1999b).

This logic leads Rotemberg and Woodford (1997, 1999b) to propose a robust control framework in which some desirable distance to the zero lower bound is added to the objective function of the central bank. Likewise, Orphanides and Wieland (1998) merely analyze how likely it is for monetary policy to hit the zero lower bound, calculating the chance to be small for modest inflation targets and realistic parameters.

This, however, begs the question of what would actually happen if monetary policy were to hit the lower bound: while efforts may be made to keep away from it, the models by the authors mentioned above will imply that monetary policy gets there eventually. Theory should provide some guidance as to what happens next.

Here, I will take a framework similar to the framework by these authors to argue that if indeed a runaway deflation could happen, then economies will, on average, spend substantial time in deflationary regimes. This insight has two interpretations. Either it means that we have been lucky so far in not running into deflationary spirals more often, and that the situation in Japan is actually the norm rather than the exception. Or it means that the danger of a deflationary spiral is more apparent than real: I therefore study a more benign version of the price dynamics in Section 3.2.

Here, suppose that the rate of price change

$$\pi_t = \log(P_t) - \log(P_{t-1})$$

evolves according to

$$\pi_{t+1} - \pi_t = -\theta\pi_t - \xi(r_{t-1} - \pi_t - \bar{r}_{\text{real}}) + \epsilon_t, \tag{1}$$

where $\bar{r}_{\text{real}}$ is some benchmark level of real interest rates, r_t is the nominal interest rate for a risk free security maturing in period $t+1$, and θ and ξ are coefficients. Thus, the expression in brackets is the difference between realized real rates and the benchmark level.

Clearly, Eq. (1) is not a good substitute for proper economic theorizing. While Goodfriend and King (1997), Walsh (1998), and Woodford (1999b) provide a more in-depth discussion of how equations of this type may result from a more full-fledged model, and while equations of this type have essentially been used by popular contributions to the literature surveyed by Clarida *et al.* (1999) such as Fuhrer and Moore (1995), Rudebusch and Svensson (1999), or Orphanides and Wieland (1999), the best defense may simply be that an equation such as (1) should simply be regarded as a starting point to provide some insights and intuition. If these insights are deemed interesting, a more in-depth investigation may be called for. It may also be more fruitful to regard the dynamics (1) as arising from the kinds of learning processes discussed in Sargent (1999) rather than some fully rational expectations equilibrium.

Importantly, Eq. (1) captures some elements which I believe are important to the analysis of monetary policy near the zero lower bound on interest rates. First, central banks do not control inflation directly: they only have indirect control over it via their monetary policy instrument. This is captured by the random shock: actual inflation may overshoot or undershoot its target. Second, there may be substantial inertia to the dynamics of price changes. Inflations or deflations are rarely one-off events: bringing these back to "normal" levels usually requires time. This is captured by the autoregressive term. Finally, it seems fairly reasonable to postulate that, e.g., inflation gets slowed down, if there is a hike in real rates, and that inflation is predetermined one period in advance. One might be tempted to call Eq. (1) a first-generation Keynesian model (see Goodfriend and King, 1997) and debate whether it is more appropriate to write this equation as an expectational forward-looking equation or to mix backward-looking and forward-looking terms. I believe that it is interesting to extend the analysis in that direction. I also believe that the essence of the argument will survive except for fairly extreme (and, some might say, incredible) alterations.

Equation (1) implies that with realized real rates equal to the benchmark level, inflation is mean-reverting to zero at rate θ. (It would be nice for future work to modify the model so that it allows for long-run neutrality: the steady state inflation rate here should be arbitrary rather than zero.) If $0 < \theta < 2$, a monetary policy which keeps the real rate of interest rates unchanged will achieve a stationary process for π_t, hovering around zero. This equation also implies that at $r_{t-1} \equiv 0$, the dynamics of the price change is given by

$$\pi_{t+1} - \pi_t = (\xi - \theta)\pi_t + \xi \bar{r}_{\text{real}} + \epsilon_t.$$

In particular, if $\xi > \theta$ and $\bar{r}_{\text{real}} = 0$, the price change variable π_t has an explosive dynamics, with either accelerating deflation or accelerating inflation as a result.

Equation (1) can be rewritten as

$$\pi_{t+1} - \pi_t = c - \theta\pi_t - \xi(r_{t-1} - \pi_t) + \epsilon_t, \tag{2}$$

where

$$c = \xi \bar{r}_{\text{real}}.$$

An OLS regression of Eq. (2), using annual U.S. data from 1961 to 1998, yields $c = 2.05(0.57)$, $\theta = 0.30(0.09)$, and $\xi = 0.31(0.09)$ with standard errors in parenthesis. The standard deviation of ϵ_t was estimated to be 1.5% and $R^2 = 0.75$. Based on these (admittedly rather crude) results[5], I cannot reject the hypothesis

[5] Another implication would be that the real rate is estimated to be $2.05/0.31 = 6.6\%$, which strikes me as too high for short-term riskless assets. Instead, average real rates of returns for treasury bills have hovered around 1%. For the subsequent analysis, I shall use this 1% level rather than the implied 6.6% figure.

TABLE I
Baseline Parameters

$\bar{r}_{\text{real}}$	1
α	1.5
π^*	2
ξ	0.3
θ	0.2
σ	1.3
$\underline{\pi}$	−5
$\bar{\pi}$	5

that $\theta = \xi$, i.e., the knife-edge case just between explosive or stable price change dynamics for $r_{t-1} \equiv 0$. In this section, I shall emphasize the explosive possibility, $\theta < \xi$, whereas I examine the more benign case of stable dynamics even for $r_{t-1} \equiv 0$ in Section 3.2. In my calculations below, I will therefore choose my baseline parameters as listed in Table I.

I now wish to calculate the steady state distribution of the rates of price changes. To that end, I make three more assumptions. First, in order to employ convenient mathematical tools, I shall use a continuous time formulation of (1),

$$d\pi_t = ((\xi - \theta)\pi_t + \xi \bar{r}_{\text{real}})\, dt + \sigma\, dW_t. \tag{3}$$

Second, I assume that interest rates are set according to a simple Taylor-rule-like formula, respecting the lower bound at zero,

$$r_t = \max\{0; \alpha(\pi_t - \pi^*) + \bar{r}_{\text{real}}\}, \tag{4}$$

where π^* is some target inflation level and where α is a parameter[6]. I do not try to derive this behavior from some optimization problem for monetary policy: essentially, because of the problem of runaway deflation, such problems may be ill-defined! However, rules such as (4) have been proposed by, e.g., Taylor (1993) and found to robustly deliver near-optimal results in many models; see Taylor (1999a,b). It therefore certainly makes sense to examine the consquences of adopting (4), notwithstanding the objections in, e.g., Reifschneider and Williams (1999).

Our third assumption deals with the problem of runaway deflation, which arises for $\xi > \theta$, as explained above. In that case, there would not be a nondegenerate steady state distribution for π_t. Instead, I therefore assume that there is some lower bound $\underline{\pi}$ for π_t which acts as a reflecting barrier. One might justify this assumption by saying that something changes as the economy experiences a rapid decline in prices and that prices at that point will no longer be set quite as mechanically as Eq. (3) would imply. In more practical terms, this allows me to draw

[6] Note that at $\pi_t = \pi^*$ and with (4), I generally expect inflation to move further toward zero, unless $\theta = 1$. One therefore needs to be a bit careful in interpreting π^* as the target inflation.

conclusions regarding the long-run average behavior of this economy, as parameters are changed. I will rather arbitrarily set $\underline{\pi} = -5\%$ in my simulations below. Additionally, I have cut off the densities at $\underline{\pi} = 5\%$; i.e., I have rescaled the densities so that they integrate to unity over the interval $\pi \in [\underline{\pi}, \overline{\pi}]$.

With these three assumptions together, the stationary distribution of inflation rates $\Phi(\pi)$ will satisfy the stationary version of the Kolmogorov forward equation, i.e., will be the solution to the differential equation

$$0 = \frac{\sigma^2}{2}\Phi''(\pi) - (p(\pi)\pi + q(\pi))\Phi'(\pi) - p(\pi)\Phi \tag{5}$$

with boundary condition

$$0 = \frac{\sigma^2}{2}\Phi'(\underline{\pi}) - (p(\underline{\pi})\underline{\pi} + q(\underline{\pi}))\Phi(\underline{\pi}), \tag{6}$$

where

$$p(\pi) = \begin{cases} \theta + \xi & \text{for } \pi \leq \pi_0 \\ \theta + \xi(1-\alpha) & \text{for } \pi > \pi_0 \end{cases}$$

$$q(\pi) = \begin{cases} \xi \overline{r}_{\text{real}} & \text{for } \pi \leq \pi_0 \\ \xi \alpha \pi^* & \text{for } \pi > \pi_0 \end{cases}$$

and where

$$\pi_0 = \pi^* - \frac{\overline{r}_{\text{real}}}{\alpha}$$

is the inflation rate at which the interest rate rule hits the zero lower boundary[7].

The differential equation above has (to my knowledge) no closed form solution, unless $r_t \equiv 0$ (for, e.g., $\pi^* = \infty$) and $\theta = \xi$. In that case, the solution is of the form

$$\Phi(\pi) = \Phi_0 \exp\left(\frac{2\xi\overline{r}_{\text{real}}}{\sigma^2}(\pi - \underline{\pi})\right),$$

where Φ_0 needs to be chosen so that the integral equals unity. For $\overline{r}_{\text{real}} > 0$, one obtains runaway inflation. For $\overline{r}_{\text{real}} < 0$, one obtains runaway deflation. To get meaningful desities at all, I rely on the lower and upper bounds on inflation.

In Fig. 1, I show the resulting densities. For this figure, I have chosen $\theta = \xi = 0.3$, varied $\overline{r}_{\text{real}}$, and have otherwise used the baseline parameters listed in Table I. As one can clearly see, the steady state density is extremely sensitive to the value of $\overline{r}_{\text{real}}$. Thus, one should treat any calculation of a low likelihood of entering a

[7] Note that $p(\pi)\pi + q(\pi)$ is continuous at π_0, even though $p(\pi)$ and $q(\pi)$ are not.

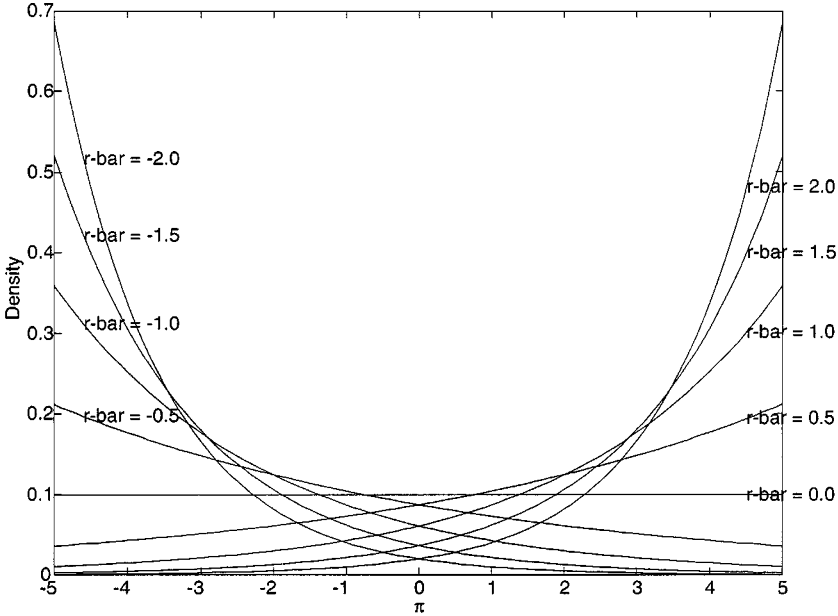

FIG. 1. This figure shows the price change densities when nominal interest rates are fixed at zero. Price changes are bounded by -5 and 5.

deflationary regime, which relies on solidly high real interest rates, with a note of caution.

For the more general case, I rely on numerical solutions to the differential equation (5). In Fig. 2 I show the steady state distribution of inflation, varying $\bar{r}_{\text{real}}$ and using the baseline parameterization of Table I for the other parameters. This figure is the counterpart to Fig. 2 except that now the deflation dynamics is explosive for $r_t \equiv 0$ and that rising inflation is combatted with increasing nominal interest rates.

Figure 3 tells a similar story about θ. As explained above, values of $\theta < \xi = 0.3$ result in runaway deflation, whereas the inflation dynamics is stable for $\theta > \xi$.

While $\bar{r}_{\text{real}}$ and θ should probably both be regarded as fundamental parameters of this economy, it may be more interesting to examine the impact of changing the policy parameters π^* and α.

In Fig. 4, I have varied π^*: occasionally, it is claimed that a higher inflation target helps in avoiding the cliff of runaway deflation. The figure shows that, indeed, mass is moved away from the deflationary regime, but not substantially so: even with an inflation target of 3%, we should see economies spending a sizeable fraction of their time in the deflationary regime! This figure therefore offers a dire prediction: while the transition probabilities between the inflationary and the deflationary regime may be small, so that deflationary regimes have luckily been observed fairly rarely, over time we will find that economies are commonly in that

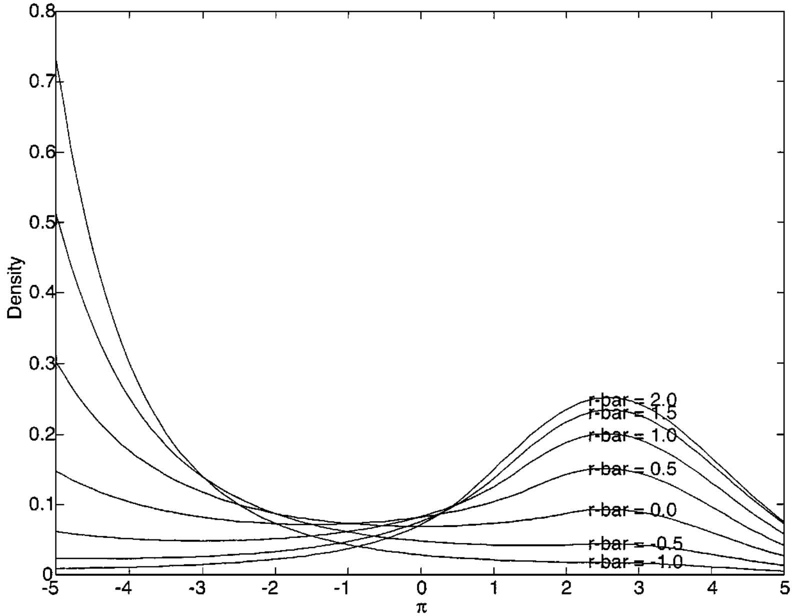

FIG. 2. Price change distribution, assuming that r follows a Taylor rule. The solutions are obtained numerically.

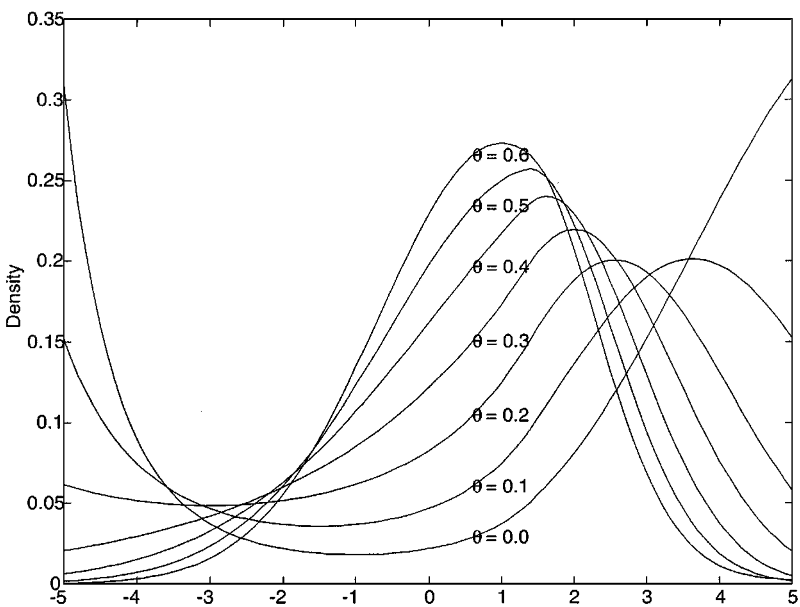

FIG. 3. Density of price changes when varying θ.

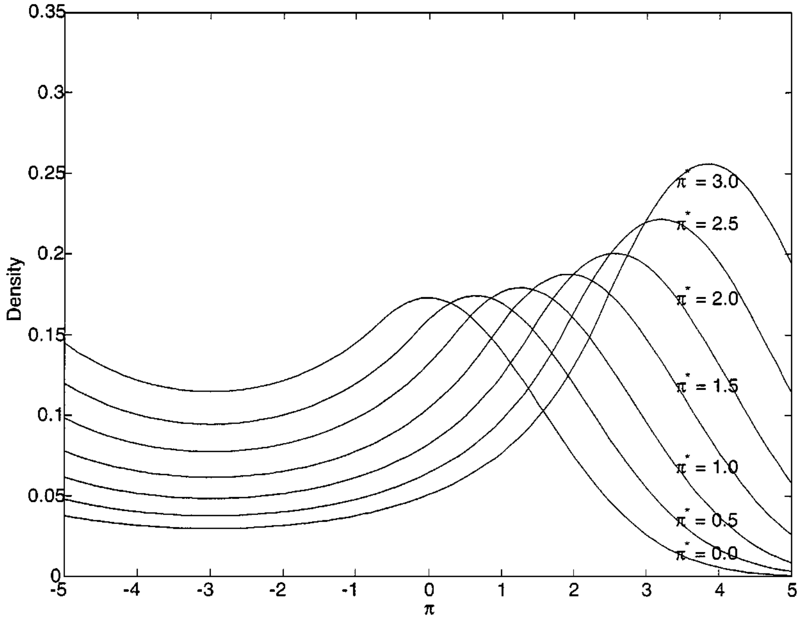

FIG. 4. Density of price changes when varying π^*.

situation. Orphanides and Wieland (1998) and Reifschneider and Williams (1999) have argued that with some appropriate monetary policy rules and even at inflation target levels close to zero, one should rarely enter deflationary regimes. While their analysis is based on more sophisticated modeling and on a more careful estimation of the underlying equations, there may be an issue how eagerly one should share their optimism. First, given the simple model here, the conclusion that the transition into the deflationary state is rare may not be sufficiently robust. Indeed, the referee to this paper wrote that "a central bank may face such a situation due to some large negative supply shock," caused in the case of Japan by "the collapse of stock prices and land prices in the early 1990s and the delay of understating their consequences on the bank and corporate balance sheets"; see Ogawa and Kitasaka (2000). Second, given that one may enter a deflationary regime once in a while, one may then stay there for a prolonged period: a low probability of entering that regime and the fact that deflationary episodes have been rarely observed so far is therefore of little comfort. It would be interesting in the context of this model to calculate the transition probabilities into the deflationary regime, and I hope that this will be pursued in subsequent work.

In Fig. 5, the result of changing α is shown: apparently, this parameter has little effect on the steady state distribution. Indeed, there are two opposing effects. On the one hand, a higher value for α means that inflation is more tightly controlled while $r_t > 0$, providing a tendency for π_t to cluster more closely to the target π^*. On the other hand, a higher value for α means that π_0 is closer to π^* and that

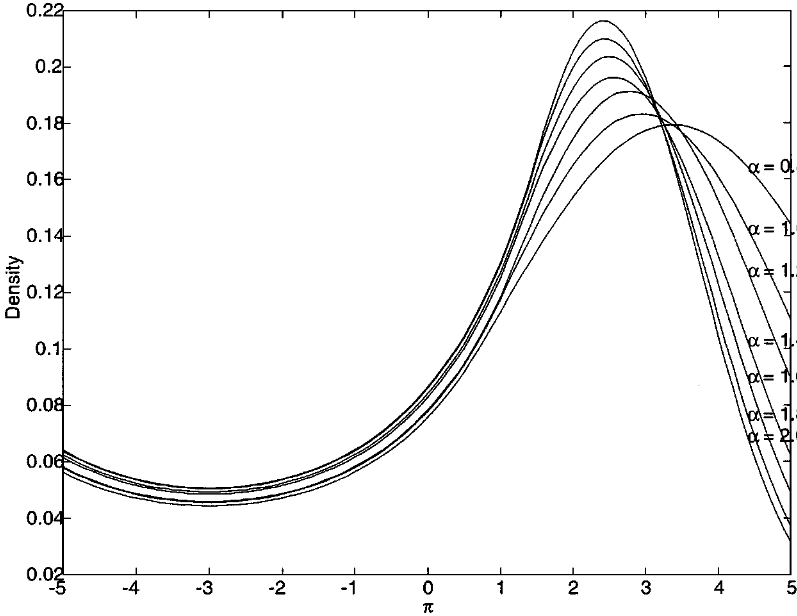

FIG. 5. Density of price changes when varying α.

the lower zero bound for r_t and thus the regime of a deflationary spiral will be entered at higher values of the price change π_t, resulting in a flattening tendency for the density[8]. Apparently, both effects are reasonably close to balance for the numerical case studied here.

3.2. Benign Price Dynamics and Maneuvering Room for Monetary Policy

Summers (1991) has argued that a positive average inflation rate and avoidance of the zero lower bound on interest rates is necessary in order to leave maneuvering room for countercyclical monetary policy. I wish to make some progress on examining this issue in this section. I use a slight generalization of Eq. (1). Our aim will be to calculate optimal monetary policy, taking into account the effect of choosing nominal interest rates on both inflation (or deflation) and the output gap. To do so, I need to rule out the possibility of a deflationary spiral: otherwise, the entire problem will solely be driven by the desire of the policymaker to avoid the deflationary spiral regime. I will therefore pick parameters which make the evolution of price changes stable, even if $r_t \equiv 0$.

More specifically, suppose that

$$\pi_{t+1} = \phi\pi_t + \gamma y_t + \epsilon_t \tag{7}$$

[8] As Henning Bohn pointed out to me, that intuition is not compelling: after all, in that region, there is still more upward pressure on inflation than for a lower value of α.

$$y_{t+1} = \psi y_t - \rho(r_t - \pi_{t+1}), \tag{8}$$

where π_t is the rate of price change from period t to $t+1$, y_t is the output gap, r_t is the nominal interest rate for safe assets maturing in period $t+1$, ϵ_t is a date-t mean zero normally distributed random shock with variance σ^2 and ϕ, γ, ψ, and ρ are parameters. These equations make an assumption, which is surely crucial in light of the preceeding Section 3.1: the benchmark real interest rate is assumed to be zero. One should therefore think of this model as capturing a rather sluggish period in time when, e.g., productivity growth is slow. The model does not apply to times of fast productivity growth as in the United States in the latter half of the 1990s, but is more applicable to the current situation in Japan: that, of course, is precisely the time when one might need to worry about the zero lower bound on nominal interest rates.

I assume that the monetary policymaker chooses r_t every period, knowing the predetermined price change π_{t+1} as well as the current output gap y_t. Further, I assume that the objective of the central bank is to maximize

$$U = -E\left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(\pi_t - \pi^*)^2}{2} + \lambda \frac{y_t^2}{2}\right)\right].$$

Except for some minor differences, this model has been taken practically verbatim from Svensson (1997) and, subsequently, Orphanides and Wieland (2000). I do not claim any originality at this point. It should be clear that this model is highly stylized and perhaps even oversimplified. The model belongs to a class of models which have recently become popular in debates and research regarding monetary policy surveyed by Clarida *et al.* (1999), see, e.g., Further and Moore (1995), Svensson (1997), Rudebusch and Svensson (1999), or Orphanides and Wieland (1999), and it should simply be regarded as a starting point to provide some insights and intuition. Goodfriend and King (1997), Walsh (1998), and Woodford (1999b) provide a more in-depth discussion of how equations of this type may result from a more full-fledged model. Here, I just notice that I have not given a reason that output gap stabilization may be desirable to begin with. Presumably, there must be some market failure which monetary policy may be able to correct: without more detailed modeling, it is hard to see why the private sector cannot do that itself. Furthermore, output is a deterministic function of lagged output, inflation, and interest rates; i.e., there are no “supply shocks” in the language of that literature. I am ignoring them for the sake of simplicity, because my focus is on the autonomous dynamics of inflation rates.

These equations have some features which one may find appealing. As in Eq. (1), central banks do not control inflation directly and actual inflation may overshoot or undershoot its target due to the stochastic term, there is inertia to the dynamics of price changes, and inflation is predetermined one period in advance. Inflation here is now no longer directly linked to real rates: the link is now via the output gap instead. Again, it should be interesting to extend this model to allow for forward-looking elements, to allow for shocks to the output gap equation, and to proceed to a more fully specified model with genuine micro foundations: I find it likely that some of the conclusions here remain intact. In fact, I believe them to depend more on the specific parameters chosen than on the particular underlying theoretical model, although such a statement must be pure speculation in the absence of actually undertaking the exercise of extending the analysis.

Note also that fiscal considerations for the determination of the price level have been left out entirely: this is an ommission in any case, as fiscal policy undoubtedly becomes an important part of the set of policy tools available at near-zero interest rates. I do not see this as a major drawback at this stage of the analysis, however, since the focus here is on monetary policy per se and not on output stabilization at large. More importantly, though, recent advances in the fiscal theory of the price level point out that the price level or perhaps even the inflation rate should be more properly viewed as being determined by fiscal policy rather than monetary policy altogether; see, e.g., Leeper (1991), Sims (1994), Woodford (1995), and Cochrane (1998). It might be interesting to puruse the implications of that framework for the question of the zero lower bound on interest rates. Doing so here would lead us too far away, however.

The benchmark real rate has been fixed at zero for this exercise (i.e., it is abent from the equations above). If the realized real rate is always equal to zero, i.e., equal to the benchmark real rate, the stability of the system is given by the eigenvalues of the matrix

$$A_{r_t-\pi_{t+1}\equiv 0} = \begin{bmatrix} \phi & \gamma \\ 0 & \psi \end{bmatrix}$$

and hence by ϕ and ψ. Thus, if both ϕ and ψ are smaller than unity in absolute value, the system is stable around the steady state values $\pi_\infty = 0$, $y_\infty = 0$ for a constant real rate. If the realized real rate is always equal to some other value $\bar{r}$, say, the steady state shifts to

$$y_\infty = \frac{-\rho}{1-\psi}\bar{r}, \qquad \pi_\infty = \frac{\gamma}{1-\phi} y_\infty.$$

I.e., to achieve a steady state of permanently positive inflation, one needs to permanently push the short-term real rate below the benchmark level: this model assumes that this is in principle possible.

If $r_t \equiv 0$, i.e., if nominal interest rates hit the zero lower bound, the stability of the system is given by the eigenvalues of

$$A_{r_t\equiv 0} = \begin{bmatrix} \phi & \gamma \\ \rho & \psi \end{bmatrix}$$

TABLE II
Parameters for the
Two-Equation Model

ϕ	0.7
γ	0.65
ψ	0.35
ρ	0.16
σ	1.3
β	0.95

and hence by the two roots ζ_i, $i = 1, 2$, to the quadratic equation

$$p(\zeta) = \zeta^2 - \zeta(\phi + \psi) + (\psi\psi - \gamma\rho).$$

I have estimated the two equations (7) and (8) with OLS using annual U.S. data from 1961 to 1998, including an intercept. To obtain the output gap, I have HP-filtered the logarithm of real GDP, removing the smooth trend. For my HP parameter, I have used the value 7, rather than the more conventional value of 100: the justification can be found in Ravn and Uhlig (1997). For Eq. (7), I obtained $\phi = 0.75(0.08)$, $\gamma = 0.80(0.16)$, and a significant intercept of 1.20(0.41). The standard error of the residual was $\sigma = 1.3$ and $R^2 = 0.8$. For Eq. (8), I obtained $\psi = 0.24(0.15)$, $\rho = 0.19(0.07)$, and an insignificant intercept of 0.41(0.25) (standard errors are in parenthesis). The standard error was 1.2 and $R^2 = 0.27$. The eigenvalues of $A_{r_t \equiv 0}$ are $\zeta_1 = 0.96$ and $\zeta_2 = 0.03$ and therefore are both stable. For the simulations below, I have used the parameters listed in Table II, which differ somewhat (but not significantly) from my empirical estimates. The roots for the parameters listed there are $\zeta_1 = 0.89$ and $\zeta_2 = 0.16$, i.e., somewhat more benignly stable.

Taking all equations together, the problem of the central bank can be rewritten as a dynamic programming problem,

$$\begin{aligned} V(\pi_{t+1}, y_t) &= \max_{r_t \geq 0} \left\{ -\frac{\pi_{t+1}^2}{2} - \lambda \frac{y_{t+1}^2}{2}' + \beta E[V(\pi_{t+2}, y_{t+1})] \right\} \\ y_{t+1} &= \psi y_t - \rho(r_t - \pi_{t+1}) \\ \pi_{t+2} &= \phi \pi_{t+1} + \gamma y_{t+1} + \epsilon_{t+1}. \end{aligned} \tag{9}$$

The crucial difference to similar models of optimal monetary policy in the literature is the restriction that nominal interest rates must be positive, $r_t \geq 0$. What I am looking for in particular is the optimal monetary policy rule,

$$r_t = f(\pi_{t+1}, y_t).$$

It is useful to first consider this problem without the zero lower bound on nominal interest rates. In that case, this is a standard linear-quadratic problem. I assume additionally that $\pi^* = 0$ as perhaps the most interesting benchmark:

PROPOSITION 4. *Without the constraint $r_t \geq 0$, and with $\pi^* = 0$, there is a solution to the dynamic programming problem, given by*

$$\begin{aligned} V(\pi_{t+1}, y_t) &= -a\frac{\pi_{t+1}^2}{2} - c \\ r_t &= \frac{\psi}{\rho} y_t + \left(\frac{\omega}{\rho} + 1\right)\pi_{t+1}, \end{aligned} \tag{10}$$

where

$$\omega = \frac{\beta\phi\gamma a}{\lambda + \beta\gamma^2 a}$$

$$c = \frac{\beta}{1-\beta}\frac{\sigma_\epsilon^2}{2}a$$

and where

$$a = \frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

with

$$p = \frac{\lambda\beta\phi^2 + \beta\gamma^2 - \lambda}{\beta\gamma^2}$$

$$q = -\frac{\lambda}{\beta\gamma^2}.$$

This proposition can be proved by the method of "guess and verify." More precisely, substitute the form (10) into the right-hand side of Bellman's Eq. (9). Take first-order conditions and calculate the envelope conditions. After some tedious calculations, the result above obtains. Alternatively, check Svensson (1997) or Orphanides and Wieland (2000), who also derive practically the same result.

There are a few things to notice. First, the decision rule for interest rates takes the form of a Taylor rule; see Taylor (1993). I.e., interest rates are set to react linearly to the (deviation in) inflation and the output gap. The coefficients have the expected sign: interest rates should be raised if the output deviations (the negative of the output gap) are particularly large or inflation is particularly strong. This is good news: there is fairly convincing evidence that monetary policy in practice comes close to following a Taylor rule; see Clarida *et al.* (1998). Our model is certainly not unique in giving rise to a Taylor rule as the optimal (or nearly optimal) decision rule for a central bank: a survey of a variety of monetary models and their implications for Taylor rules is in Taylor (1999a,b).

Second, one can actually bypass interest rates r_t in the model above completely and directly assume that the central bank controls y_t. Even ψ then does not matter anymore. One finds that

$$y_t = -\omega\pi_t.$$

From this, it follows that

$$\omega = \frac{\sigma_y}{\sigma_\pi},$$

i.e., the ratios of the observed standard deviations of output and inflation yield the (negative of the) reaction cofficient of the output gap to inflation. A more extensive discussion, in particular in the relationship to the data on unemployment volatility and inflation volatility across countries, can be found in Uhlig (1999). Finally, I note that there is no systematic inflation bias when $\pi^* = 0$.

To solve the dynamic problem with the zero lower bound on nominal interest rates, I use numerical techniques[9] in an exercise similar in spirit to the one performed by Orphanides and Wieland (1999). The parameters have been taken from Table II.

A plot of the decision rule for the "benchmark" parameterization $\pi^* = 2$ and $\lambda = 1$ can be seen in Fig. 6. Naturally, the nominal interest rate is set equal to zero for large ranges of the state space. Nonetheless, interest rates are raised quickly if the output gap or the inflation rate is too large.

A comparison to the benchmark situation of no lower bound is in Fig. 7. As one can see, the nominal interest rate is held at zero for somewhat higher levels of inflation (which should be clear from an "option value" perspective), but is otherwise not substantially different.

Simulation results when varying π^* and λ can be found in Tables III for the price changes π_t, IV for the output gap, and V for the nominal interest rates. Furthermore, Figs. 8 and 9 contain histograms for the nominal interest rates r_t: Fig. 8 shows the histograms when fixing $\lambda = 1$, but varying $\pi^* = 0, 0.5, \ldots, 3$, whereas Fig. 9 fixes $\pi^* = 2$ and varies $\lambda = 0, 1, 10, \infty$.

A clear and striking feature emerges from all these results: optimal monetary policy in this setting keeps nominal interest rates close to zero in a substantial fraction of the periods. Indeed, one may want to regard the results here as a

[9] I have used a grid method on equally spaced grids and value function iteration. I have discretized the shock ϵ_t, calculating an appropriate Markov transition matrix for inflation rates on the grid once. For each state on the grid, the optimal decision was used by bisection methods, using a linear interpolation of the expected value function, given on the grid. With the decision rules calculated this way, I have then performed simulations of length 1500 with normally distributed shocks, truncated to fit the range of $\pi_t' s$ on the grid, discarding the first 500 observations and linearly interpolating the decision rule between grid points.

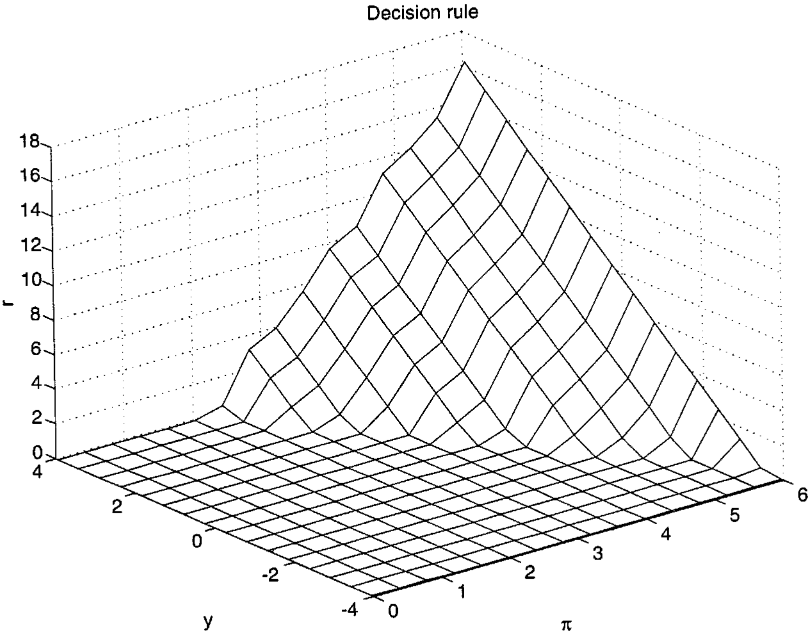

FIG. 6. A plot of the decision rule using $\pi^* = 2$ and $\lambda = 1$.

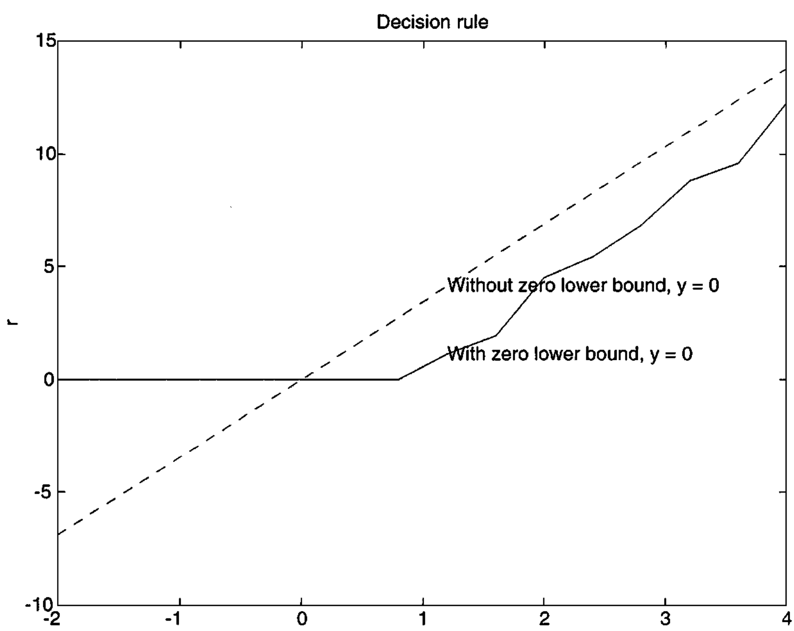

FIG. 7. This figure compares the no-constraint decision rule and the decision rule constrained by a lower bound of zero on the nominal interest rate. For the parameters, we have used $\pi^* = 0$ and $\lambda = 1$.

TABLE III
Results for π

$\lambda =$	0	1	10	∞
$\pi^* = -0.5$	−1.09 (1.90)	−0.94 (1.94)	−0.62 (2.06)	−0.47 (2.12)
$\pi^* = 0.0$	−0.75 (1.99)	−0.68 (2.02)	−0.53 (2.08)	−0.47 (2.12)
$\pi^* = 0.5$	−0.50 (2.07)	−0.47 (2.08)	−0.45 (2.11)	−0.47 (2.12)
$\pi^* = 1.0$	−0.27 (2.16)	−0.29 (2.16)	−0.38 (2.13)	−0.47 (2.12)
$\pi^* = 1.5$	−0.14 (2.23)	−0.17 (2.22)	−0.32 (2.16)	−0.47 (2.12)
$\pi^* = 2.0$	−0.04 (2.30)	−0.08 (2.28)	−0.26 (2.18)	−0.47 (2.12)
$\pi^* = 2.5$	0.01 (2.33)	−0.02 (2.31)	−0.21 (2.20)	−0.47 (2.12)

Note. Means and standard deviations are in parenthesis.

practical means to implement the Friedman rule. In words, the recipe would be to keep nominal interest rates at zero all the time, except if prices start to rise too fast: in that case, swiftly react with high nominal interest rates to bring inflation down again.

The intuition for the choice of a zero nominal interest rate is clear. If inflation is below the target level or if output is below trend, the central bank would want to choose low nominal interest rates anyhow and ends up choosing zero nominal interest rates here. The monetary policymaker is not worried about runaway deflation, because I have assumed that even deflationary processes are stable. Should inflation reach levels which are deemed too high, then nominal interest rates are raised just as most monetary models would recommend: there is no surprise here.

Interestingly, the inflation target level matters little for the actual average rate of inflation achieved: because of the inherently stable price change dynamics, it is too costly to try to keep inflation high (and it is impossible to force prices to go up, when the economy is in the deflationary regime). Indeed, if anything, a higher inflation target results in adopting a zero nominal interest rate more often! This result casts doubt on the recommendation of some observers to avoid deflationary zero-interest-rate episodes by adopting positive levels of inflation as the inflation target.

TABLE IV
Results for y

$\lambda =$	0	1	10	∞
$\pi^* = -0.5$	−0.51 (0.40)	−0.44 (0.33)	−0.30 (0.32)	−0.22 (0.36)
$\pi^* = 0.0$	−0.35 (0.41)	−0.32 (0.36)	−0.25 (0.34)	−0.22 (0.36)
$\pi^* = 0.5$	−0.24 (0.41)	−0.23 (0.38)	−0.22 (0.36)	−0.22 (0.36)
$\pi^* = 1.0$	−0.13 (0.44)	−0.15 (0.42)	−0.19 (0.38)	−0.22 (0.36)
$\pi^* = 1.5$	−0.07 (0.47)	−0.09 (0.45)	−0.16 (0.40)	−0.22 (0.36)
$\pi^* = 2.0$	−0.03 (0.50)	−0.04 (0.49)	−0.13 (0.41)	−0.22 (0.36)
$\pi^* = 2.5$	−0.01 (0.52)	−0.02 (0.51)	−0.11 (0.43)	−0.22 (0.36)

Note. Means and standard deviations are in parenthesis.

TABLE V
Results for r

$\lambda =$	0	1	10	∞
$\pi^* = -0.5$	0.99 (2.72)	0.86 (2.07)	0.58 (1.22)	0.45 (0.93)
$\pi^* = 0.0$	0.69 (2.39)	0.63 (1.80)	0.50 (1.15)	0.45 (0.93)
$\pi^* = 0.5$	0.47 (1.84)	0.45 (1.53)	0.43 (1.07)	0.45 (0.93)
$\pi^* = 1.0$	0.28 (1.49)	0.30 (1.23)	0.38 (0.99)	0.45 (0.93)
$\pi^* = 1.5$	0.16 (1.07)	0.19 (0.98)	0.32 (0.91)	0.45 (0.93)
$\pi^* = 2.0$	0.08 (0.75)	0.11 (0.74)	0.27 (0.85)	0.45 (0.93)
$\pi^* = 2.5$	0.04 (0.46)	0.06 (0.54)	0.23 (0.78)	0.45 (0.93)

Note. Means and standard deviations are in parenthesis.

4. BABY-SITTING COOPS

Some, in particular Sweeney (1978) and Krugman (1998b, 1999a), have argued that monetary policy is best understood using the parable of a baby-sitting coop. In fact, Krugman (1999a) uses it throughout his book as a key device to analyze the current situation in Japan and to discuss remedies.

The story is probably familiar to many. A community decided at some point to organize mutual baby-sitting by trading coupons. Each family would be endowed with an initial number of coupons. If a family requested baby-sitting services from

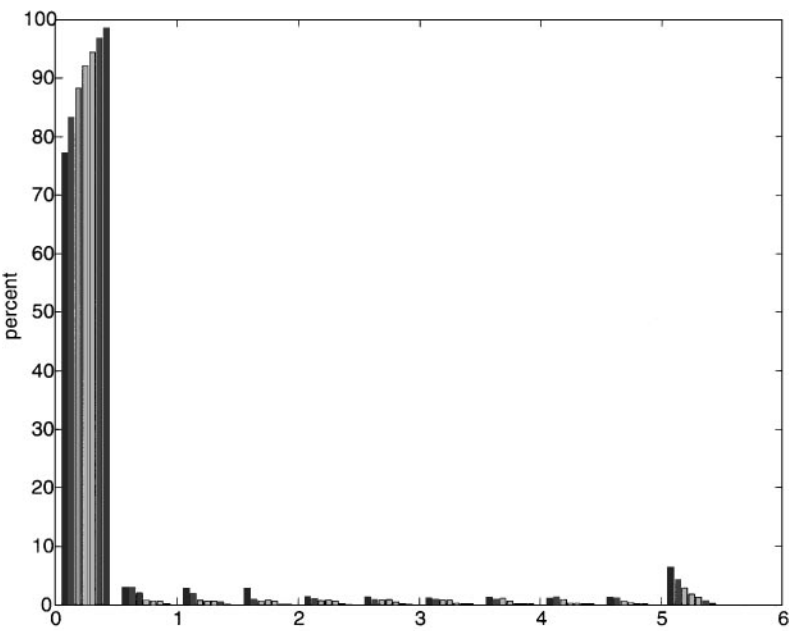

FIG. 8. Histogram of nominal interest rates when varying π^*.

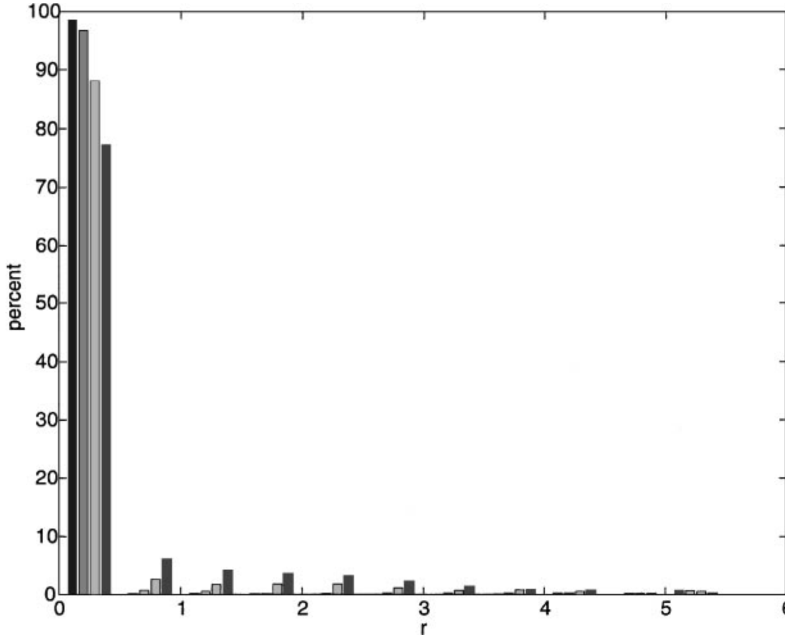

FIG. 9. Histogram for nominal interest rates when varying λ.

someone within the community, it would give that person or family a coupon. Likewise, for baby-sitting services rendered, it would receive a coupon. At some point, this system of mutual baby-sitting came to a halt. Krugman (1999a) writes on p. 9, that "one couple's decision to go out was another's opportunity to baby-sit; so opportunities to baby-sit became hard to find, making couples even more reluctant to use their reserves except on special occasions. . . ." The remedy turns out to be a helicopter drop of money. On p. 11, Krugman (1999a) states that "eventually, however, the economists prevailed, and the supply of coupons was increased. The results were magical: with larger reserves of coupons to baby-sit more plentiful, making couples even more willing to go out, and so on. . . . The monetary screwup had been rectified. Recession, in other words, can be fought simply by printing money—and sometimes (usually) be cured with surprising ease."

While Krugman told me in private conversation that he intended this parable only as a means of explanation to the general public how monetary policy might have real effects, it is hard to read his description above from a more informed perspective without giving it a multiple equilibrium interpretation of moving from a good to a bad equilibrium and back through the injection of liquidity. I shall proceed to examine his argument with the rigor of a formal model of a baby-sitting coop. I shall show that it indeed can give rise to multiple equilibria. However, the injection of liquidity is unlikely to enable the economy to move back to a good equilibrium: rather, it may move the economy to a region where equilibria, in which money is valued, cease to exist altogether. The intuition is simple: if multiple equilibria exist, the "bad" equilibrium branch does not disappear for higher values of coupons per person, as one might think, given the baby-sitting coop parable above. Rather, both branches coexist except for a single maximal value of coupons, at which monetary equilibria are sustainable. Beyond that, existence breaks down as there are too many coupons chasing too few baby-sitters.

The formal analysis here thus offers a dire warning regarding Krugman's recommendation. Given the attention that his parable and recommendations have received, more formal research should be devoted to clarifying its precise logic, separating the pitfalls from the genuine insights. The analysis here should be understood as a step in that direction and as pointing out that a casual treatment of the baby-sitting coop parable as a guide to monetary policy can be dangerously misleading.

The model is developed in Section 4.1 and solved in Section 4.2. Insights into the nature of multiple equilibria as well nonexistence are provided in Section 4.3.

4.1. The Model

There is a continuum of agents who live in an infinite number of discrete periods $t = 0, 1, 2, \ldots$. They own coupons exchangable for baby-sitting services. I assume that agents are not allowed to borrow coupons, but there is no upper limit to holding coupons. Receiving baby-sitting services yields random utility u_t at date t, drawn iid according to some continuous distribution function F. Agents themselves may be asked in turn to baby-sit, in which case they receive a coupon. I assume for simplicity that they do not experience any disutility for rendering this service. Furthermore, the selection of the baby-sitting agents is assumed to be random. In each period, both events can take place. I assume that agents know the utility u_t generated by the baby-sitting service but do not know whether they in turn will be asked to baby-sit "later" in that period, when making the decision whether or not to make use of a coupon. Perhaps it is a bit absurd to assume that couples can go out as well as baby-sit in the same period, but it simplifies the analysis considerably (and it may not be so absurd after all, judging from my own experience). I suppose that the average number $\bar{n}$ of coupons per agent is given.

Solving for the social planners' solution is simple: demand that every opportunity to go out is taken, i.e., that there is baby-sitting all the time. This result is but an artifact of the simplifying assumption that baby-sitting is painless: while reasonable as a simplifying device, this is certainly not reasonable for providing a welfare analysis, from which I will therefore refrain. The assumption of painless baby-sitting also delivers the artificial equilibrium if there is at least one coupon per agent, in which all agents request baby-sitting every period, always using one coupon and receiving one in return.[10] While agents might as well participate, given that they are indifferent between providing baby-sitting services and not baby-sitting, this equilibrium is rather "perverse": it is as if agents are willing to

[10] I am grateful to Kyoji Fukao, my discussant, for pointing this out.

deliver a free good for receiving valueless money and therefore do not mind holding the extra reserves forever, if the number of coupons exceeds one per agent. It is likely that this equilibrium will disappear with a small cost for baby-sitting, although this shall not be shown formally. Instead, I will simply proceed with my fingers crossed, focussing on the "reasonable" equilibria instead, in which agents perceive the coupons as scarce and valuable.

Focusing on the stationary environment, I shall now describe the dynamic programming problem faced by the agents. When making their decisions, agents knows the probability p that they will be asked later in that period to render the baby-sitting services themselves. I shall therefore take p as given: eventually, I will need to solve for p in equilibrium.

Let $v(n, u)$ be the value of having n coupons, provided that receiving the service generates utility u this period. Let β be the discount factor, $0 < \beta < 1$. I have

$$\begin{aligned} n = 0: &\quad v(0, u) = \beta w(0) \\ n \geq 1: &\quad v(n, u) = \max\{u + \beta w(n-1); \beta w(n)\}, \end{aligned}$$

where $w(n)$ is the expected continuation value for n coupons before being possibly asked to render the service oneself,

$$w(n) = (1-p)v^e(n) + pv^e(n+1), \tag{11}$$

where

$$v^e(n) = E[v(n, u)] = \int v(n, u)F(du).$$

It is clear that the optimal decision rule is of the threshold type, i.e., to request the baby-sitting service iff

$$u \geq \bar{u}(n) = \beta(w(n) - w(n-1)), \quad n \geq 1 \tag{12}$$

and $\bar{u}(0) = 1$ (note that I do not particularly worry about the case of equality as I have assumed the distribution for u to be continuous). Hence,

$$\begin{aligned} v^e(n) &= \beta w(n) + \int_{u \geq \bar{u}(n)} u - \bar{u}(n)F(du) \\ &= \beta w(n-1) + \int \max\{u, \bar{u}(n)\}F(du). \end{aligned} \tag{13}$$

With these decision rules, I can now calculate the population dynamics. Focusing on the stationary case, let λ_n be the mass of agents with n coupons. An agent who has n coupons this period has experienced one of exactly four different scenarios last period, provided $n \geq 1$:

1. The agent started with $n+1$ coupons, used one, but did not receive one.
2. The agent started with n coupons, did not use one, and did not receive one.
3. The agent started with n coupons, used one, and also received one.
4. The agent started with $n-1$ coupons, did not use one, but received one.

Formally, for $n \geq 1$:

$$\begin{aligned}\lambda_n &= \lambda_{n+1}(1 - F(\bar{u}(n+1)))(1-p) + \lambda_n F(\bar{u}(n))(1-p) \\ &\quad + \lambda_n(1 - F(\bar{u}(n)))p + \lambda_{n-1}F(\bar{u}(n-1)))p. \end{aligned} \tag{14}$$

For $n = 0$, one gets

$$\lambda_0 = \lambda_1(1 - F(\bar{u}(1)))(1-p) + \lambda_0(1-p). \tag{15}$$

The weights λ need to be probability weights,

$$1 = \sum_{n=0}^{\infty} \lambda_n. \tag{16}$$

Furthermore, it needs to be the case that $0 \leq \lambda_n \leq 1$.

Finally, it needs to be the case that the average number of coupons spent per agent equals the probability p of receiving a coupon and that the average number of coupons held equals the average endowment $\bar{n}$,

$$p = 1 - \sum_{n=0}^{\infty} \lambda_n F(\bar{u}(n)) \tag{17}$$

$$\bar{n} = \sum_{n=0}^{\infty} \lambda_n n. \tag{18}$$

DEFINITION 1. An *equilibrium* is a vector $((w(n), v^e(n), \bar{u}(n), \lambda_n)_{n=0}^{\infty}, p)$ such that

1. the dynamic programming is solved, i.e., Eqs. (11)–(13) are satisfied,
2. the distribution $(\lambda_n)_{n=0}^{\infty}$ is stationary, i.e., satisfies Eqs. (14)–(16),
3. aggregate feasibility holds, i.e., Eqs. (17) and (18) are satisfied.

It should already be clear that this model is essentially a model of search and thus closely related to the search-theoretical models of money as a medium of exchange; see Kiyotaki and Wright (1989, 1993) or Trejos and Wright (1993,

1995) and others. In the model here, agents can hold several units of the medium of exchange (i.e., coupons). Money here comes in discrete, but multiple, units whereas the service does not. One might want to follow Trejos and Wright (1995) and cast this into a bargaining framework, i.e., introduce bargaining about the cost of baby-sitting in terms of the number of coupons each time somebody seeks the service. In the environment here, one may then want to take into account questions of observability. Here, instead, we go the opposite and perhaps simpler route. There is no bargaining over the price level—the price level is instead fixed at one coupon per one unit of baby-sitting. Furthermore, there is no problem in finding agents willing to render the service. Instead, the "search" feature here is with respect to the urgency of need in receiving the service. Nobuhiro Kiyotaki pointed out to me that there is a paper by Pesek, Saving, Li, and Runero, which I unfortunately have not been able to locate (and which therefore does not appear in the list of references), but which, I am told, also has the feature that making money too attractive may slow down the exchange, because the money holder becomes too picky or too lazy in his or her search effort.

4.2. Analysis

Counting equations and unknowns in the definition above, one can see that there are two equations too many. One superfluous equation is easily spotted: because Eqs. (14) and (15) in λ_n are homogenous of degree 1, a rescaled version of a solution still statisfies these equations. Still, there needs to be one more. The following lemma provides the answer.

LEMMA 1. *Equation* (17) *is superfluous.*

Proof. Suppose that all other equations hold. Imagine taking "one step" from t to $t+1$ in this economy, starting from this distribution, having agents spend coupons according to their optimal decision rules, i.e., the threshold levels $\bar{u}(n)$ and a fraction p of the population receiving coupons. Since $(\lambda_n)_{n=0}^{\infty}$ is a stationary distribution, the total number of coupons will not change, implying that the average number of coupons spent must equal the average number of coupons received. But this is exactly what Eq. (17) states. ■

It is also useful to recognize that $v^e(n)$ can be eliminated:

LEMMA 2. *Equations* (11) *and* (13) *can be replaced by*

$$0 = -(1-\beta)w(n) + (1-p)\int_{u\geq\bar{u}(n)} u - \bar{u}(n)F(du) \tag{19}$$

$$+\, p\int \max\{u, \bar{u}(n+1)\}F(du). \tag{20}$$

Proof. Substitute. ■

To proceed further, I need a convenient assumption on the distribution of the utility u:

Assumption A.1. u is uniformly distributed between 0 and 1, i.e., $F(u) = u, 0 \leq u \leq 1$.

I can now proceed in three steps. First, I calculate the solution to the dynamic programming problem. Next, I show how the stationary distribution can be calculated. Finally, I check for aggregate feasiblity to determine the probability p, which I have treated as a parameter in the first two steps.

4.2.1. Solving the dynamic programming problem. With the assumption of a uniform distribution, the dynamic equation (19) turns out to have a particularly tractable form:

PROPOSITION 5. *With assumption* (1), *Eq.* (19) *can be rewritten as*

$$0 = p\beta w(n+1) - (2-\beta)w(n) + \beta(1-p)w(n-1) + 1 \tag{21}$$

for $n > 0$ *and*

$$0 = p\beta w(1) - (2-(2-p)\beta)w(0) + p. \tag{22}$$

The solution is given by

$$w(n) = w^* - a\nu^n, \tag{23}$$

where

$$w^* = \frac{1}{2(1-\beta)} a = \frac{\nu}{\beta(1-\nu)}$$

and where ν *is the nonexplosive solution to the quadratic equation*[11] (24) *stated in the proof.*

Proof. Equations (19) and (12) and a bit of algebra, exploiting the uniform distribution assumption, directly yield Eqs. (21) and (22). Note, that (21) is a second order linear difference equation. It is well known that these equations have the generic solution

$$w(n) = w^* - a\nu_1^n - b\nu_2^n,$$

[11] This claim is subject to the caveat that I need that the other root solving the same quadratic equation is explosive, but have not formally shown this to be the case in the proof. However, this was always the case in the numerical examples, so I skip this step.

where w^* is the steady state of (21), where a and b are coefficients and where ν_1 and ν_2 are solutions to the characteristic equation

$$0 = p\beta\nu^2 - (2-\beta)\nu + \beta(1-p) \tag{24}$$

provided the two solutions are distinct. First, check that $w^* = 1/(2(1-\beta))$ is indeed the solution of Eq. (21), if one were to set $w(n+1) = w(n) = w(n-1) = w^*$ there. The quadratic equation (24) has the two solutions

$$\nu_{1,2} = \frac{1}{p\beta} - \frac{1}{2p} \pm \sqrt{\left(\frac{1}{p\beta} - \frac{1}{2p}\right)^2 - \frac{1-p}{p}}.$$

While I have not formally shown this yet, for all my numerical examples, one of the roots turned out to be explosive, whereas the other one turned out to be positive and stable. Let $\nu = \nu_1$ be the stable and ν_2 be the unstable root. It is clear that $w(n)$ is bounded above from always consuming the maximal possible utility, i.e., by $1/(1-\beta)$, and is bounded below from never consuming anything, i.e., by 0. This implies that the coefficient b on the explosive part must equal zero. Finally, to calculate a, I exploit the initial condition (22). That equation can now be rewritten as

$$1 - p = a(2 - 2\beta + p\beta - p\beta\nu).$$

Finally nothing that (24) implies

$$2 - \beta - p\beta\nu = \beta(1-p)\frac{1}{\nu}$$

yields the stated solution for the coefficient a. ■

The previous lemma with (12) immediately implies that the threshold values $\bar{u}(n)$ are given by

$$\bar{u}(n) = \nu^n.$$

4.2.2. The stationary distribution. While I cannot obtain a single formula for λ_n, the population weights can easily be calculated recursively as follows. I continue to uphold the assumption (1) of a uniform utility distribution.

PROPOSITION 6. *Given the threshold values $\bar{u}(n)$, let $\tilde{\lambda}_o = 1$, let*

$$\tilde{\lambda}_1 = \frac{p}{(1-p)(1-\bar{u}(1))},$$

and recursively calculate

$$\tilde{\lambda}_{n+1} = \frac{((1-p) - (1-2p)\overline{u}(n))\tilde{\lambda}_n - p\overline{u}(n-1)\tilde{\lambda}_{n-1}}{(1-p)(1-\overline{u}(n+1))}$$

for $n \geq 1$. *The population weights are now given by*

$$\lambda_n = \left(\sum_{j=0}^{\infty} \tilde{\lambda}_j\right)^{-1} \tilde{\lambda}_n.$$

Proof. Direct. ■

4.2.3. Closing the model. I finally have to check the feasibility relationship

$$\overline{n} = f(p),$$

where

$$f(p) = \sum_{n=0}^{\infty} \lambda_n(p)n,$$

where I have indicated the dependence of λ_n on the value for p, which I have so far treated as a parameter. This equation can be solved by "brute force": for all $p \in (0; 1)$, trace out the function $f(p)$ on the right-hand side of this equation. By construction, $f(p)$ can be shown to be continuous. Thus, if $\inf f(p) \leq \overline{n} \leq \sup f(p)$, then a solution exists, otherwise, it will not.

In fact, note that $f(p)$ does not depend on $\overline{n}$ itself! I can therefore calculate $f(p)$ once and then immediately provide solutions p for any $\overline{n}$ this way.

4.3. Discussion

In the last section, I made the assumption of a uniform distribution for the utilities u, and I then proceed to characterize the solution to my model. It takes the form

$$\overline{n} = f(p).$$

This formulation immediately allows the discussion of the number of equilibria

$$p \in \Phi(\overline{n}) = f^{-1}(\overline{n}) = \{p \mid \overline{n} = f(p)\}.$$

The only parameter left to vary is the discount factor β: I shall therefore write the equation above as

$$\overline{n} = f(p; \beta).$$

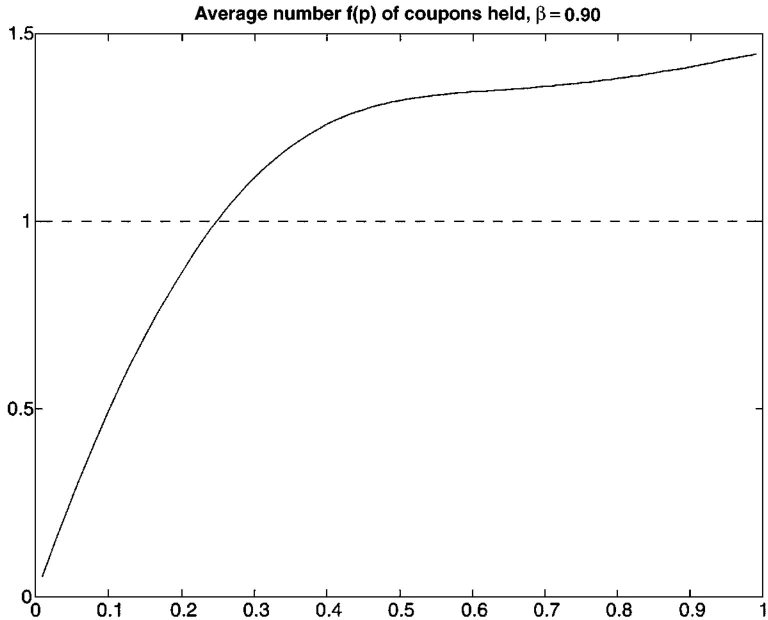

FIG. 10. This figure shows $f(p;\beta)$ for $\beta = 0.90$. The function looks monotonic.

For $\beta = 0.9$, the function is plotted in Fig. 10. The function appears to be rising monotonically: for each $\bar{n}$ there is therefore at most one solution $p(\bar{n})$ with $\bar{n} = f(p)$ for any given $\bar{n}$. In this economy, I may think of p as the level of economic activity of GNP. Clearly, economic activity $p(\bar{n})$ rises with the total amount of coupons in circulation. I interpret the comments by Paul Krugman during a private conversation that this may be his desired interpretation of the baby-sitting coop parable. If so, it is perhaps a bit tricky to motivate why economic activity would collapse here—a reduction in the number of coupons in circulation? A shift in tastes? Some "government" interference in markets?—and one would probably like to understand the specific cause before suggesting a specific remedy. In any case, note, that there are no (stationary) equilibria if $\bar{n}$ is too large, so there is a problem in injecting too much liquidity, an issue which we shall discuss in greater detail toward the end of the paper.

Figure 11 is similar to the preceeding figure, except that the consumers are now more patient: $\beta = 0.99$ instead of $\beta = 0.90$. For, e.g., $\bar{n} = 2$, one can now see that there are two equilibria: a low-activity equilibrium and a high-activity equilibrium. For the low-activity branch, economic activity behaves as expected: more coupons $\bar{n} = f(p)$ result in a higher p (which one has to look up on the x-axis). For the high-activity branch, economic activity behaves "perversely," though: an additional injection of coupons actually lowers economic activity. And, again, if there are too many coupons in circulation, no stationary equilibrium exists (except for the perverse equilibrium of always providing baby-sitting services): theory, at least

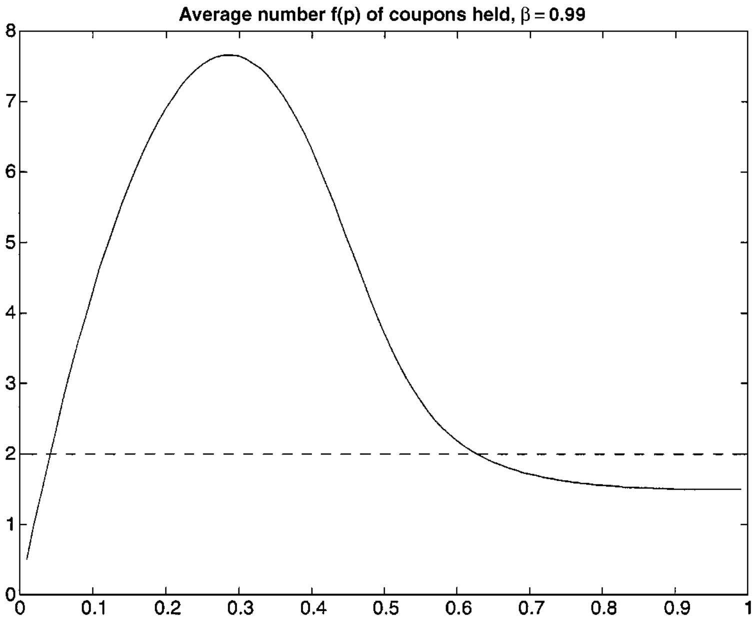

FIG. 11. This figure shows $f(p; \beta)$ for $\beta = 0.99$. Notice the hump-shape, indicating multiple equilibria.

developed so far, offers little guidance as to what would happen then, although I shall feel free to speculate a bit further below.

The fact that multiple equilibria only emerge if agents are more patient suggests that this problem is more likely to arise in countries in which patience is seen as a relatively greater virtue. It is conceivable that this helps explain why this type of problem has arisen in Japan rather than, say, in the United States, if indeed this analysis is appropriate for the situation there.

To understand how the multiple equilibria arise, it may be useful to examine the changing shape of the stationary distributions, as p is varied for $\beta = 0.99$: this can be seen in Fig. 12. When looking at this figure, keep in mind, that the total number of coupons in circulation varies according to $f(p; \beta)$, as I vary p: in fact, $f(p; \beta)$ results from integrating the number of coupons, using these distributions. Apparently, both at low levels of p and at high levels of p, the distribution concentrates on low levels of n, but for different reasons: for low levels of p, there simply is not a large supply of coupons per agent, so most of them "dry up." For high levels of p, agents do not mind spending their coupons as quickly as possible, because they can rest assured that they will receive another coupon very shortly. It is only in the intermediate range that agents typically wish to hold a number of coupons: while there is ample supply so that it will not "dry up," coupons nonetheless arrive sufficiently infrequently so that agents wish to carry a buffer stock. The result of

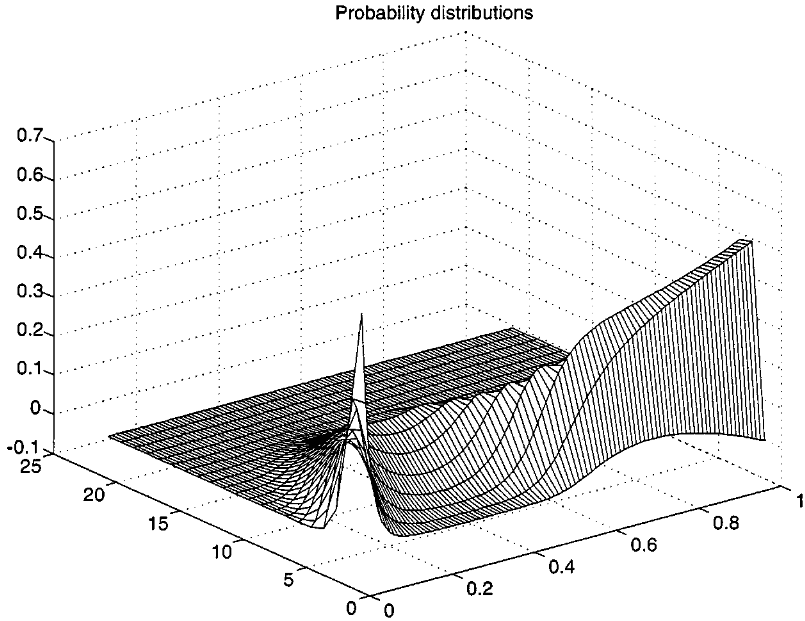

FIG. 12. This figure shows the distribution (λ_n) as p varies. At a given p, the "slice" of this figure shows the distribution.

this logic is the banana-shape of this three-dimensional picture of the population distribution.

The evolution of $f(p;\beta)$ can be seen in Fig. 13 or, as a three-dimensional "hill," in Fig. 14. Whenever there are peaks, there are multiple equilibria, when $\bar{n}$ is sufficiently high: a high-activity equilibrium and a low-activity equilibrium.

Taking the contour-plot of the hill in Fig. 14 yields Fig. 15, which finally provides the sets $\Phi(\bar{n})$ as β varies.

Some more speculative remarks after this analysis may be in order. First, the story told in the Introduction may be a story of an economy falling from a high-activity stationary equilibrium into a low-activity equilibrium. The theory here certainly does not provide a rationale for how this might happen, but it may not be too hard to envision this possibility. For example, perhaps all the families in the coop woke up one morning, each individually believing that now everybody would coordinate on the low-activity equilibrium, whereas this was thought impossible just yesterday. What, then, would happen if additional coupons would be injected in order to stimulate economic activity? The pictures suggest perhaps nothing much. If the additional injection of coupons resulted in the stationary steady state moving "locally" (again, this is quite a stretch of the theory—there is nothing in the theory that would allow me to make this leap of faith! But it may noetheless be useful to guide intuition in future, more fully specified models), then economic activity would simply inch up a bit, but stay on the low-activity branch.

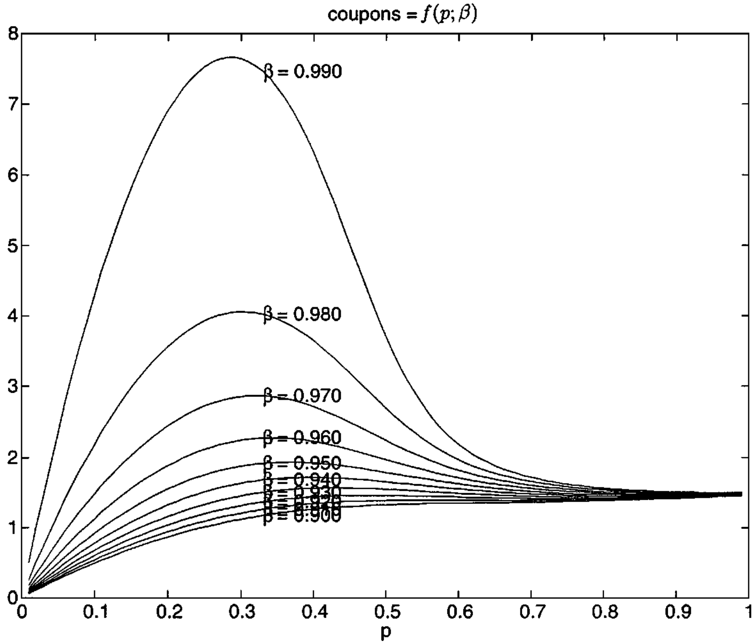

FIG. 13. $f(p;\beta)$ for several values of β are shown.

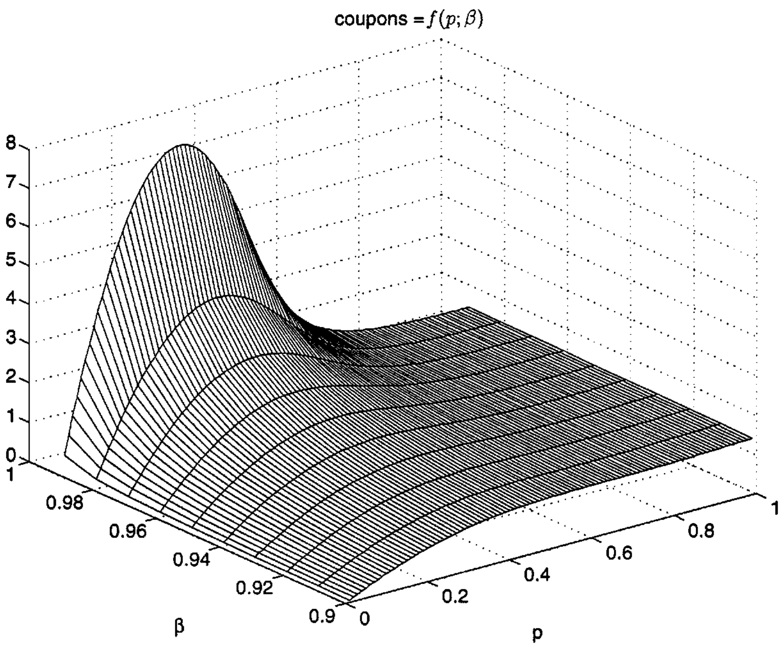

FIG. 14. Here, one can see the function $n = f(p;\beta)$ on its two-dimensional domain. The hill-shape indicates the existence of multiple equilibria.

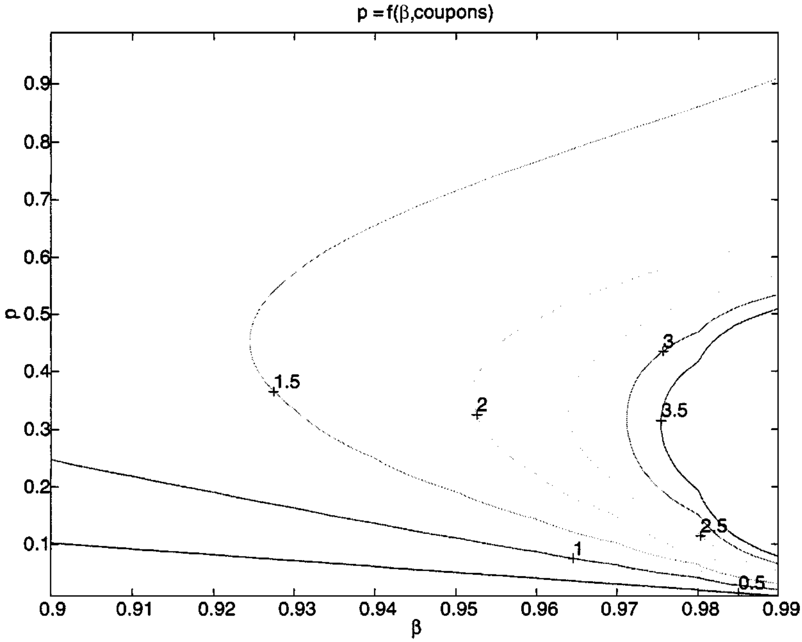

FIG. 15. For each β and each $\bar{n}$ (on the contour curves), one can read off this figure the solution set of all p's, satisfying $\bar{n} = f(p; \beta)$.

Perhaps, then, remembering the "good old times," the coop leaders might become desperate and inject a lot more coupons. For example, somebody might come up with the idea of measuring the change in economic activity due to the recent injection of extra coupons and extrapolating the result to figure out how many additional coupons would be needed to restore the old level of activity. The result would be disastrous. Rather than restoring that old level of activity, the economy would instead enter the range of nonexistence of a stationary equilibrium. More precisely, only the perverse equilibrium of always baby-sitting remains, but presumably disappears when assuming a small cost to delivering the baby-sitting service.

What, exactly, will happen then? Again, the theory as described so far is of little assistance. Intuition suggests that the coupons simply become totally valueless: there will be too many of them circulating for anybody to be interested in accepting them as payment for services rendered, again supposing some disutility to baby-sitting to rule out the perverse baby-sit-always equilibrium. The families in the coop would surely find other ways to agree on mutual baby-sitting arrangements, i.e., they would resort to some form of barter and pairwise contracting instead. As a parable of monetary policy, this suggests that inflating a country out of a liquidity trap may actually result in runaway inflation and a complete change in the payment system rather than restoration of the old level of economic activity.

This may be the most important of the overlooked implications of the baby-sitting coop analogy to monetary policy making.

Now, one may object that surely the comparison here has been taken too far. Monetary policy is different from raising or lowering the number of coupons in a baby-sitting coop. Sure it is. But if that is so, why should one attach much weight to the claim that monetary policy can get a country out of deep recession simply by rekindling inflation, as Krugman (1999) has claimed, using the baby-sitting coop parable in support of his thesis? A deeper analysis is called for.

5. DISCUSSION AND CONCLUSION

I have discussed three different frameworks in order to gain a few more insights into the phenomenon of a liquidity trap, of Friedman's rule, and of runaway deflation. None of these models are sufficiently satisfactory.

In the first section, I examined a cash-in-advance economy, restating some results by Cole and Kocherlakota (1998). It turns out that Friedman's rule can be implemented by shrinking the money stock at the appropriate rate in the long run. As this leaves ample room in the short run, one important lesson to be drawn here is that increases in the money stock—be they through outright helicopter drops, any form of open market operation, or foreign exchange intervention—change nothing as long as the economy remains in the equilibrium of a long-run shrinkage of the money stock. I believe that this lesson is probably correct and a general one. However, the cash-in-advance structure is also perhaps too rigid in its implications. How should markets know whether or not the Bank of Japan is bent on implementing Friedman's rule in the long run? More appropriately then, near-zero interest rates and deflationary tendencies in prices are more appropriately viewed as medium-term developments, due to the capricious nature of the price process and economic activity itself.

This view was taken up in the next section, which examined two simple models of the inflationary process. One was inherently instable, unless controlled by sufficiently drastic action in nominal interest rates. But as the economy spiraled into deflation, nominal interest rates hit the zero lower bound, thus accelerating the deflationary tendencies. This model offered a dire warning: if these dynamics are indeed inherently unstable, then one should expect to see economies spending a substantial fraction of time in deflationary regimes.

While this prediction seems to be perhaps too dire, the prediction emerging from a model in which a central bank maximizes some objective function in inflation and the output gap, facing an inherently stable price change process, is perhaps too panglossian: in this scenario, the central bank will set the nominal interest rate to zero a large fraction of times, thus erring on the side of deflation rather than inflation.

It is clear that both models do not go deep enough. What is needed is a clearer understanding of exactly what will happen as economies enter deflationary regimes, forcing central banks to zero nominal interest rates. So far, there is too little historical experience to know and too little theorizing to allow any safe predictions. Perhaps, the two models offered here straddle the reasonable middle ground.

Finally, even if prices and nominal interest rates remain unchanged, it may make sense to inject liquidity: by putting liquidity in the hands of agents, they may be more willing to use it to acquire goods, because they expect to soon receive cash by others on a similar shopping spree: an injection of liquidity may move the economy from a bad equilibrium to a good one. Krugman has popularized the parable of a baby-sitting coop as an analogy to the workings of an economy in trouble, and the multiple equilibrium interpretation of his parable is strikingly reasonable, even if not intended. Here, I have examined a formal baby-sitting coop model. While the model generates multiple equilibria, I find the conclusion about the injection of liquidity unwarranted: in fact, it can make matters even worse.

The baby-sitting model also leaves many questions unanswered. To begin with, it is a model in which the medium of exchange is measured in real rather than in nominal units. The analysis only fits the case of money to the degree that one regards the price level as unchanged. The model is essentially silent on what would happen if prices were to change. Second, the analysis so far is static, comparing across equilibria. A dynamic analysis is called for.

In sum, are zero nominal interest rates to be recommended? Is the dreaded liquidity trap instead an implementation of the benign Friedman rule? Is the Scylla lurking at the lower bound of nominal interest rate a frightening multiheaded sea monster or a beautiful nymph? Central bankers may not have the luxury to decide and probably should prefer to err on the side of caution. Academics, such as I, should at least question the conventional wisdom that a liquidity trap is really such a terrible situation. Hopefully, this paper has thrown some question marks into the usual analysis and has shown the need to model deflationary processes with deeper and greater caution. This time, more research really is needed.

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