

Physics 385, Spring 2006.

Advanced mathematical methods of physics: representations of matrix groups and Lie algebras.

General Information.

Hours: TuTh 9am–10:20am

Location: KPTC 103

Instructor: Ilya Gruzberg, RI 227

Office hours: Tu 2:00pm–3:00pm

Grader: Hai Qian, RI-204

This course is a review of techniques applied in a variety of physical problems when the studied system has some degree of symmetry. The main focus will be on continuous symmetries in their infinitesimal form (Lie algebras), and the main goal will be to understand the representations of Lie algebras. Discrete groups will be treated (if at all) as a special case of compact groups.

Syllabus.

We will try to cover the following (later items on the list may have to be skipped because of lack of time):

1. Basic concepts.

Classical (matrix) groups with examples. Topology: compactness, connectedness and simply-connectedness. Homomorphisms and isomorphisms: $SU(2)$ and $SO(3)$. Polar (Cartan) decomposition.

2. Lie algebras.

Matrix exponential and logarithm. Lie algebras of classical groups. Complexifications. Relationship between groups and algebras. Baker-Campbell-Hausdorff formula.

3. Structure of Lie algebras.

$su(2)$, $su(3)$ and their representations. General complex Lie algebras. Adjoint representation and Killing form. Cartan subalgebras and Cartan-Weyl basis. Roots and Dynkin diagrams. Cartan's classification.

4. Representations of Lie algebras.

Weights. Weyl group and Weyl character formula. Clebsch-Gordan decomposition of tensor products. Universal enveloping algebras

and Casimir operators. Young tableaux and tensor methods. Realization of representations in fermionic and bosonic Fock spaces. Subalgebras and branching rules.

5. Compact classical groups.

$SU(N)$, $SO(N)$, $Sp(N)$ and their representations. Covering groups and spinors.

6. Non-compact groups and symmetric spaces.

Real forms of Lie algebras. Group actions on manifolds. Homogeneous and symmetric spaces. Fourier analysis on groups and symmetric spaces.

Prerequisites.

Basic group theory, linear algebra, some quantum mechanics.

Homework and grades.

There will be homework assignments once every one or two weeks. Grades will be determined by the homework and participation in class.

References.

Two main texts for the course will be:

B. C. Hall, Lie Groups, Lie Algebras, and Representations. Springer, 2003.

R. N. Cahn, Semi-simple Lie algebras and their representations. (Available online at <http://www-physics.lbl.gov/~rncahn/book.html>)

The subject of group theory and representation theory is truly enormous. I will borrow additional material from various books. Here I provide a list of additional books that I found useful or interesting.

Books written for or by physicists.

1. A. O. Barut, R. Raczka, Theory of group representations and applications, 2nd edition, World Scientific, 1986.

2. J. F. Cornwell, *Group theory in physics*, 3 v., Academic Press, 1984–1989.
3. L. Frappat, A. Sciarrino, P. Sorba, *Dictionary on Lie algebras and superalgebras*, Academic Press, 2000.
4. J. Fuchs, C. Schweigert. *Symmetries, lie algebras and representations: a graduate course for physicists*, CUP, 1997.
5. H. Georgi. *Lie algebras in particle physics*, 2nd edition, Westview Press, 1999.
6. R. Gilmore. *Lie groups, Lie algebras, and some of their applications*, Wiley, 1974.
7. M. Hamermesh. *Group theory and its application to physical problems*, Dover, 1990.
8. H. F. Jones. *Groups, representations, and physics*, Insitute of Physics Pub., 1998.
9. L. D. Landau, E. M. Lifshitz. *Quantum mechanics: non-relativistic theory*, Pergamon Press, 1977.
10. H. J. Lipkin. *Lie groups for pedestrians*, Dover, 2002.
11. G. I. Liubarskii, *The application of group theory in physics*, Pergamon Press, 1960.
12. A. M. Perelomov, *Generalized coherent states and their applications*, Springer, 1986.
13. M. I. Petrashen, E. D. Trifonov, *Applications of group theory in quantum mechanics*, M.I.T. Press, 1969.
14. R. D. Richtmyer. *Principles of advanced mathematical physics*, vol. 2, Springer, 1978.
15. S. Sternberg. *Group theory and physics*, Cambridge University Press, 1994.
16. Wu-Ki Tung. *Group theory in physics*, World Scientific, 1985.
17. H. Weyl, *Theory of groups and quantum mechanics*, Dover, 1984.
18. B. G. Wybourne, *Classical groups for physicists*, Wiley, 1974.
24. E. B. Dynkin, *The structure of semi-simple algebras*, AMS, 1950.
25. W. Fulton, J. Harris, *Representation theory: a first course*, Springer, 1999.
26. R. Goodman, N. R. Wallach, *Representations and invariants of the classical groups*, Cambridge University Press, 2000.
27. M. Goto, F. D. Grosshans, *Semisimple Lie algebras*, New York: M. Dekker, 1978.
28. S. Helgason. *Differential geometry, Lie groups, and symmetric spaces*, AMS, 2001.
29. S. Helgason. *Groups and geometric analysis: integral geometry, invariant differential operators, and spherical functions*, AMS, 2000.
30. J. E. Humphreys, *Introduction to Lie algebras and representation theory*, Springer, 1980.
31. N. Jacobson. *Lie algebras*, Dover, 1979.
32. A. A. Kirillov. *Elements of the theory of representations*, Springer, 1976.
33. A. W. Knap. *Lie groups beyond an introduction*, Boston: Birkhauser, 2002.
34. A. W. Knap. *Representation theory of semisimple groups, an overview based on examples*, Princeton University Press, 1986.
35. A. L. Onishchik, *Lectures on real semisimple Lie algebras and their representations*, European Mathematical Society, 2004.
36. A. L. Onishchik, editor, *Lie groups and Lie algebras*, *Encyclopaedia of mathematical sciences* v. 20, 21, 41, Springer, 1993.
37. J. P. Serre, *Lie algebras and Lie groups*, Springer, 1992.
38. B. Simon. *Representations of finite and compact groups*, AMS, 1996.
39. V. S. Varadarajan. *Lie groups, Lie algebras, and their representations*, Springer, 1984.
40. N. J. Vilenkin, *Special functions and the theory of group representations*, AMS, 1968.
41. N. J. Vilenkin, A. U. Klimyk, *Representation of Lie groups and special functions*, 3 v., Kluwer Academic Publishers, 1991-1993.
42. N. J. Vilenkin, A. U. Klimyk, *Representation of Lie groups and special functions: recent advances*, Kluwer Academic Publishers, 1995.
43. E. B. Vinberg, A. L. Onischik, *Lie groups and algebraic groups*, Springer, 1990.
44. H. Weyl, *The classical groups, their invariants and representations*, Princeton University Press, 1997.
45. D. P. Zhelobenko, *Compact Lie groups and their representations*, AMS, 1973.

More mathematical books.

19. N. Bourbaki, *Lie groups and Lie algebras*, Springer, 1989.
20. D. Bump. *Lie groups*, Springer, 2004.
21. C. W. Curtis, I. Reiner, *Representation theory of finite groups and associative algebras*, Interscience Publishers, 1962.
22. J. J. Duistermaat, J. A. C. Kolk. *Lie groups*, Springer, 2000.
23. E. B. Dynkin, *Selected papers of E.B. Dynkin with commentary*, AMS, 2000.